Friction on faults and scaling laws: hypotheses, dynamic models, and comparisons against lab experiments

Luca Malagnini[1], Irene Munafo'[1], Massimo Cocco[1], Stefan Nielsen[1], Elena Spagnuolo[1], Seung-Hoon Yoo[2], and Kevin Mayeda[3]

[1]Istituto Nazionale di Geofisica e Vulcanologia, Roma, Italy
[2]University of California Berkeley, Berkeley, CA, USA
[3]Weston Geophysical Corporation, Lexington, MA, USA

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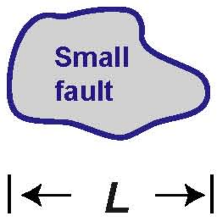
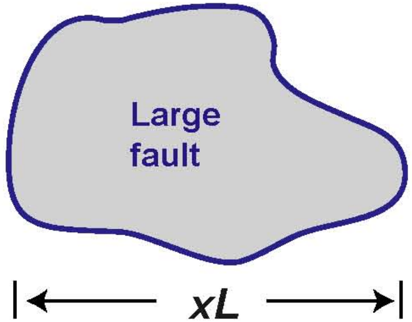
Self-Similar Earthquake Scaling

Quake

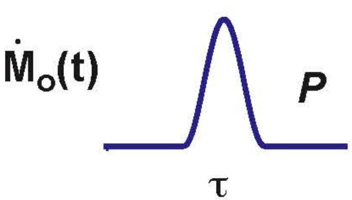


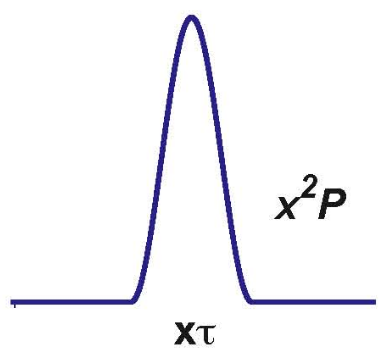
Scaled Quake



Area = $A \sim L^2$
Slip = $\bar{u} \sim \frac{\Delta\sigma}{\mu} L$
$M_o = \mu \bar{u} A \sim \Delta\sigma L^3$
Duration = $\tau \sim \bar{u}/V \sim L$
Energy = $E \sim P^2/\tau$

Area = $x^2 A$
Slip = $x\bar{u}$
$M_o = x^3 M_o$
Duration = $x\tau$
Energy = $x^3 E$

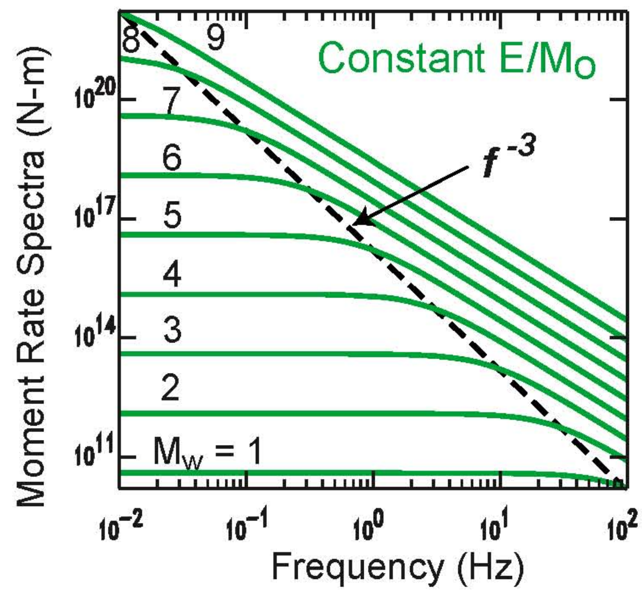




Implicit $\Delta\sigma$ and V are constant

$$M_o \sim \Delta\sigma L^3 \sim \Delta\sigma V^3 \tau^3 \sim (\Delta\sigma V^3) f_c^{-3}$$

Product $\Delta\sigma V^3$ is constant



Aki (1967) and Brune (1970) models, any model invariant under f^{-3} scaling

(see also Prieto et al 2004; Kanamori and Rivera, 2004)

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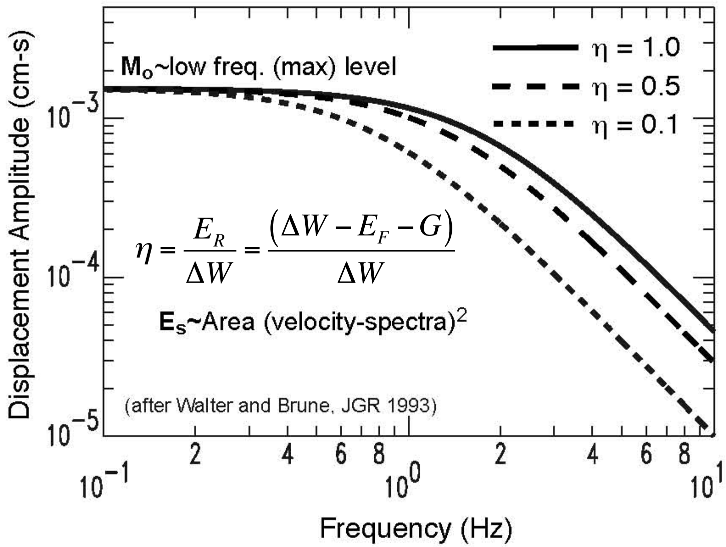
Non-self-similar earthquake scaling: Case of same spectral shape but different corner frequency behavior

1) Changes in $\Delta\sigma$ and/or V with size
(e.g. large have higher $\Delta\sigma$ and/or V than small)

2) Changes in efficiency with size
(large more efficient than small)

Constant moment shear crack spectra for different seismic efficiencies

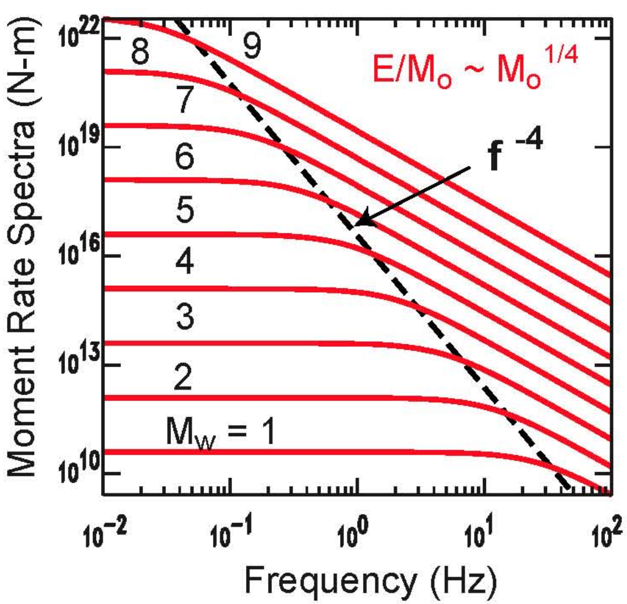


Kanamori and Rivera (2004):

$$M_o \sim f_c^{-(3+\varepsilon)}$$

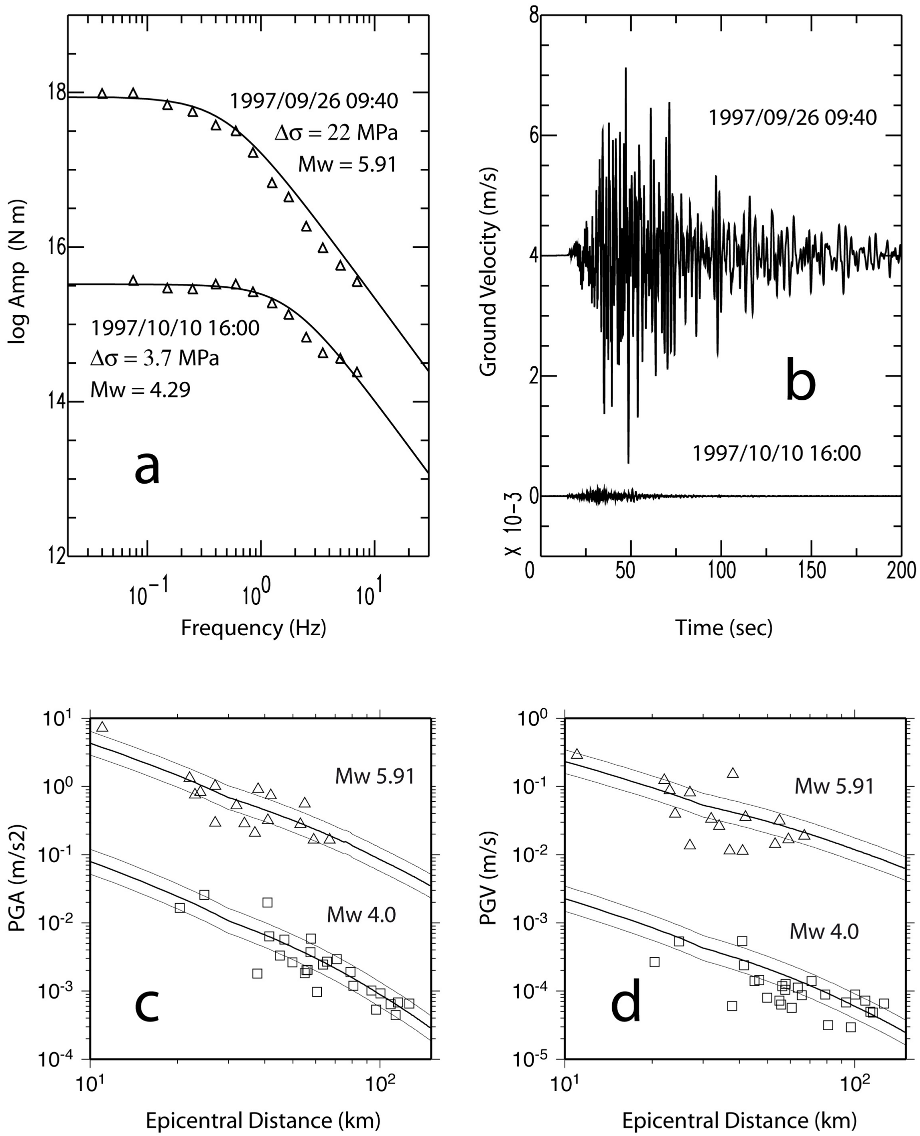
$$\tilde{e} \sim M_o^{\varepsilon/(3+\varepsilon)} \sim (\Delta\sigma V^3)$$

An Example where $M_o \sim f_c^{-4}$

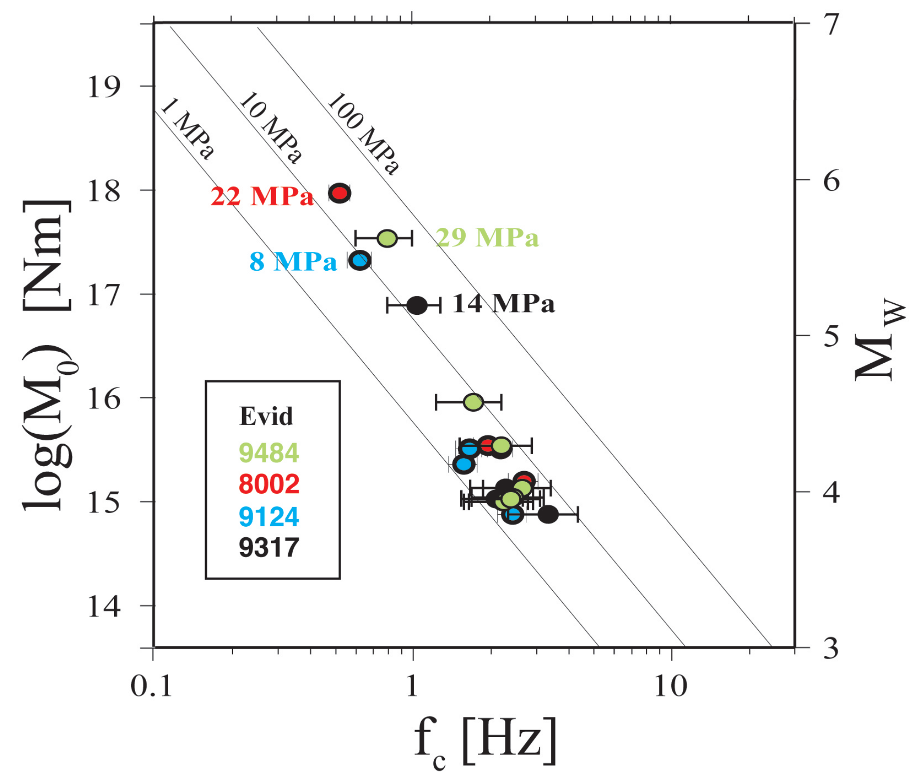


Spectral shape invariant as f^{-4}

Ground motion studies require source scaling…



Central Apennines: observed and predicted ground motions of two events of the Colfiorito sequence (M_W 5.91 & 4.29). A regional attenuation (geometric & anelastic) and a magnitude-dependent stress parameter ($\Delta\sigma_B$) are used to match the observed ground motion through RVT (from Malagnini et al., 2008).



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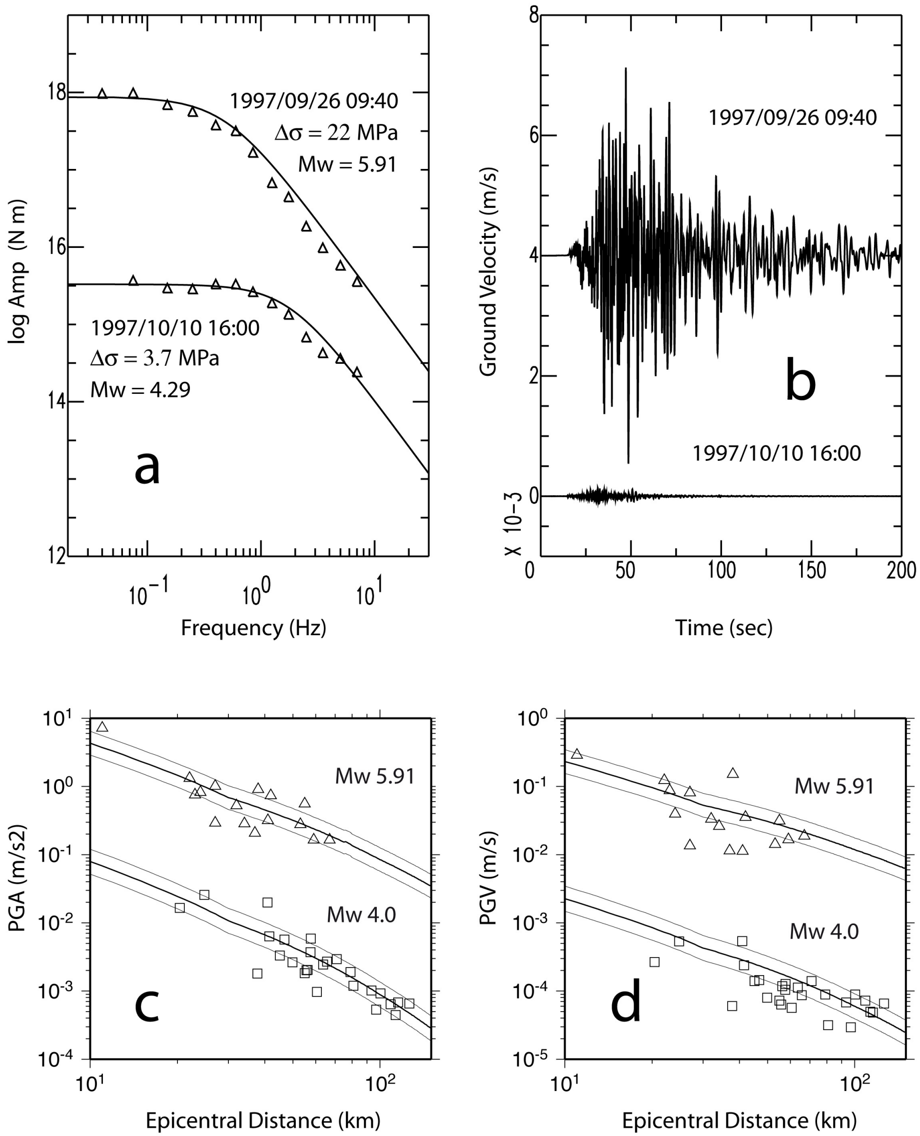

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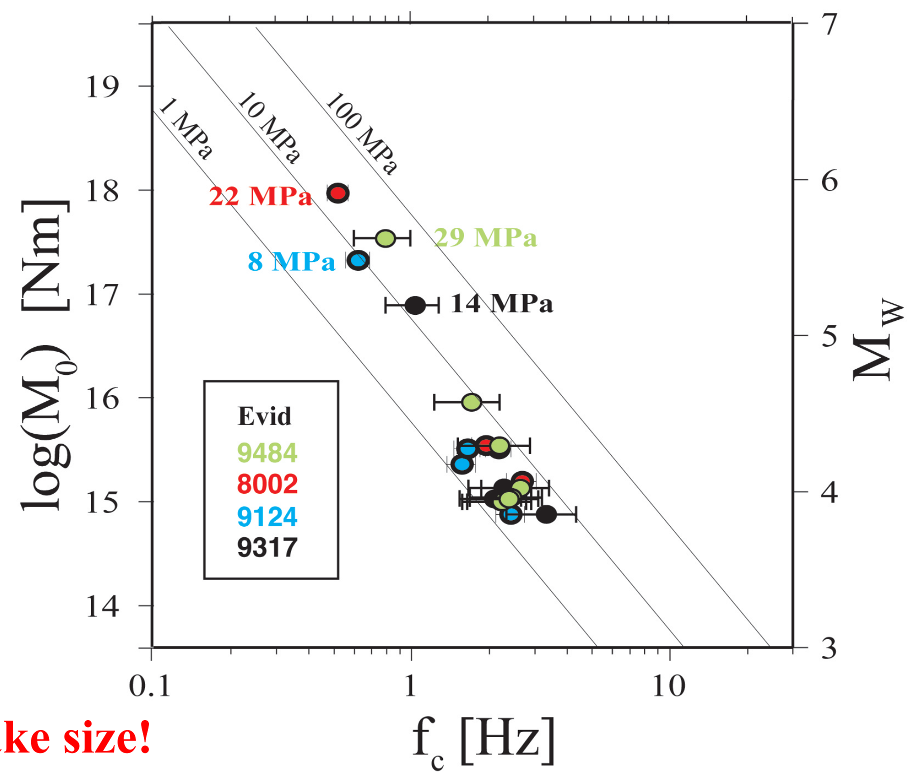
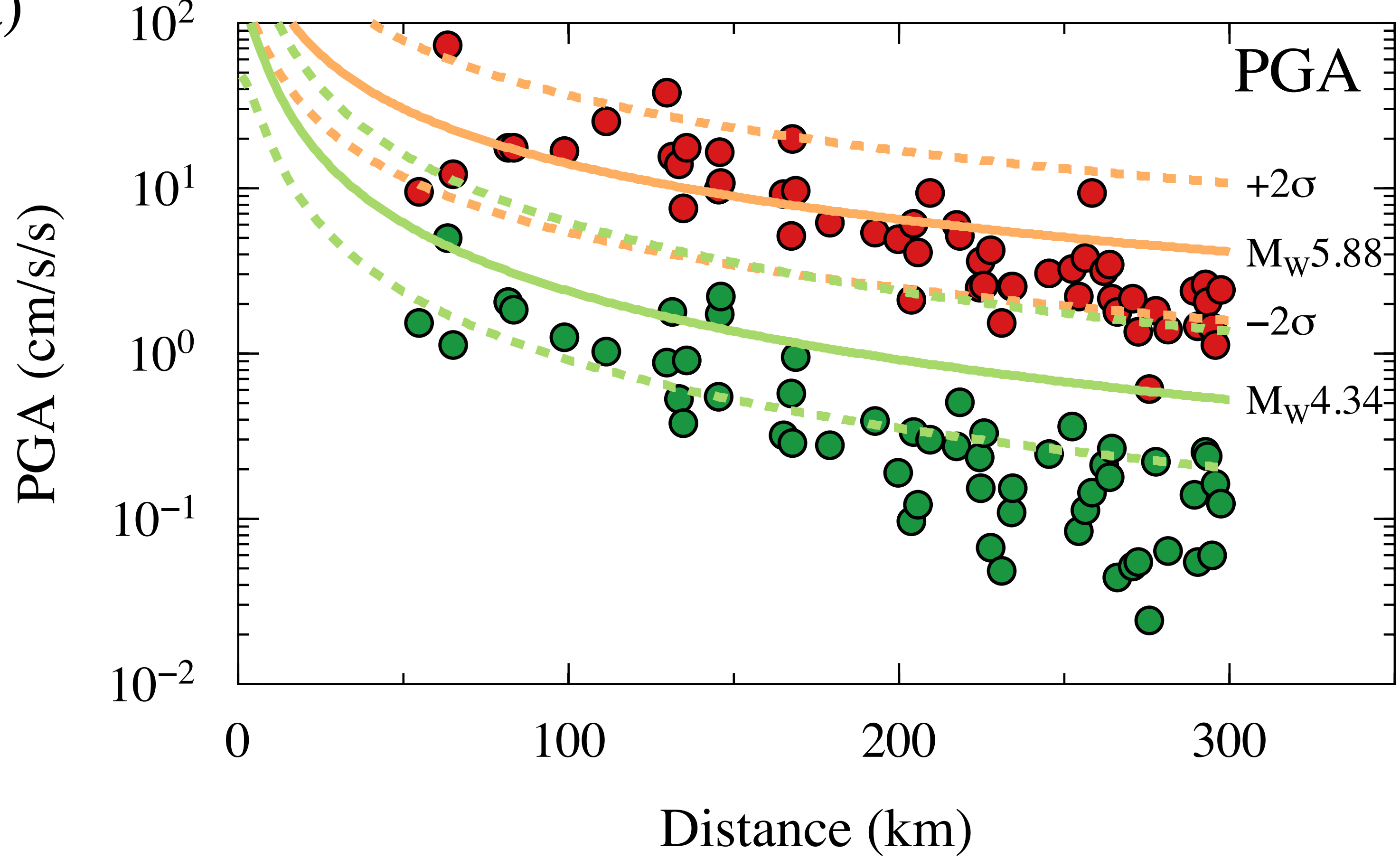


The stress parameter increases with earthquake size!

Ground motion studies require source scaling…

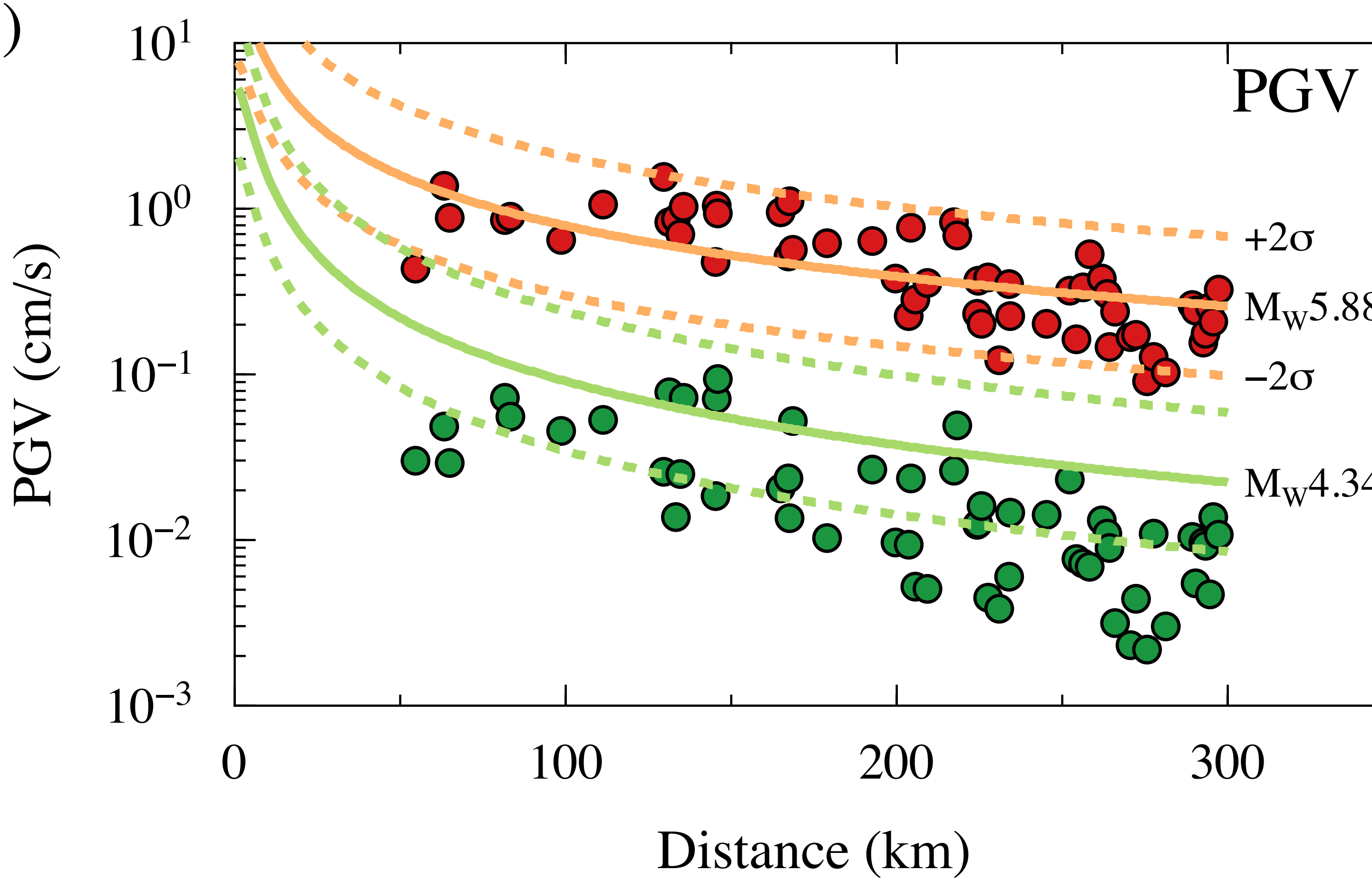
a)



Here is what happens when you try to predict the ground motion using self-similar scaling (Wells, NV)…

Ground motion studies require source scaling…

b)

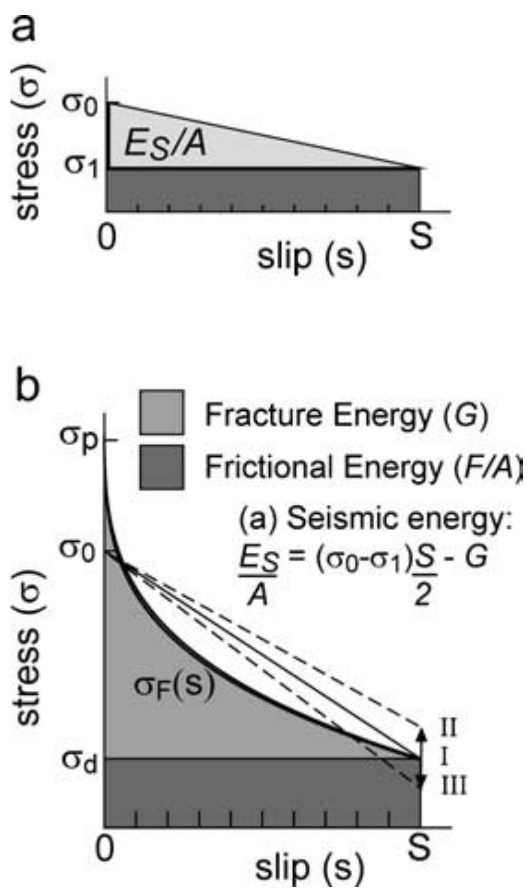


Here is what happens when you try to predict the ground motion using self-similar scaling (Wells, NV)…

Scaling and stress history…

How can we relate seismic observations to fault dynamics?
Hypothesis of constant rupture velocity…
Increased stress drop means:

a) more energy available for radiation;
b) less dynamic friction on fault…



From Abercrombie and Rice (2005)

Defining the fracture energy, G:

$$G(S) = \int_0^S \left(\sigma_F(u) - \sigma_d\right) du$$

$$G(S) = \frac{1}{2}\left(\Delta\sigma - 2\sigma_a\right) S$$

From now on, we assume no overshoot or undershoot, and that peak stress is equal to initial stress…

Scaling and stress history…

$$\log_{10}(G(S)) = \log_{10}(G_0) + n\log_{10}(S)$$

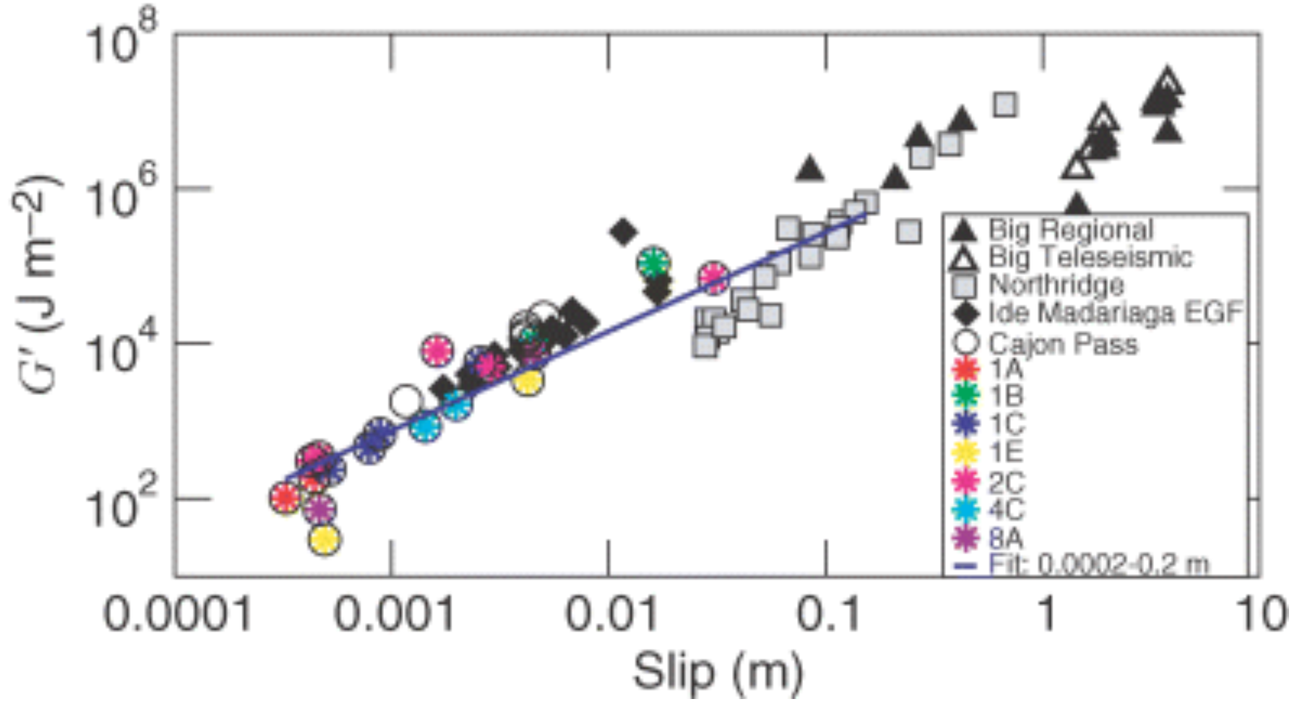
$$G(S) = G_0 S^n$$

In terms of stress:

$$\frac{dG(S)}{dS} = -S\frac{d\sigma_F(S)}{dS}$$

$$\sigma_F(S) = -\int_0^S \frac{1}{u}\frac{\partial G(u)}{\partial u}d$$

$$\sigma_F(S) = \sigma_0 - \frac{n}{n-1}G_0 S^{n-1}$$



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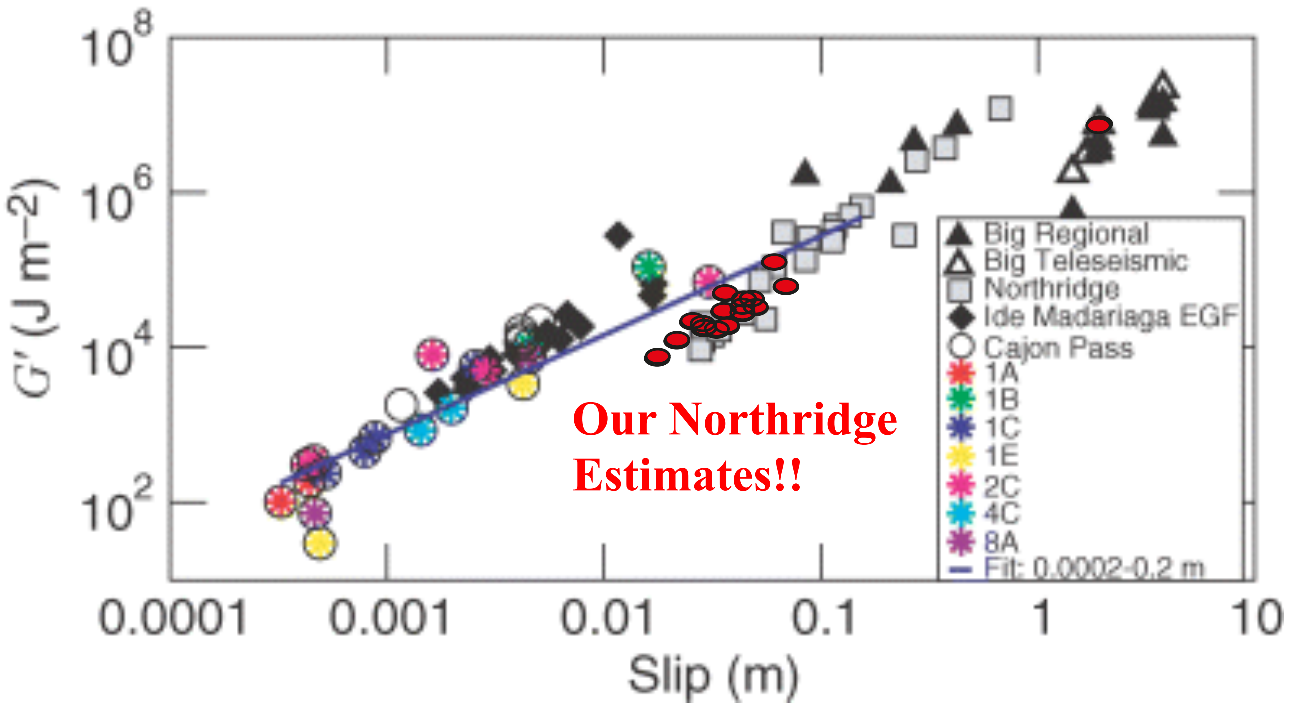

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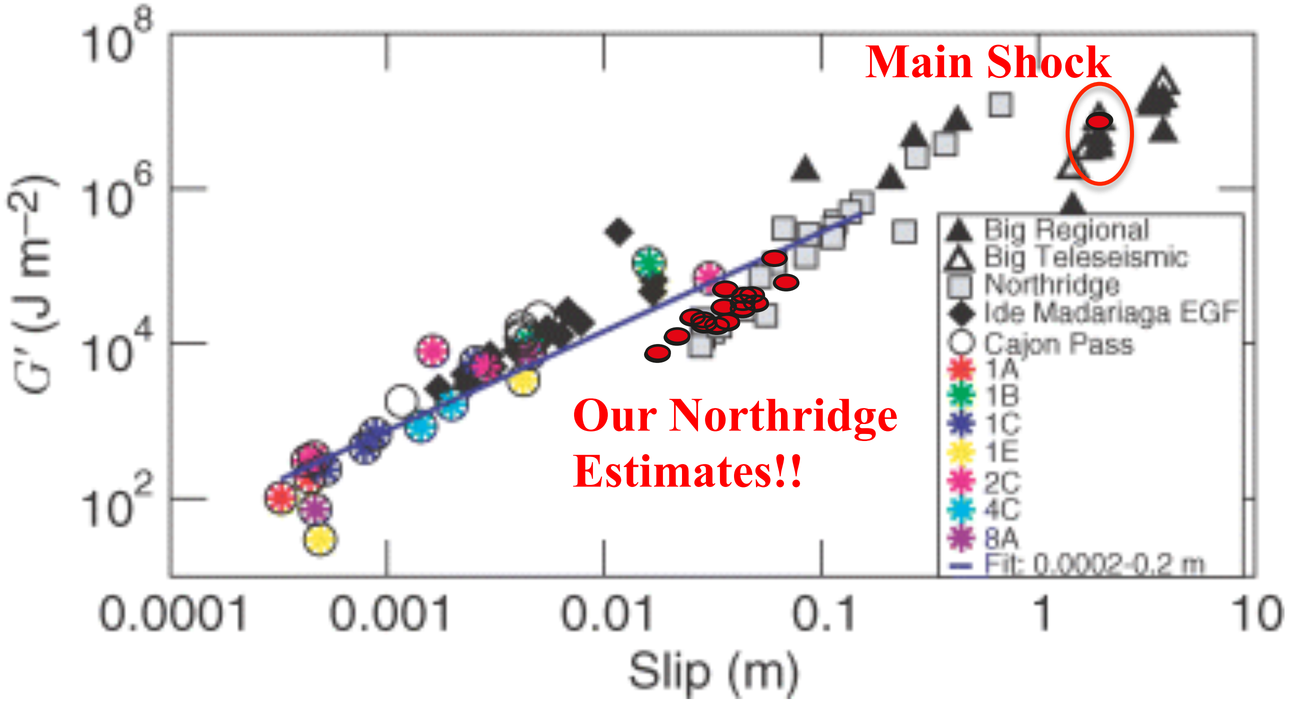

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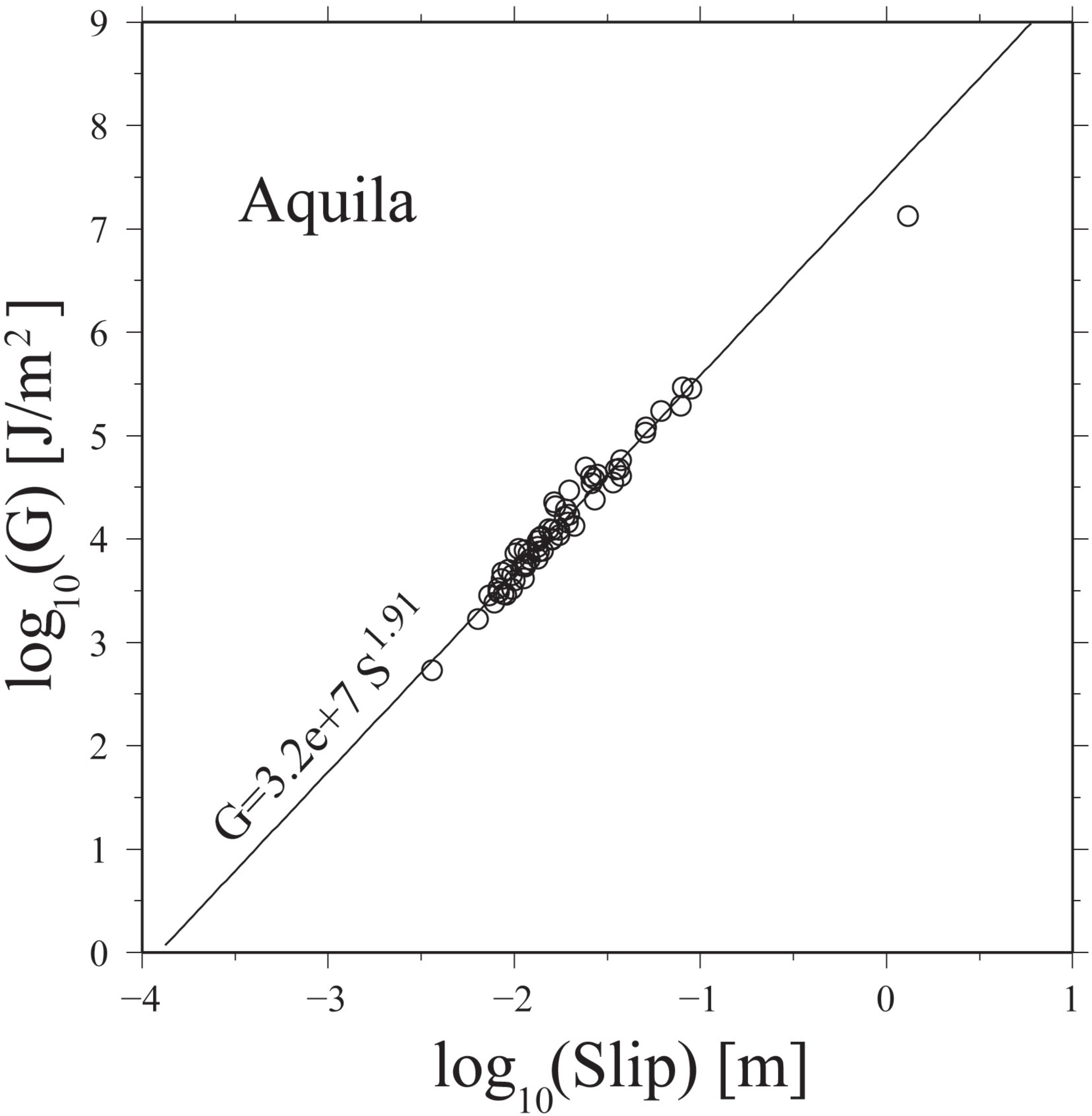
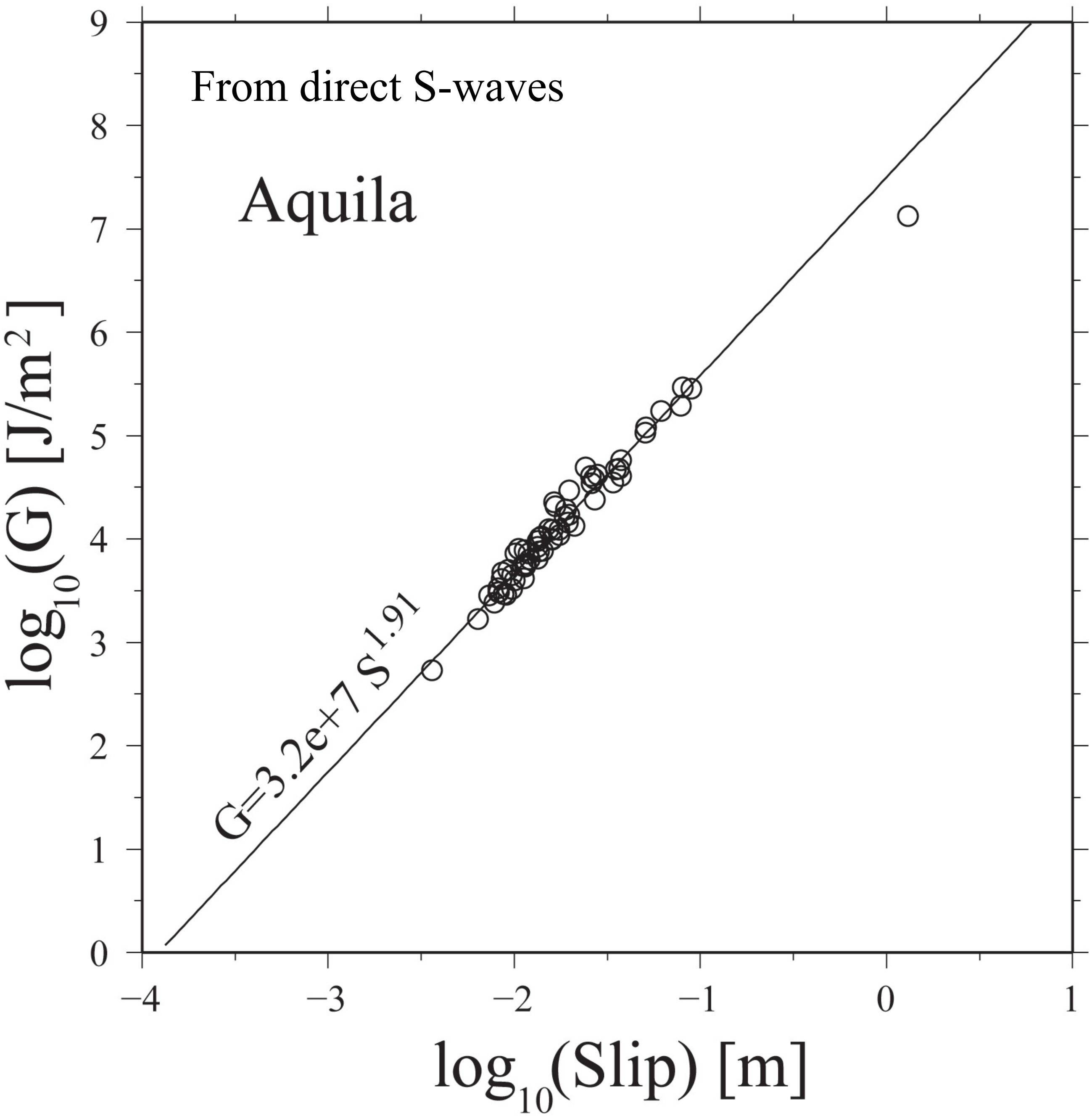
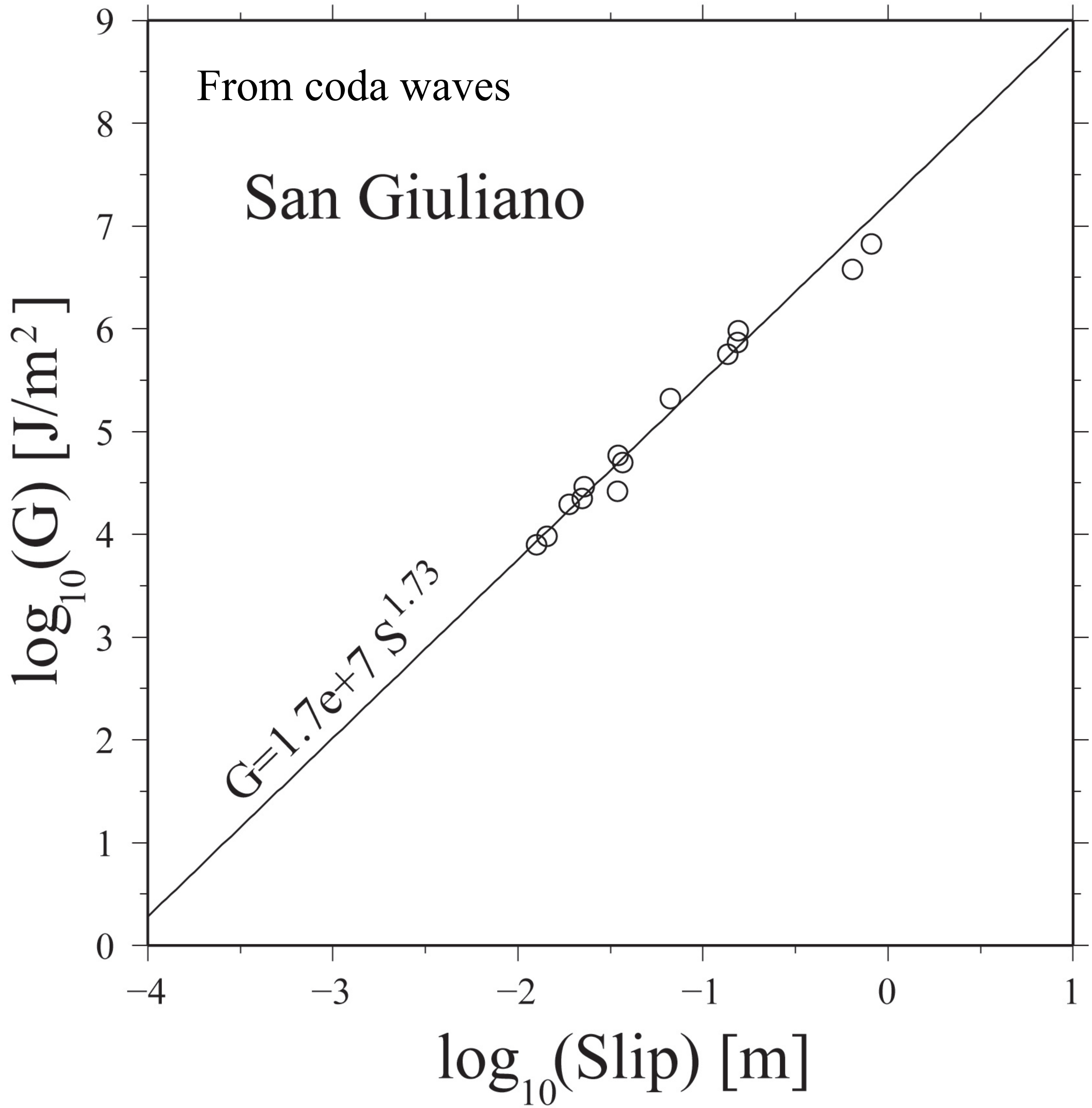


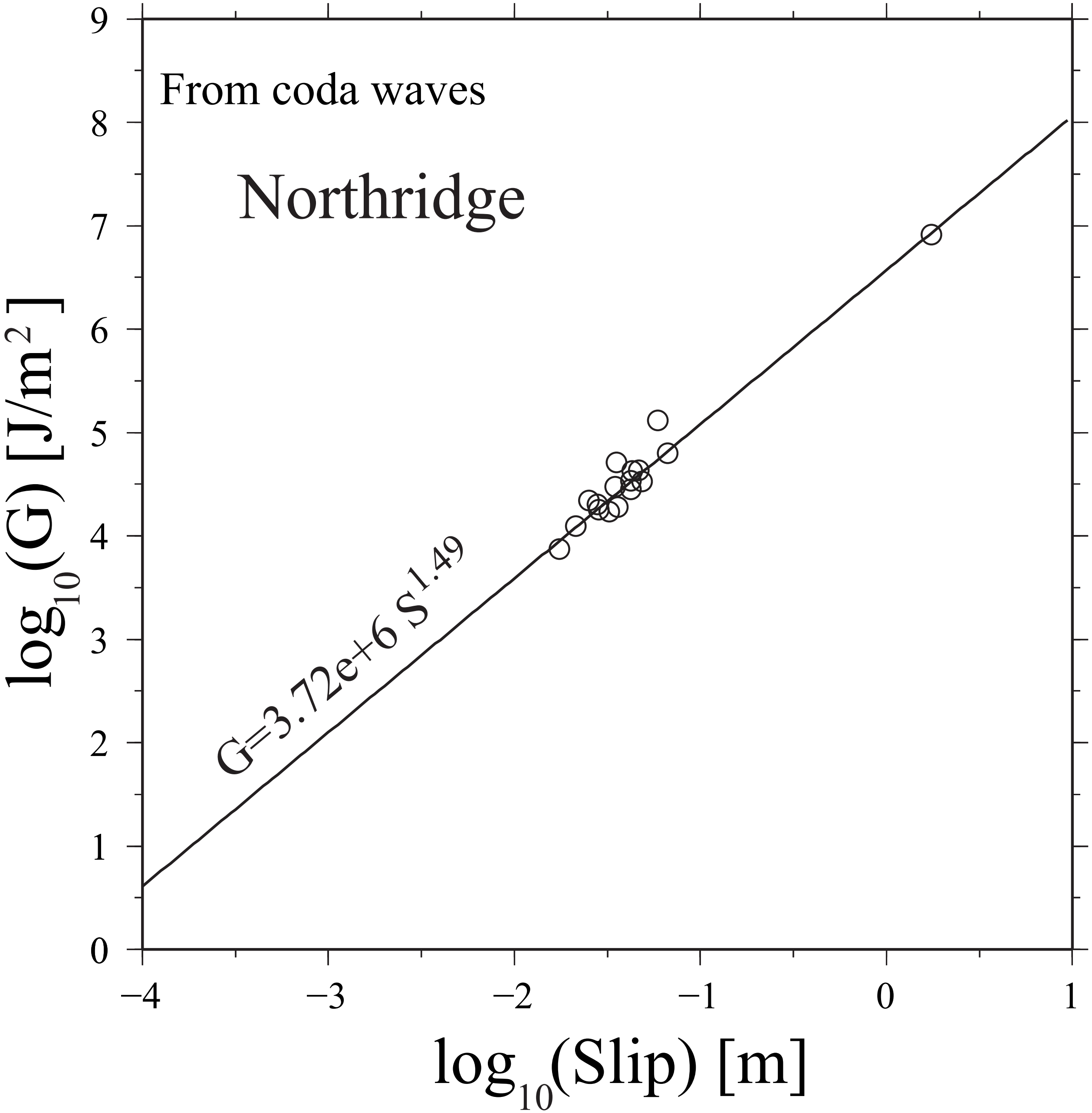
Aquila
G=3.2e+7 S^1.91
log10(G) [J/m2]
log10(Slip) [m]
9
8
7
6
5
4
3
2
1
0
−4
−3
−2
−1
0
1



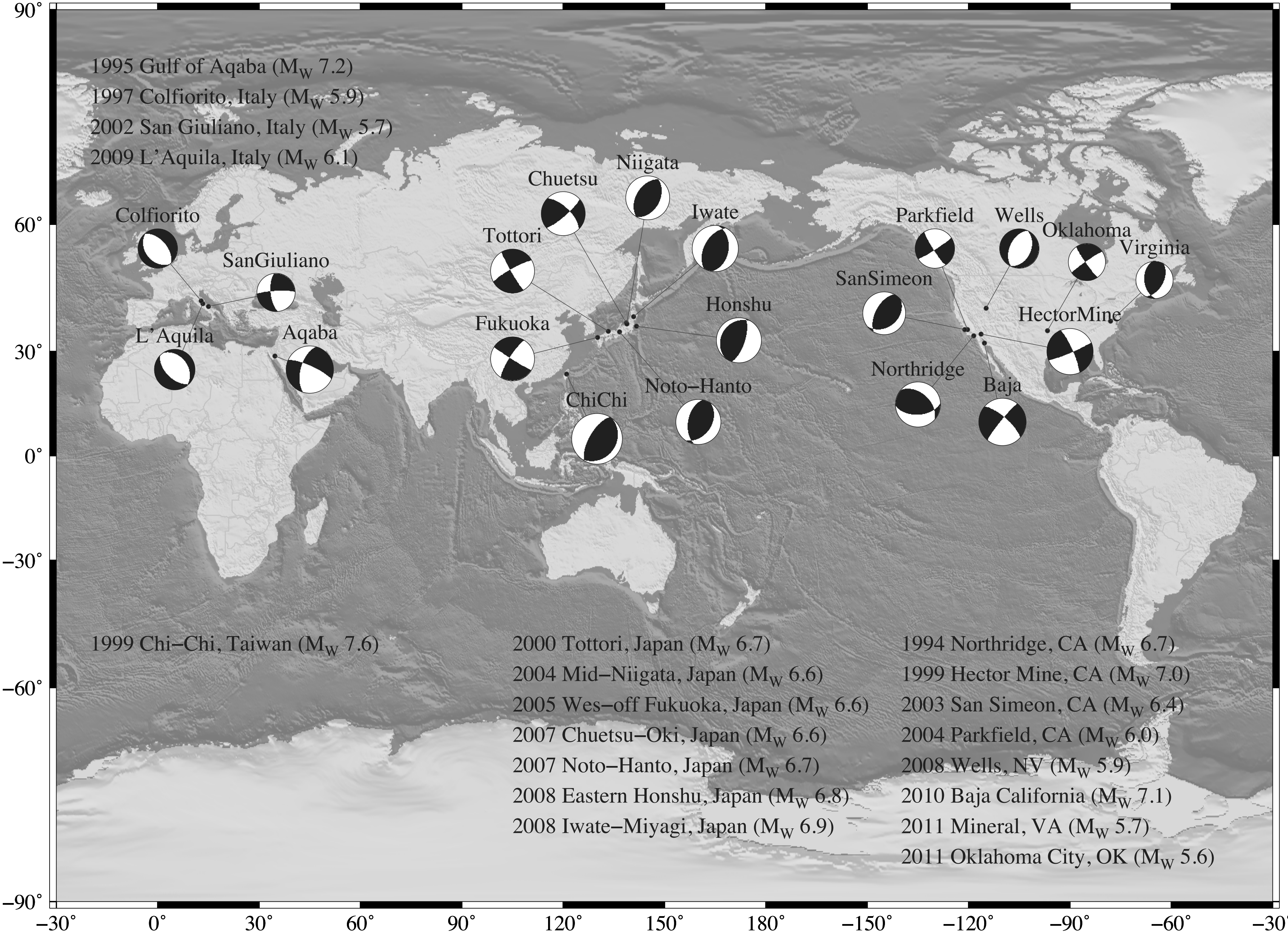
From direct S-waves
Aquila
G=3.2e+7 S^1.91
log10(G) [J/m2]
log10(Slip) [m]
0
1
2
3
4
5
6
7
8
9
−4
−3
−2
−1
0
1



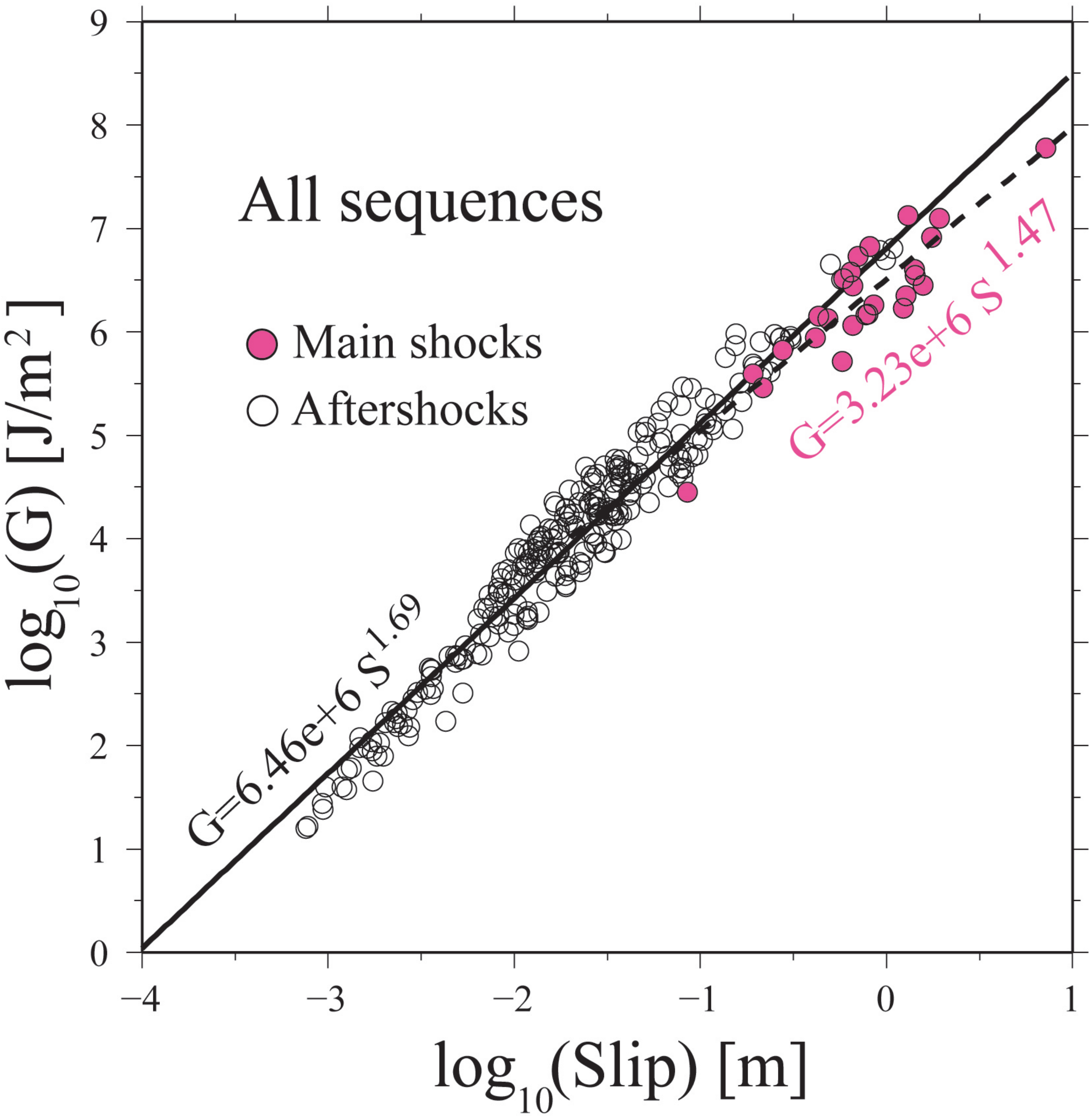
From coda waves
San Giuliano
G=1.7e+7 S^1.73
log10(G) [J/m2]
log10(Slip) [m]
0
1
2
3
4
5
6
7
8
9
−4
−3
−2
−1
0
1



From coda waves
Northridge
G=3.72e+6 S^1.49
log10(G) [J/m2]
log10(Slip) [m]
0
1
2
3
4
5
6
7
8
9
−4
−3
−2
−1
0
1



1995 Gulf of Aqaba (M_W 7.2)
1997 Colfiorito, Italy (M_W 5.9)
2002 San Giuliano, Italy (M_W 5.7)
2009 L'Aquila, Italy (M_W 6.1)
Colfiorito
SanGiuliano
L'Aquila
Aqaba
Chuetsu
Niigata
Tottori
Iwate
Fukuoka
Honshu
ChiChi
Noto−Hanto
Parkfield
Wells
Oklahoma
Virginia
SanSimeon
HectorMine
Northridge
Baja
1999 Chi−Chi, Taiwan (M_W 7.6)
2000 Tottori, Japan (M_W 6.7)
2004 Mid−Niigata, Japan (M_W 6.6)
2005 Wes−off Fukuoka, Japan (M_W 6.6)
2007 Chuetsu−Oki, Japan (M_W 6.6)
2007 Noto−Hanto, Japan (M_W 6.7)
2008 Eastern Honshu, Japan (M_W 6.8)
2008 Iwate−Miyagi, Japan (M_W 6.9)
1994 Northridge, CA (M_W 6.7)
1999 Hector Mine, CA (M_W 7.0)
2003 San Simeon, CA (M_W 6.4)
2004 Parkfield, CA (M_W 6.0)
2008 Wells, NV (M_W 5.9)
2010 Baja California (M_W 7.1)
2011 Mineral, VA (M_W 5.7)
2011 Oklahoma City, OK (M_W 5.6)
90°
60°
30°
0°
−30°
−60°
−90°
−30°
0°
30°
60°
90°
120°
150°
180°
−150°
−120°
−90°
−60°
−30°



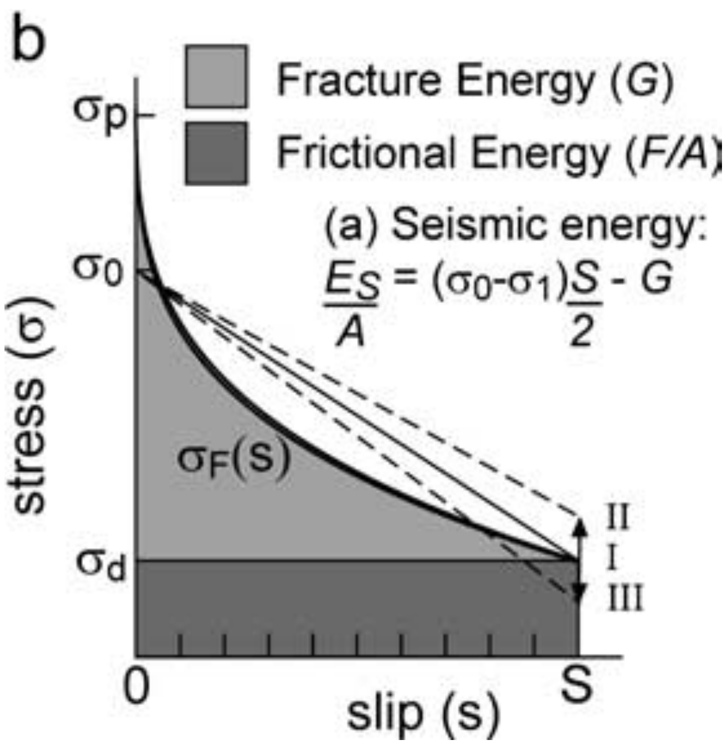
All sequences
Main shocks
Aftershocks
G=6.46e+6 S^1.69
G=3.23e+6 S^1.47
log10(G) [J/m^2]
log10(Slip) [m]
0
1
2
3
4
5
6
7
8
9
−4
−3
−2
−1
0
1

From fracture energy to stress history… (two weakening models)

$$G(S) = \int_0^S \left(\sigma_F(u) - \sigma_d\right) du$$

$$\frac{dG(S)}{dS} = -s\,\frac{d\sigma_F(S)}{dS}$$

$$\sigma_F(S) = -\int \frac{1}{S}\frac{\partial G(S)}{\partial S}\, dS$$



From Abercrombie and Rice (2005)

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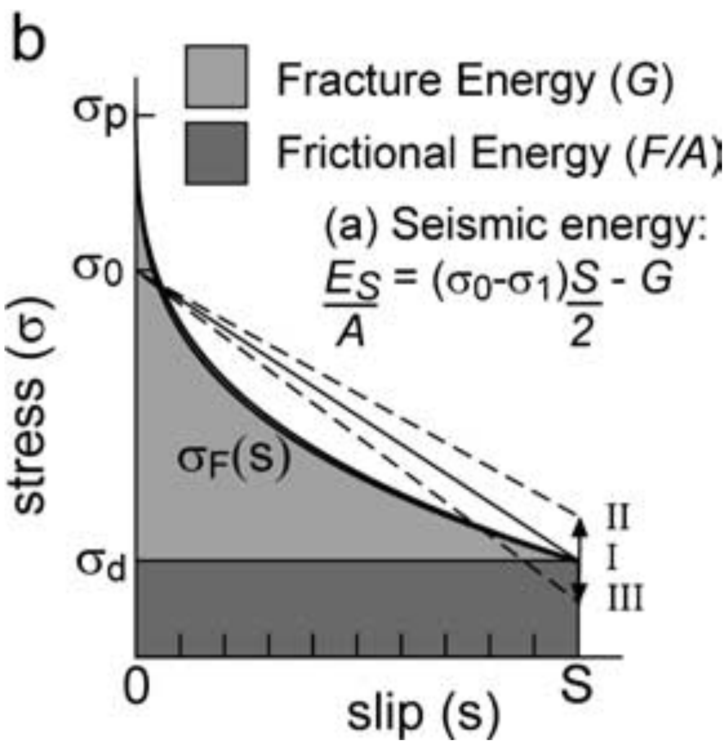

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$$\frac{dG(S)}{dS} = -s\frac{d\sigma_F(S)}{dS}$$

$$\sigma_F(S) = -\int \frac{1}{S}\frac{\partial G(S)}{\partial S} dS$$

1) Polynomial weakening
(Abercrombie & Rice, 2005):

$$\sigma_F(S) = \sigma_0 - \frac{n}{n-1} G_0 S^{n-1}$$

$$G(S) = G_0 S^n$$

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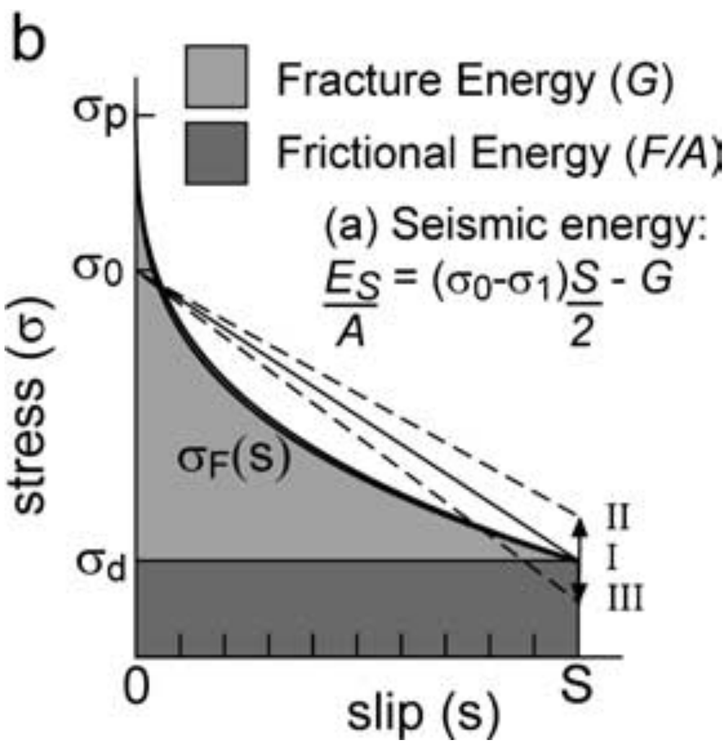

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$$\sigma_F(S) = \sigma_0 - \frac{n}{n-1} G_0 S^{n-1}$$

$$G(S) = G_0 S^n$$

2) Exponential weakening
(e.g., Rice, Sammis & Parson, 2005):

$$\sigma_F = \sigma_{S-S} + (\sigma_0 - \sigma_{S-S})\exp\left(-\frac{S}{D_C}\right)$$

$$G(S) = \int_0^S \left(\sigma_F(u) - \sigma_F(S)\right) du$$

$$G(S) = \left[D_C - \exp\left(-\frac{S}{D_C}\right)(D_C + S)\right]\Delta\sigma$$

From fracture energy to stress history…

Polynomial weakening:

$$G(S) = G_0 S^n$$

Exponential weakening:

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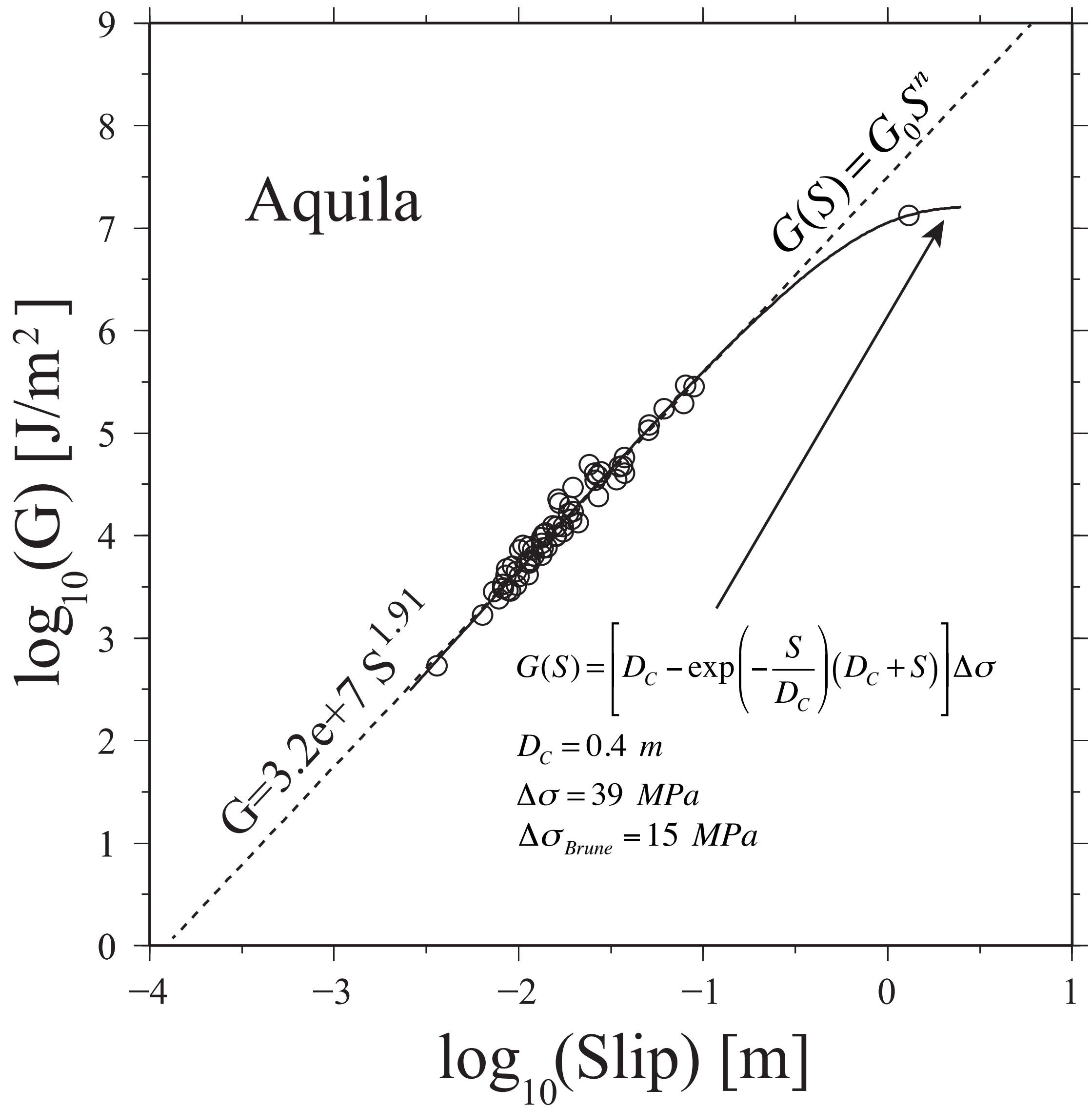

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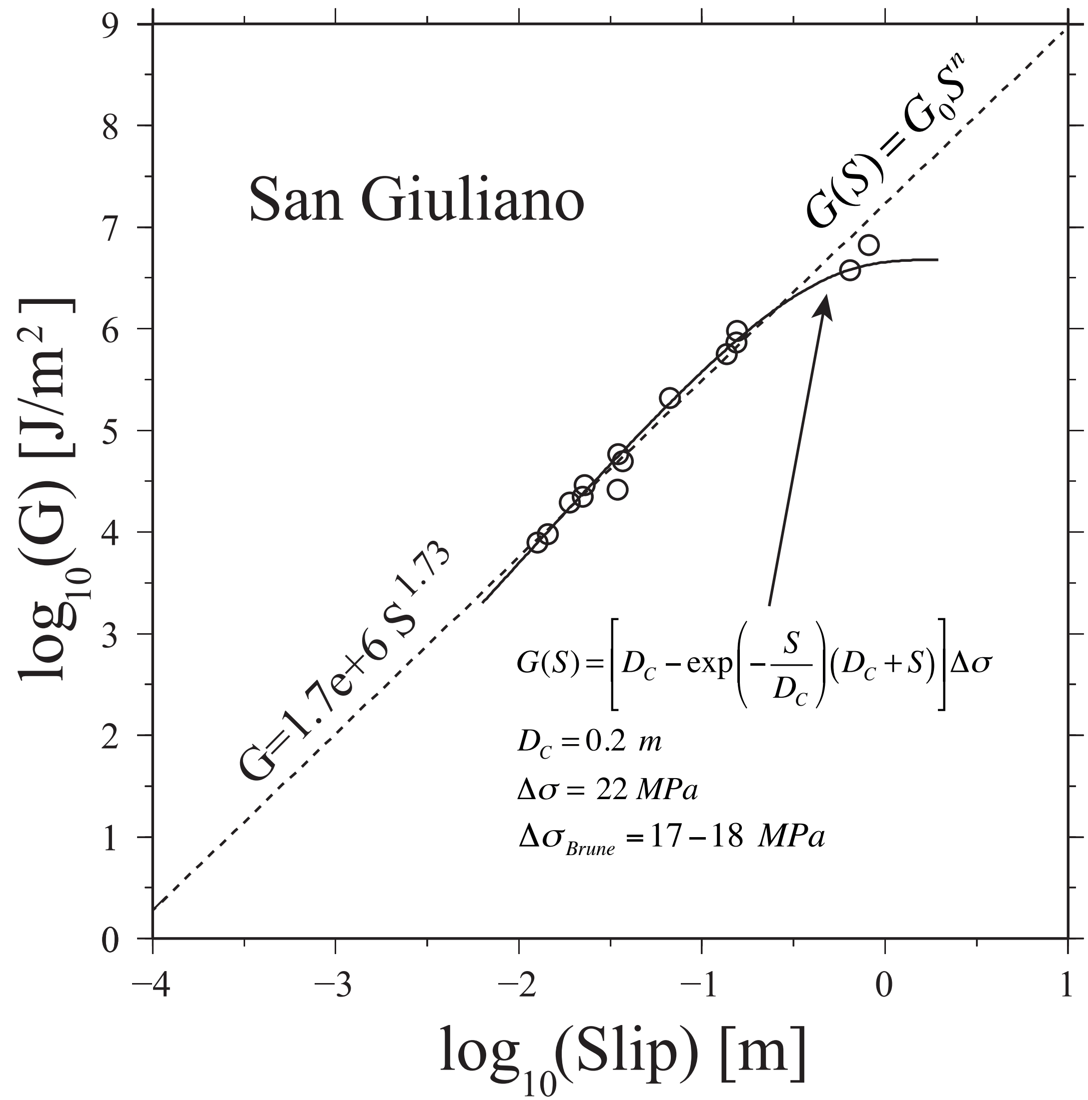

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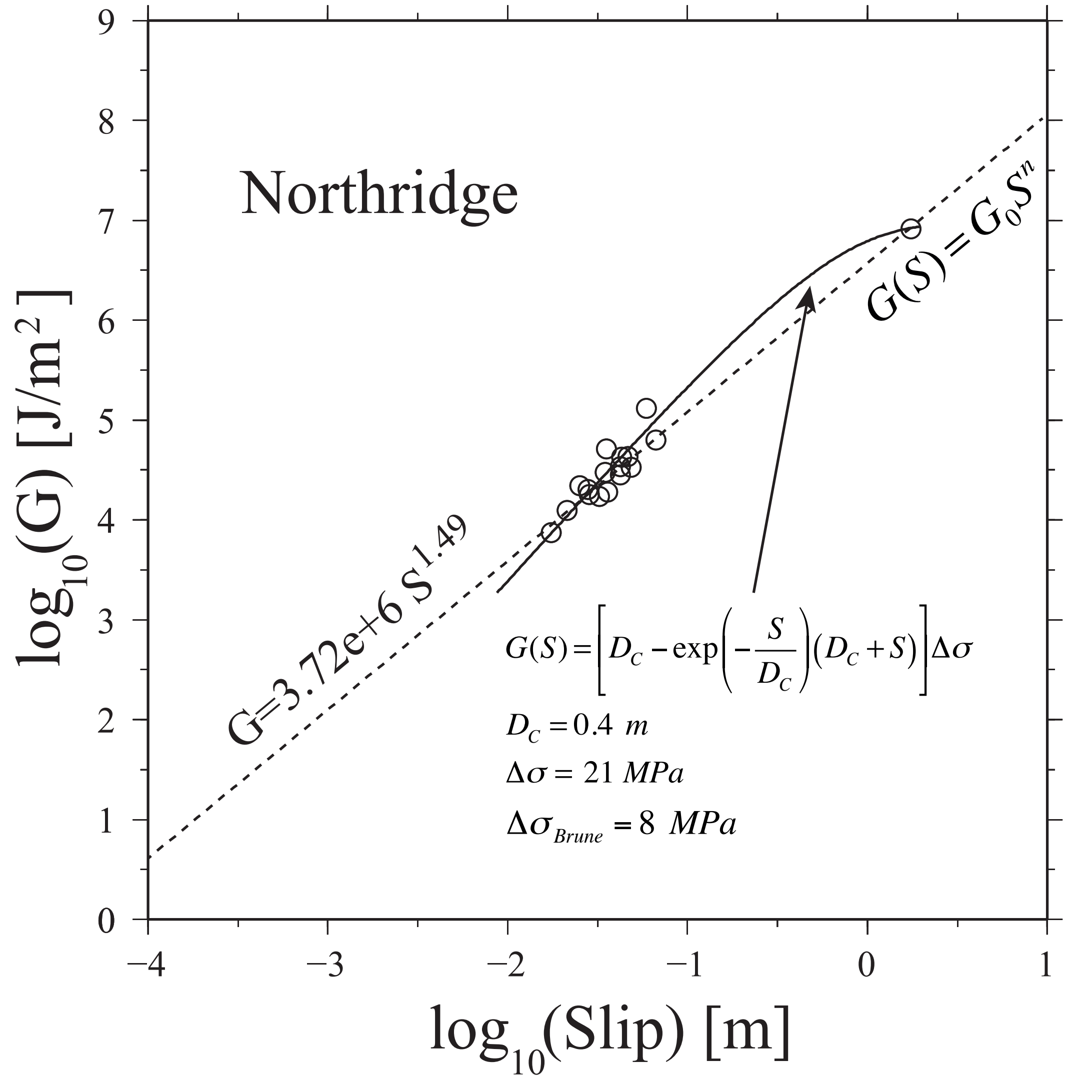


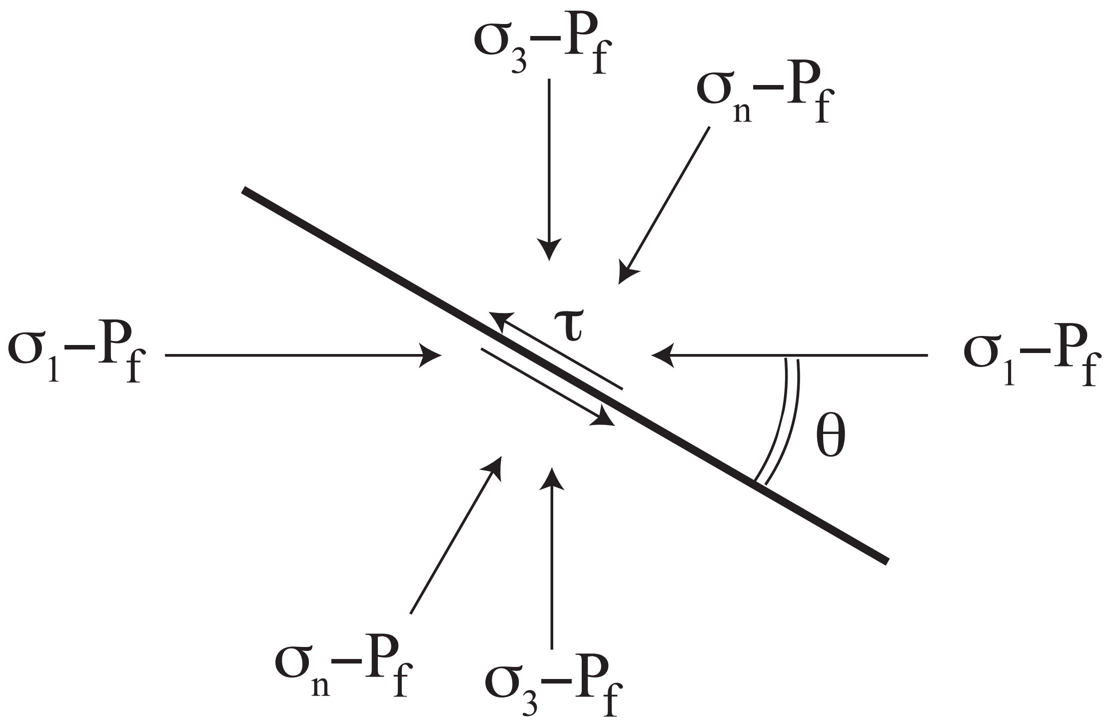
Computing normalized shear stress for earthquakes…

Initial shear stress on fault: $\sigma_0 = \mu_S\left(\sigma_n - P_f\right)$

Normal stress on fault: $\sigma_n = \sigma_n\left(\theta, P_f, h\right)$

Shear stress during sliding: $\tau = \sigma_0 - \Delta\sigma$

Normalized shear stress: $\dfrac{\tau}{\sigma_0} = 1 - \dfrac{\Delta\sigma}{\sigma_0}$



Using the crack model by Malagnini et al. (JGR 2010):

$$\Delta\sigma = \kappa\mu\left(\frac{E_R}{M_0}\right)$$

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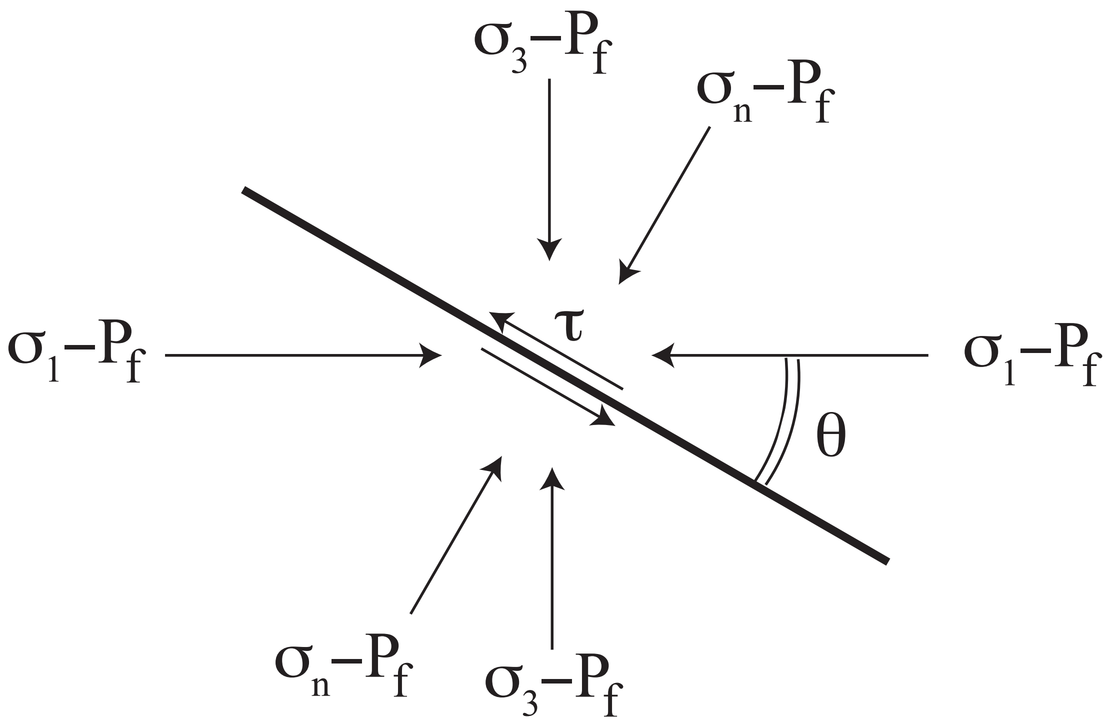


Using the crack model by Malagnini et al. (JGR 2010):

$$\Delta\sigma = \kappa\mu\left(\frac{E_R}{M_0}\right)$$

(in what follows, κ is used to fit the specific mainshock "stress"…)

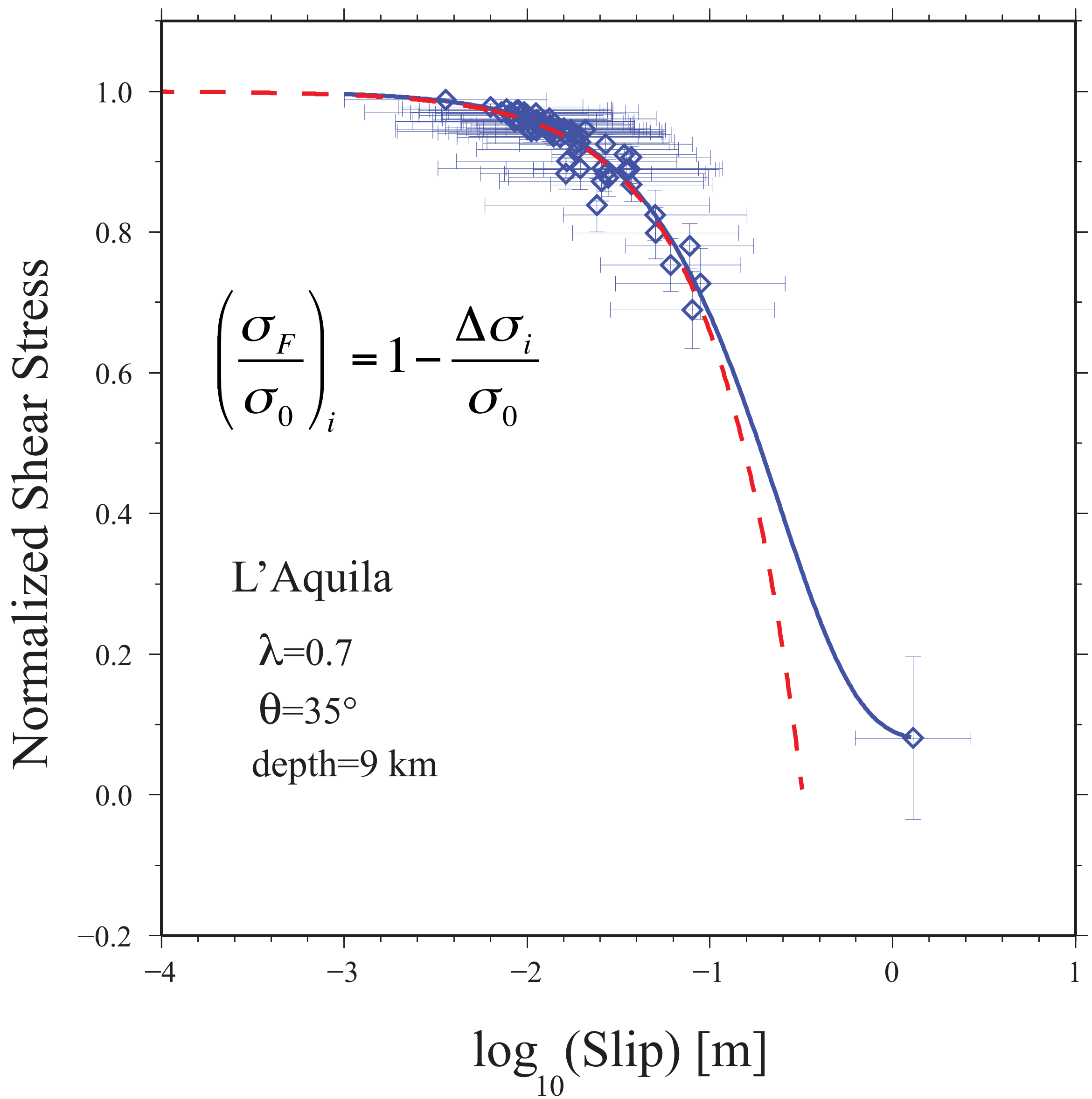
Weakening curves…

Polynomial weakening:

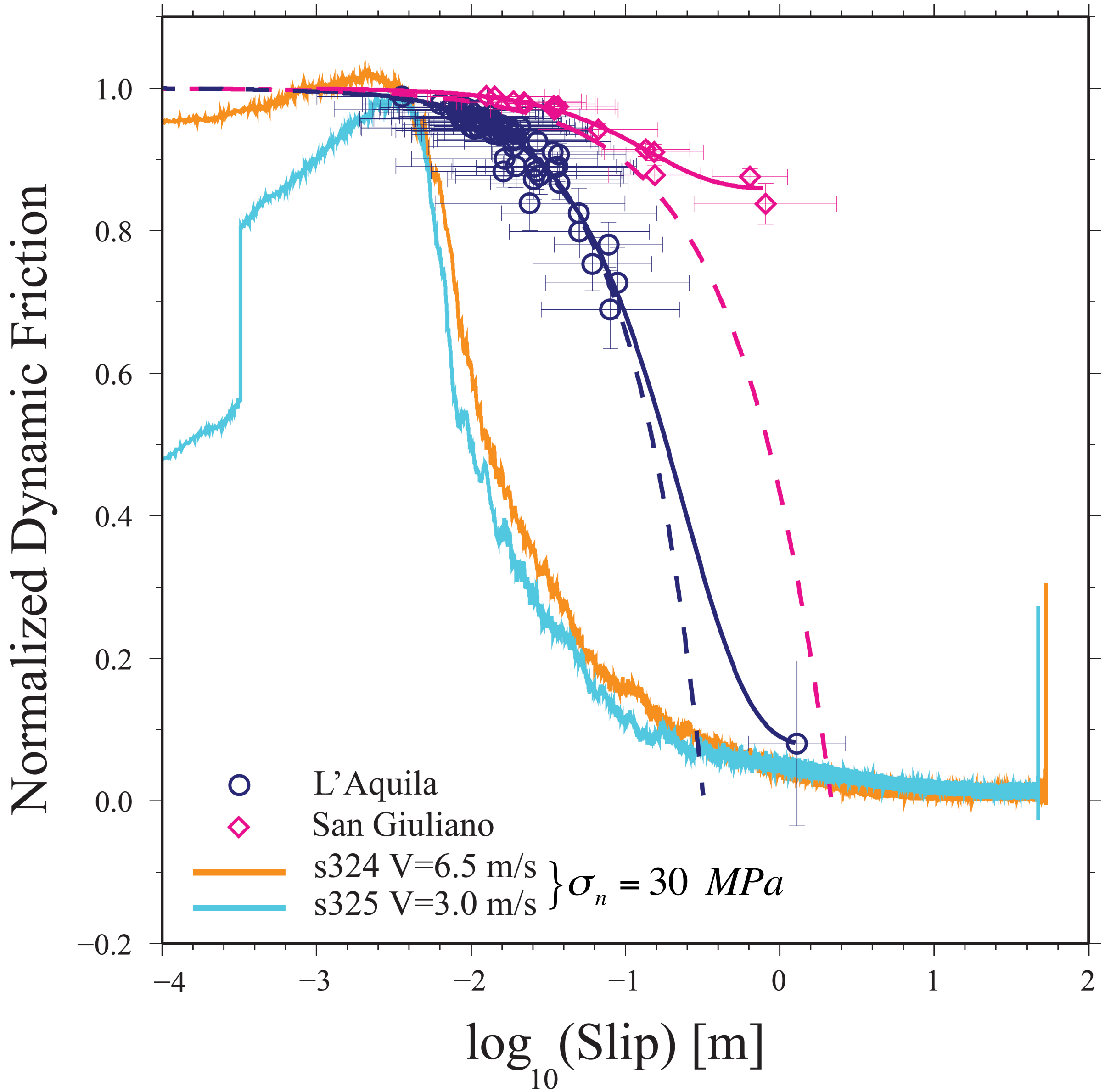
$$\frac{\sigma_F(S)}{\sigma_0} = 1 - \frac{n}{\sigma_0(n-1)} G_0 S^{n-1}$$

Exponential weakening:

$$\frac{\sigma_F(S)}{\sigma_0} = \frac{\sigma_{S-S}}{\sigma_0} + \frac{\Delta\sigma}{\sigma_0} \exp\left(-\frac{S}{D_C}\right)$$

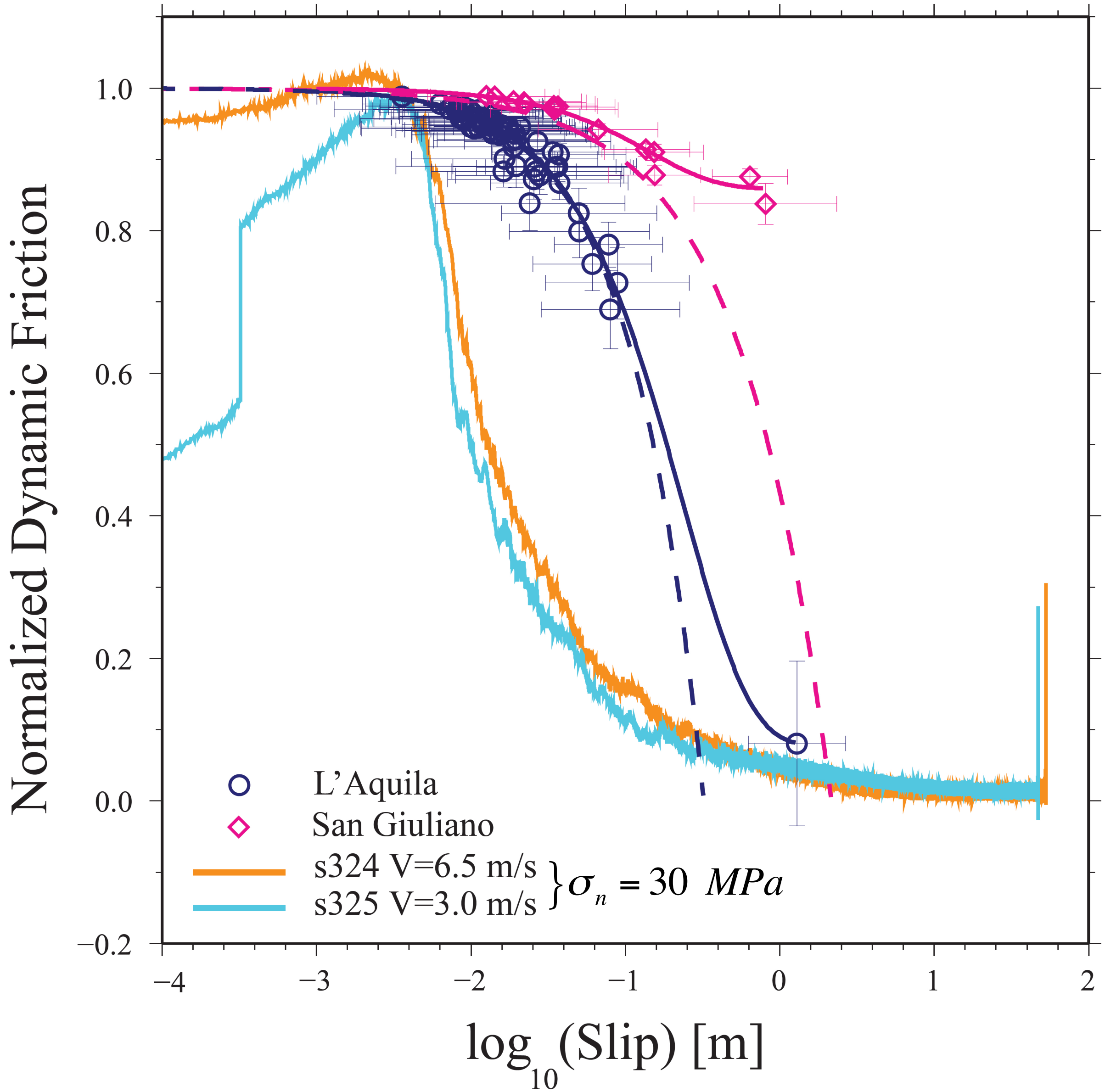


Comparison against lab experiments…



Comparison against lab experiments…

?



Summary & Conclusions

We computed fracture energy vs. slip $\left(G(S)\right)$ for ~300 earthquakes from 20 seismic sequences occurred worldwide;

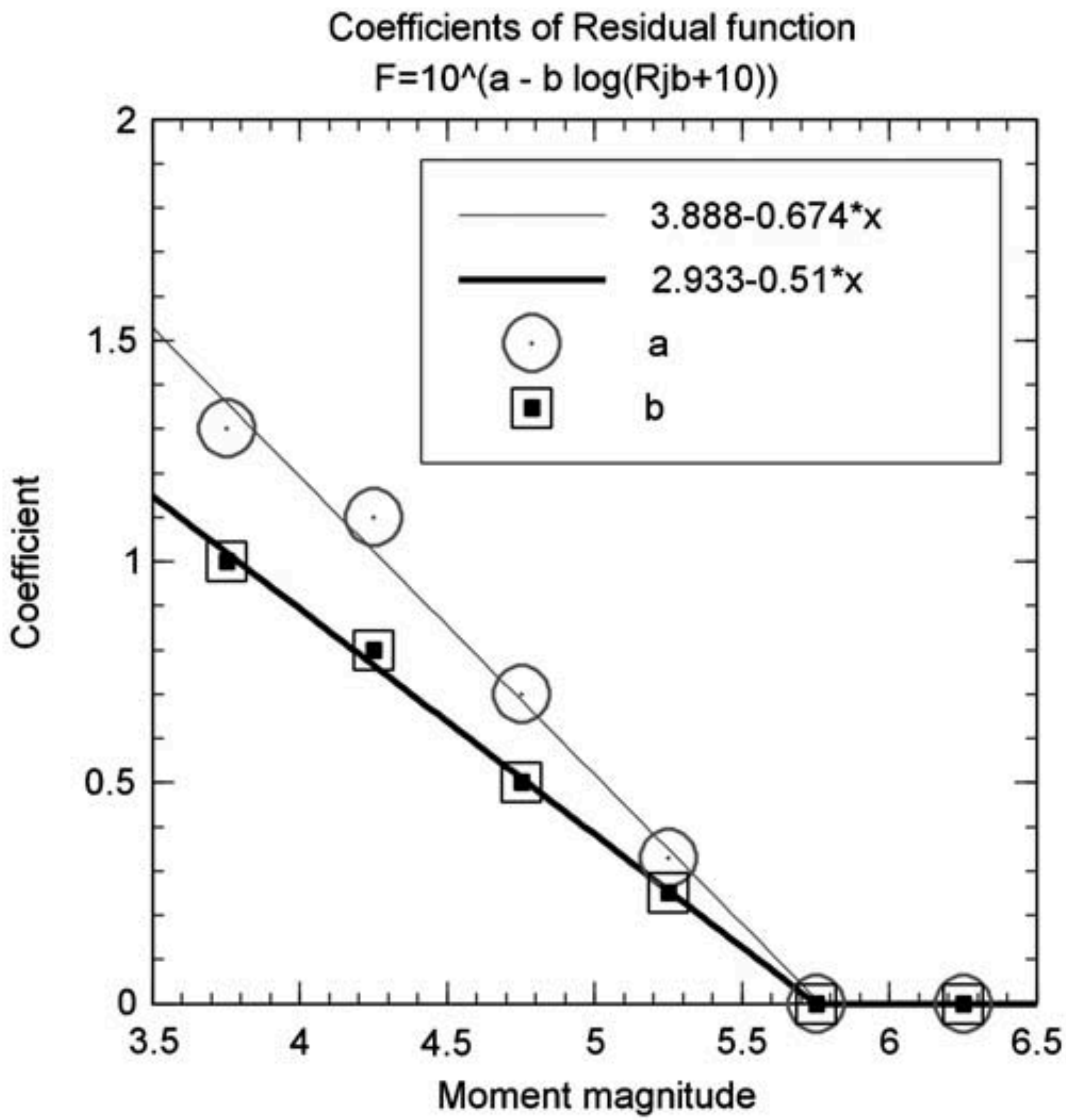
Within an individual sequence:

- Fracture energies of small events: power law & polynomial weakening $\left(G(S)=G_0 S^n;\;\; \sigma_F(S)=\sigma_0-\frac{n}{n-1}G_0 S^{n-1}\right)$;
- Fracture energies of large earthquakes suggest shear stress saturation;
- Polynomial stress-slip function: first approximation of exponential weakening $\left(\sigma_F=\sigma_{S-S}+\left(\sigma_0-\sigma_{S-S}\right)\exp\left(-\frac{S}{D_C}\right)\right)$;

Seismology & Lab:

- Interesting comparison…

Magnitude-dependent correction parameter…



Atkinson and Boore (2011) discussed the use of a correction parameter for NGA's for the GM prediction of small events…

$$Y' = YF_{BA08}$$

Note that log(F)=0 for M>5.75 (i.e., no corrections for large events)

(from Atkinson and Boore, 2011)

Wells: evidence of non self-similar scaling from ground motion analysis