Stress Drop Variability

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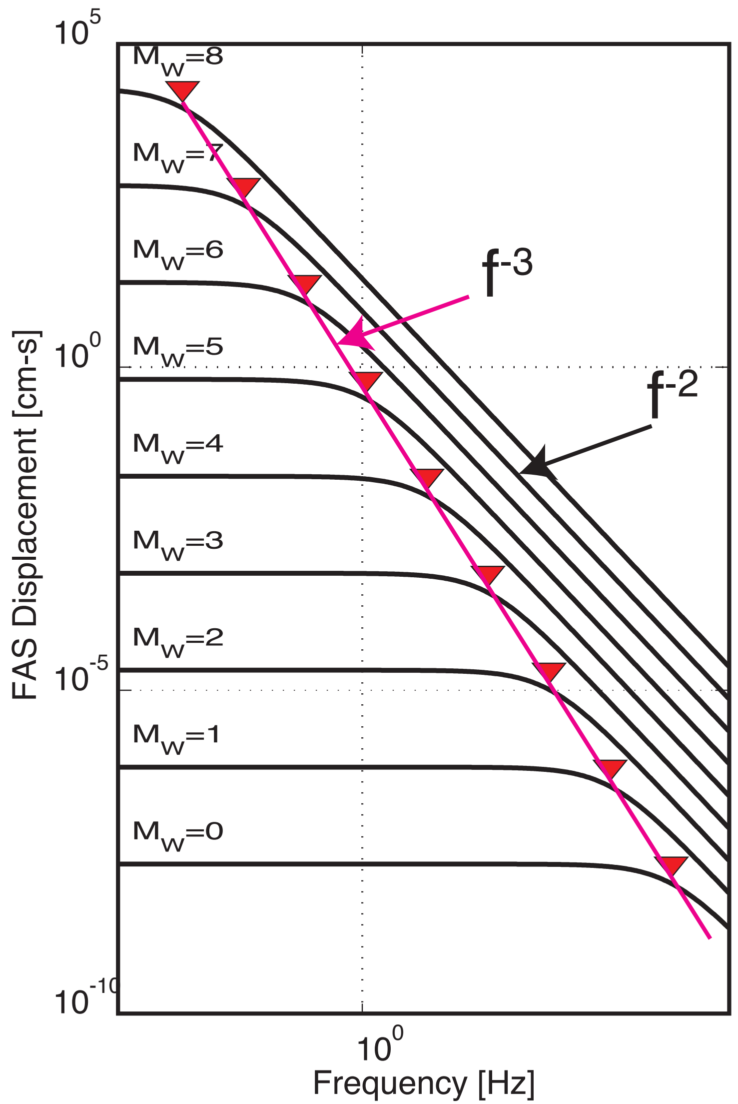
Self-Similarity: Constant Stress Drop Aki (1967)

Aki's ω^{-2} Spectrum



$$|A(\omega)| = \frac{wD_0L}{\left\{1+\left(\frac{\cos\theta}{C}-\frac{1}{V}\right)^2\left(\frac{\omega}{k_L}\right)^2\right\}^{1/2}\left\{1+\left(\frac{\omega}{k_L}\right)^2\right\}^{1/2}}$$

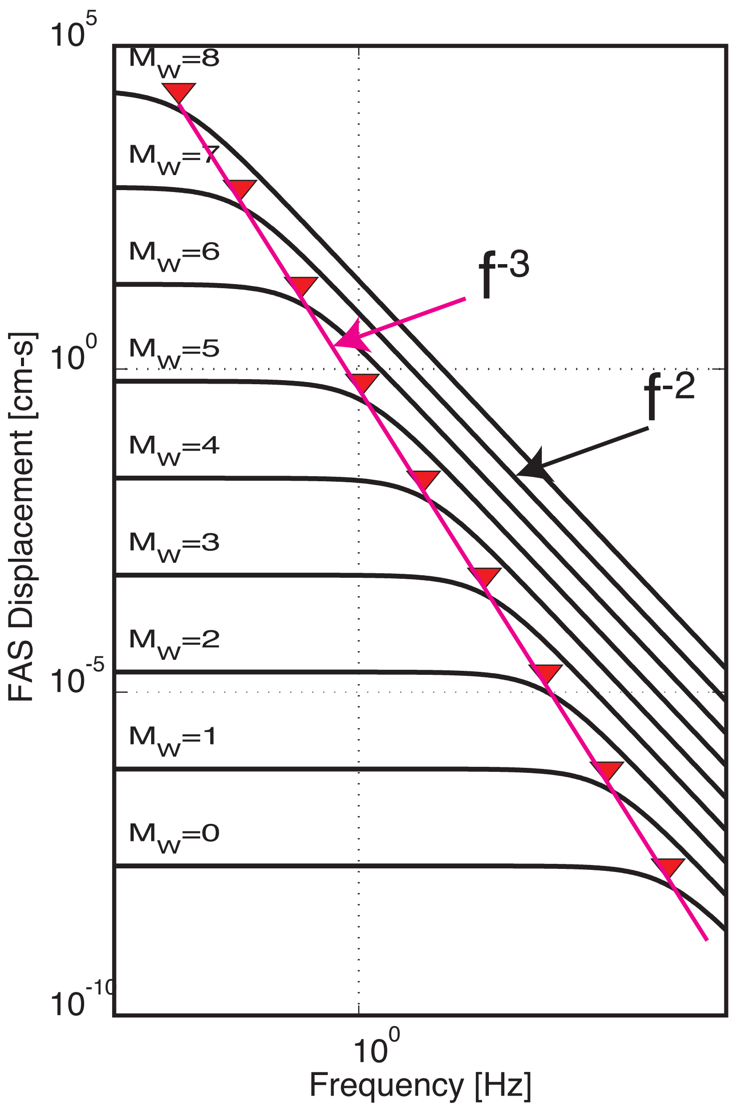
"Further, in order to specify an earthquake by a single source parameter, 'magnitude,' we must reduce to one the number of parameters appearing in (30) and (31) by assuming they are related to each other in some manner.

The simplest of such assumptions may be that large and small earthquakes are similar phenomena. If any two earthquakes are geometrically similar, the fault width w is proportional to the length L. If they are physically similar, all the nondimensional products formed by the source parameters will be the same. The average dislocation D_0 will be proportional to L and, consequently to w. This implies that if an earthquake is a Starr fracture, the pre-existing stress or strength is constant and independent of source size [Tsuboi, 1956]."

Aki, 1967, JGR

Self-Similarity: Constant Stress Drop Aki (1967)

$Aki's\ \omega^{-2} Spectrum$



"Since the wave velocity is practically independent of of source and may be considered constant for our present purpose, all the quantities having the dimension of velocity must also be constant and independent of source size. Thus, the similarity assumptions imply that the rupture velocity v is a constant and that all the quantities having the dimension of time, such as , k_T^{-1} and $(vk_L)^{-1}$ are proportional to L."

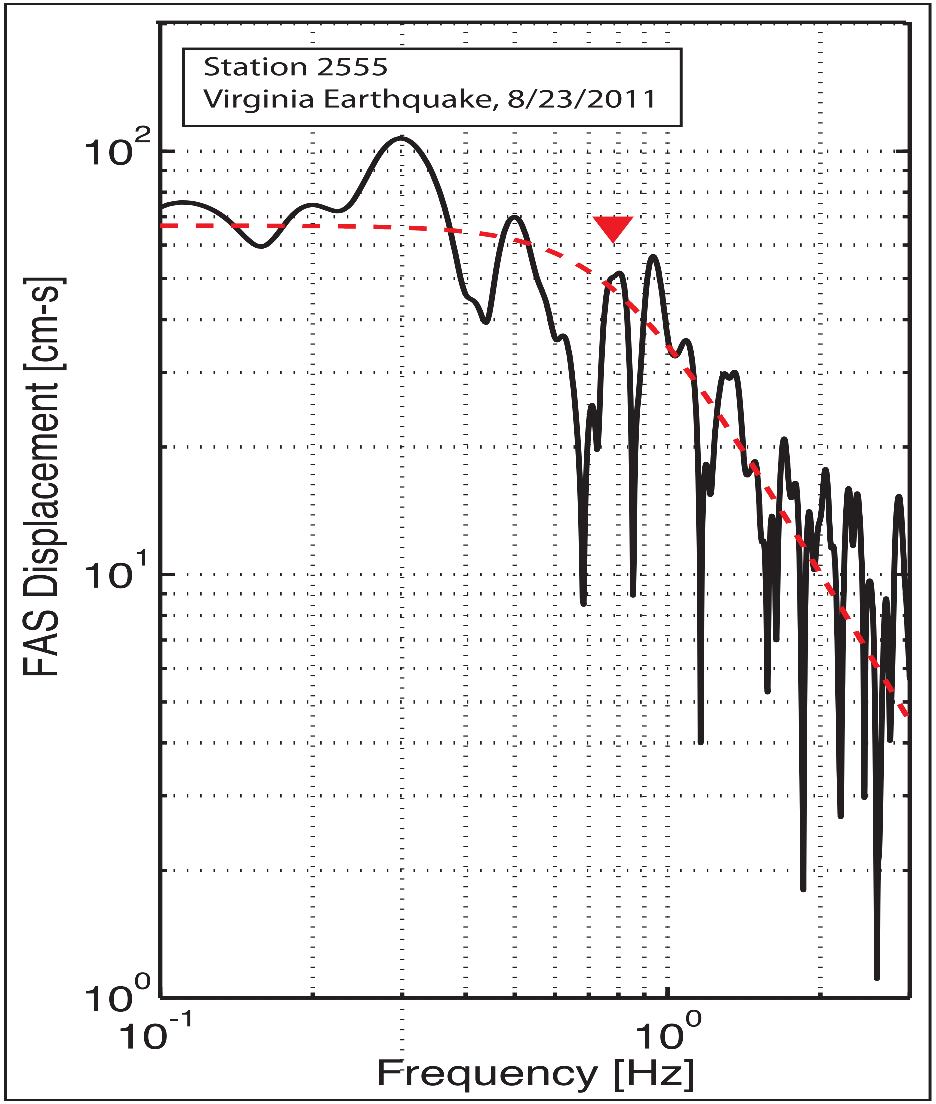
From dimensional analysis

$$\frac{D}{L} = \gamma \frac{\Delta\tau}{\mu}$$

$$\frac{\dot{D}}{L} = \zeta \frac{\Delta\tau}{\mu} V_S$$

Aki, 1967, JGR

Spectra and Corner Frequency: Brune (1970)



The average spectrum is given by:

$$\langle \Omega_s(\omega) \rangle = \langle R_{\theta\vartheta} \rangle (\sigma\beta/\mu)(r/R)(\omega^2 + (2.36\beta/r)^2)^{-1}$$

The corner frequency $f_c = (1/2\pi)(2.36\beta/r) = 0.37\beta/r$

At zero frequency, the spectrum must be the same as that from the double-couple dislocation.

$$\langle \Omega_s(0) \rangle = \langle R_{\theta\vartheta} \rangle M_0 \big/ (4\pi\rho\beta^3 R)$$

or

$$M_0 = (4\pi\rho\beta^3 R)\langle \Omega_s(0) \rangle \big/ \langle R_{\theta\vartheta} \rangle$$

Brune (1970, 1971): $f_c = 0.37\beta/r$

Madariaga(1976) $f_c = 0.21\beta/r$

Madariaga(1979) $f_c = 0.28\beta/r$

Brune, 1970, 1971, JGR

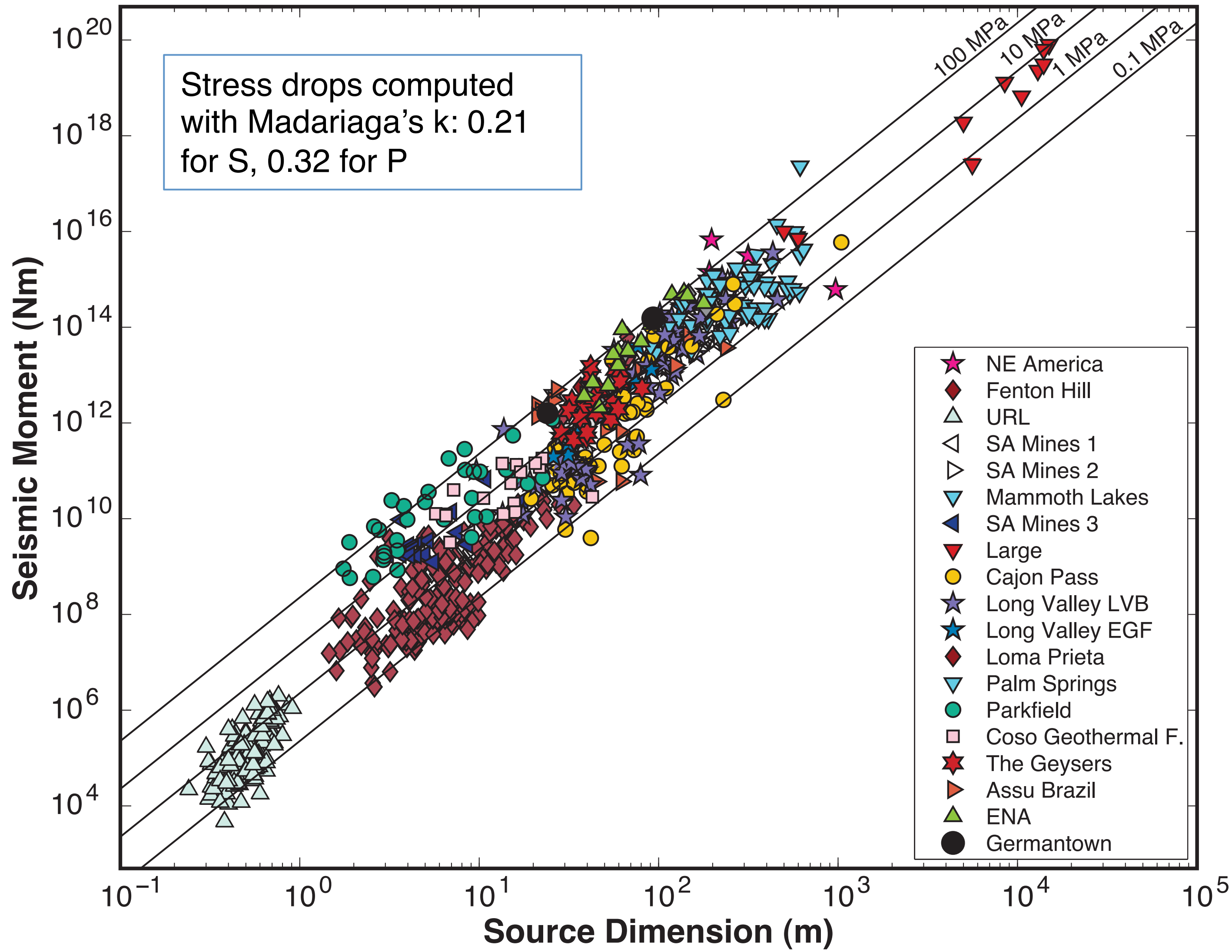
Stress Drop and Moment

Static Stress Drop and Seismic Moment for Three Fault Geometries			
	Circular (Radius, r)	Strike-Slip (Width, W)	Dip-Slip (Width, W)
$\Delta\tau$	$\left(\frac{7\pi}{16}\right)\mu\left(\frac{\bar{D}}{r}\right)$	$\left(\frac{2}{\pi}\right)\mu\left(\frac{\bar{D}}{W}\right)$	$\left(\frac{4}{\pi}\right)\left(\frac{\lambda+\mu}{\lambda+2\mu}\right)\mu\left(\frac{\bar{D}}{W}\right)$
M_0	$\left(\frac{16}{7}\right)\Delta\tau r^3$	$\left(\frac{\pi}{2}\right)\Delta\tau W^2 L$	$\left(\frac{\pi}{4}\right)\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\Delta\tau W^2 L$

Moment: Aki (1966)		
Slip related to a source dimension:		
Eshelby (1957)	Knopoff (1958)	Starr (1928)

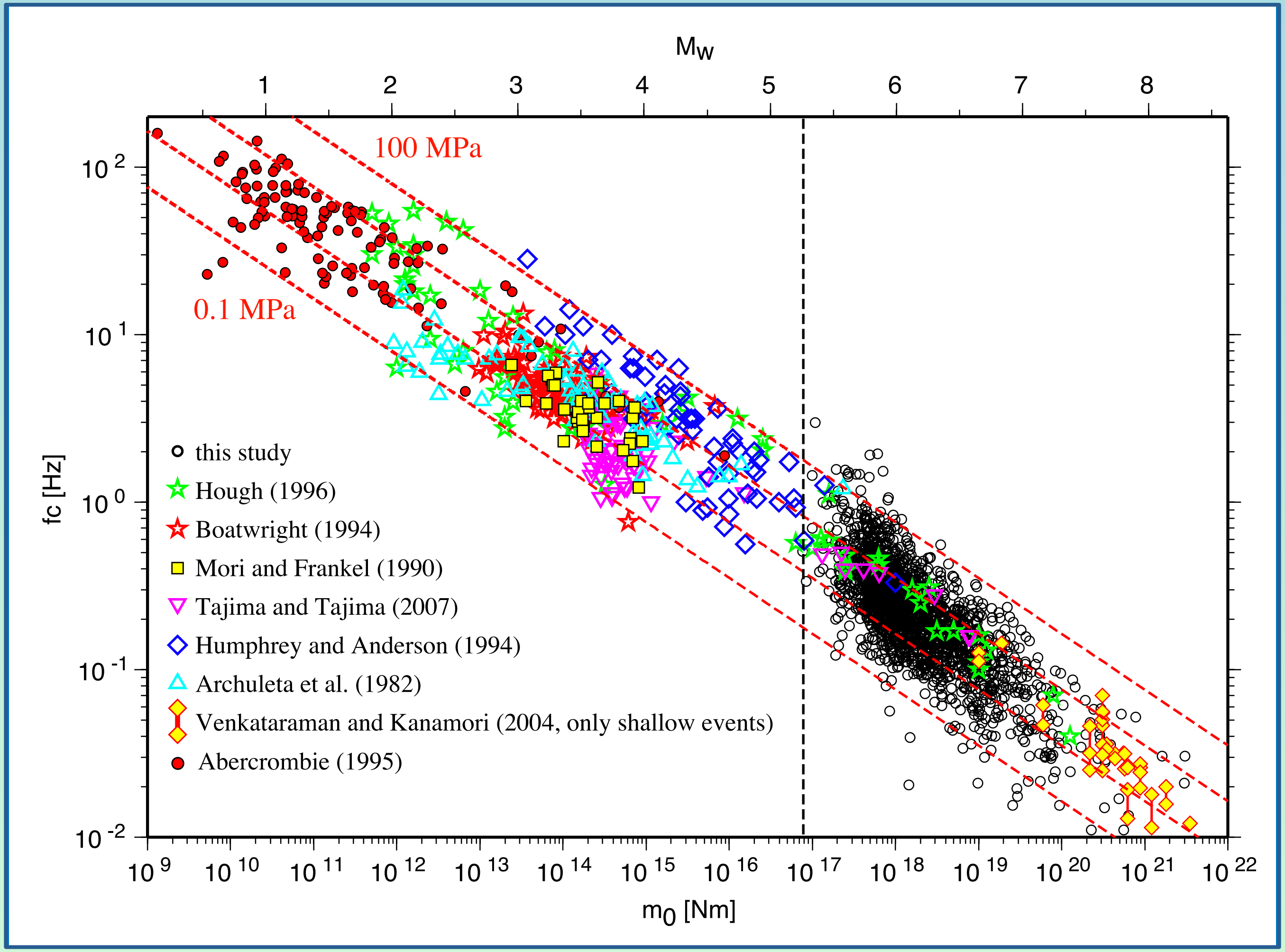
Lay and Wallace, 1995

Stress Drops



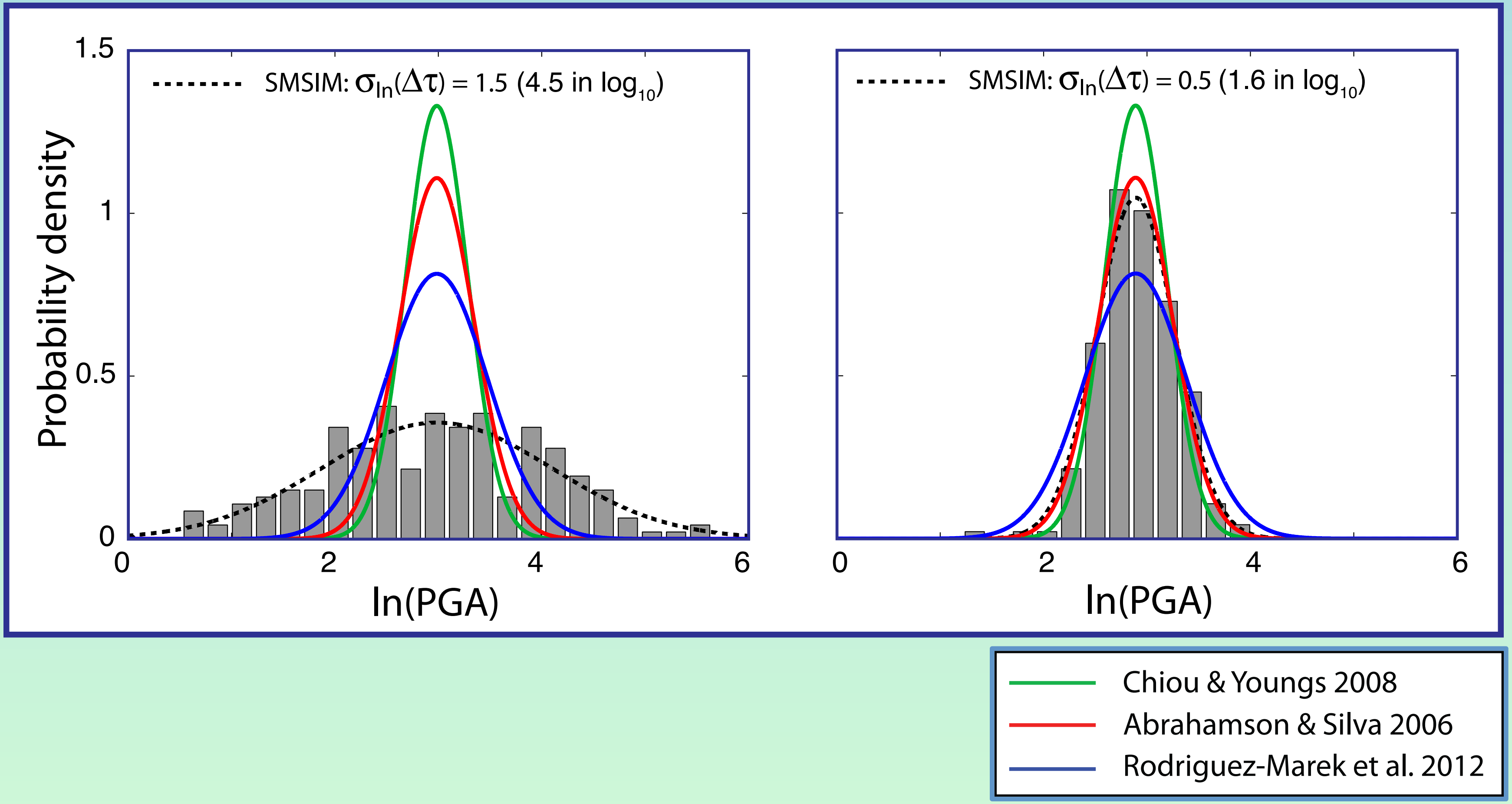
Viegas, SRL 2012; Abercrombie and Leary, 1993 GRL

Stress Drop: $10^9 \le M_0 \le 10^{22}$ Nm



Allmann and Shearer, 2009 JGR

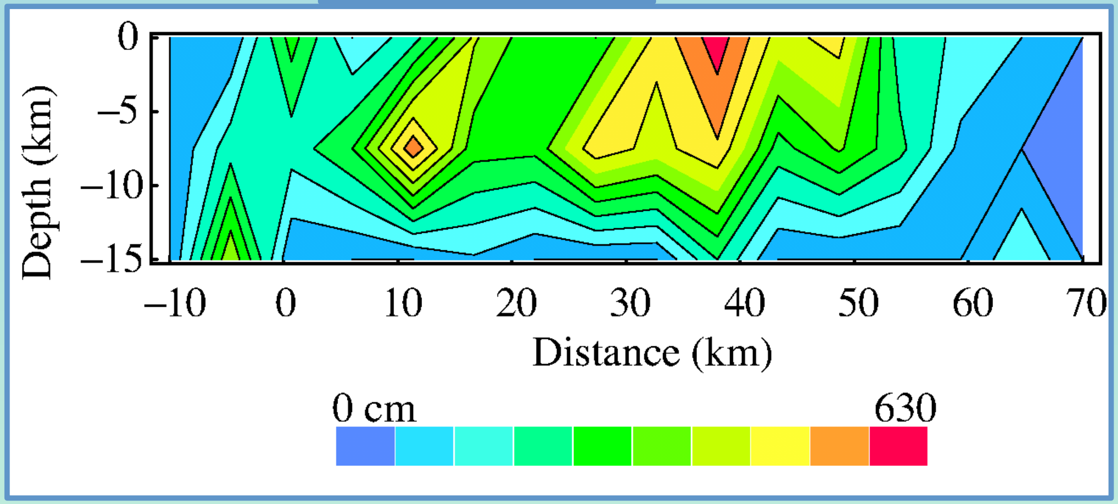
Sigma: Its Effect on Ground Motion



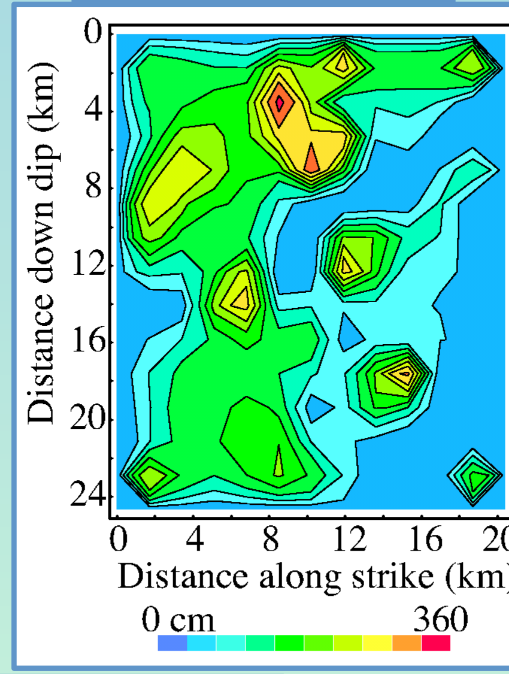
Cotton, Archuleta, Causse, SRL 2012

Heterogeneous Slip (Stress)

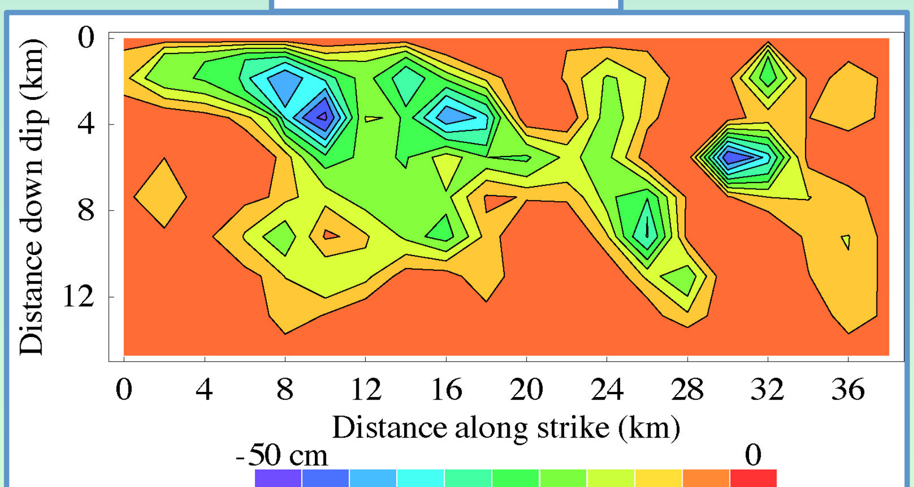
Landers 1992



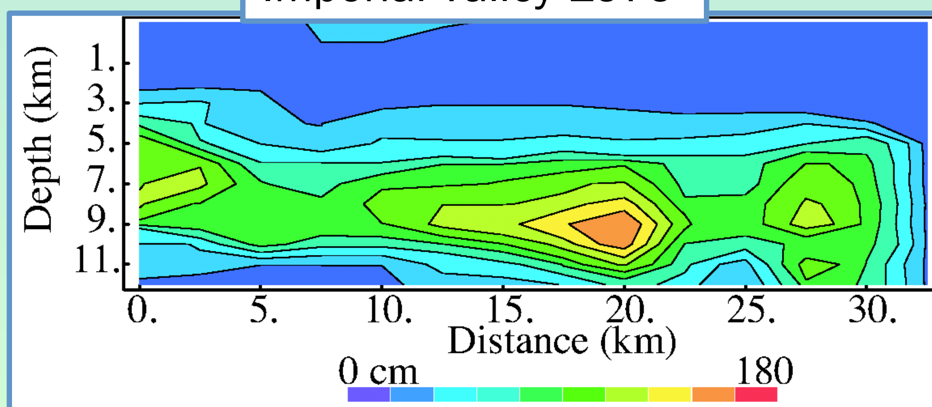
Northridge 1994



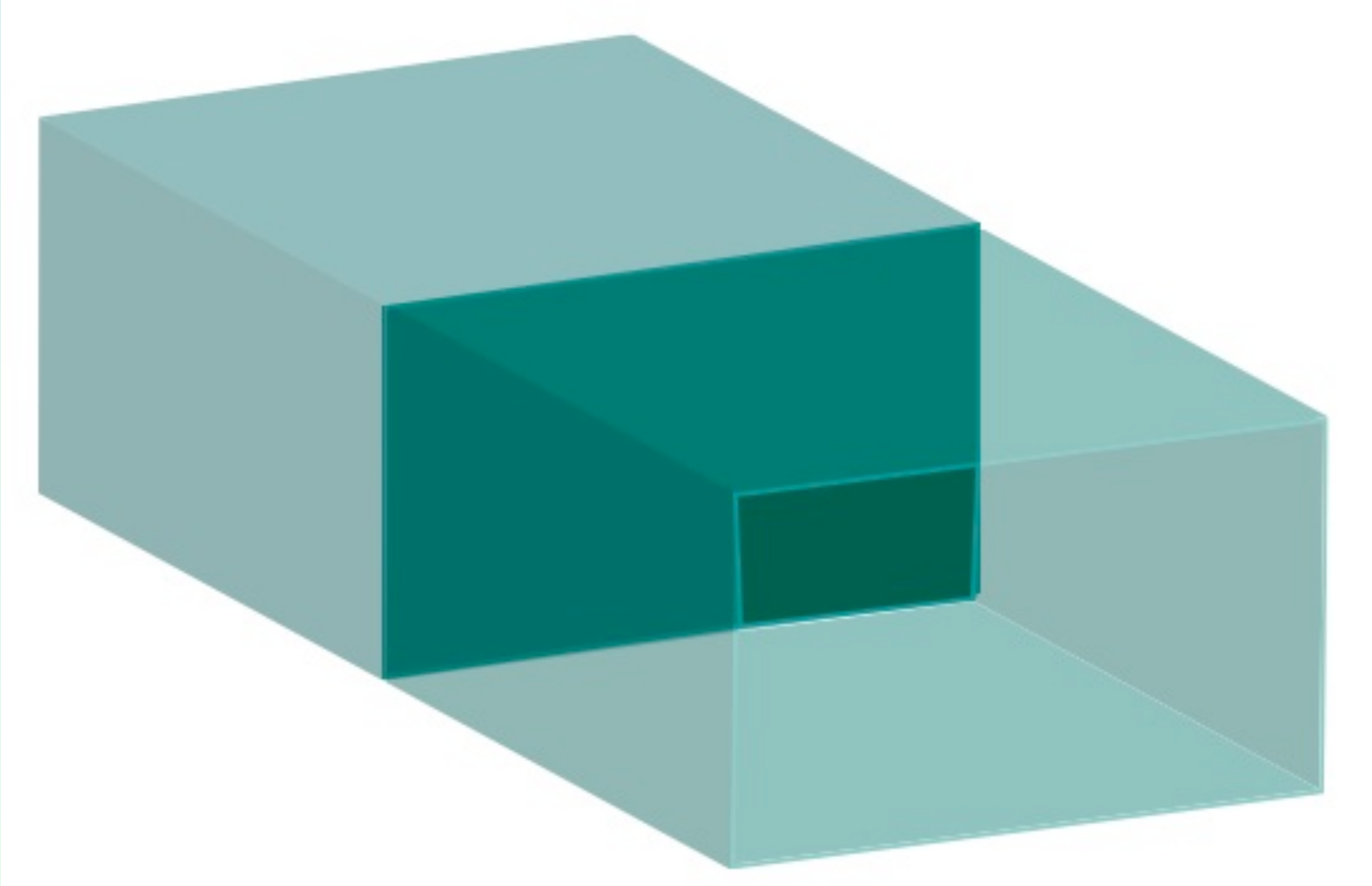
Parkfield 2004



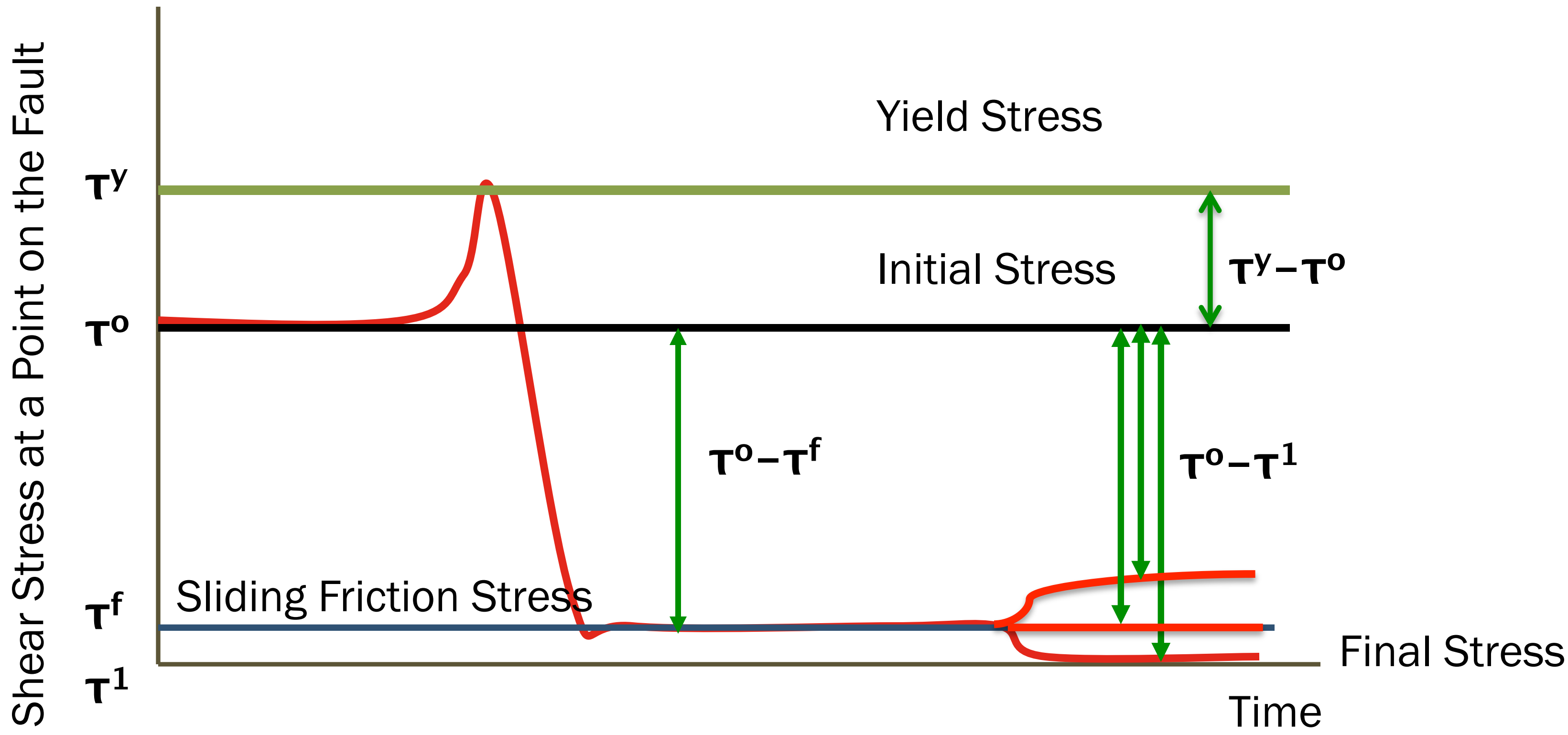
Imperial Valley 1979



Fault in Homogeneous Medium

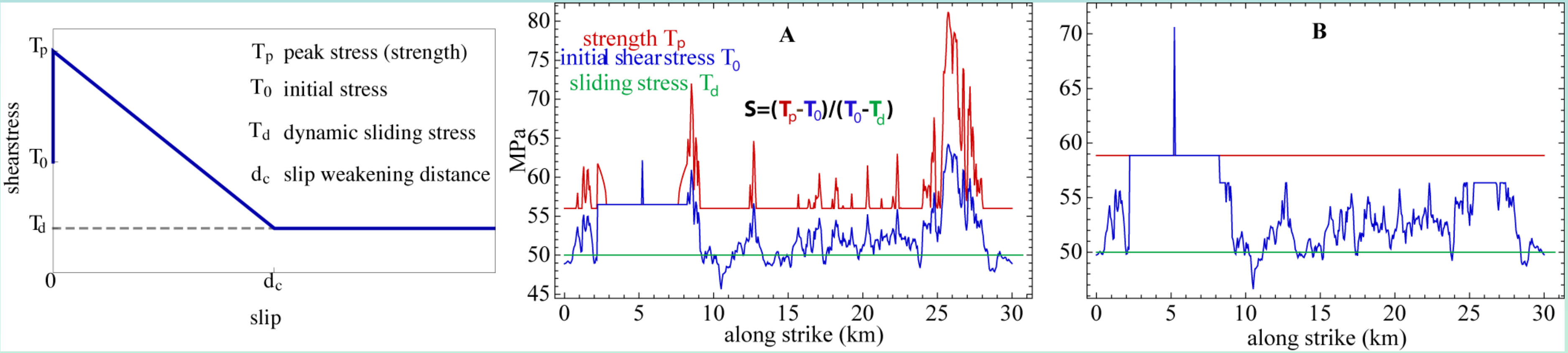


Stress at a Point on the Fault



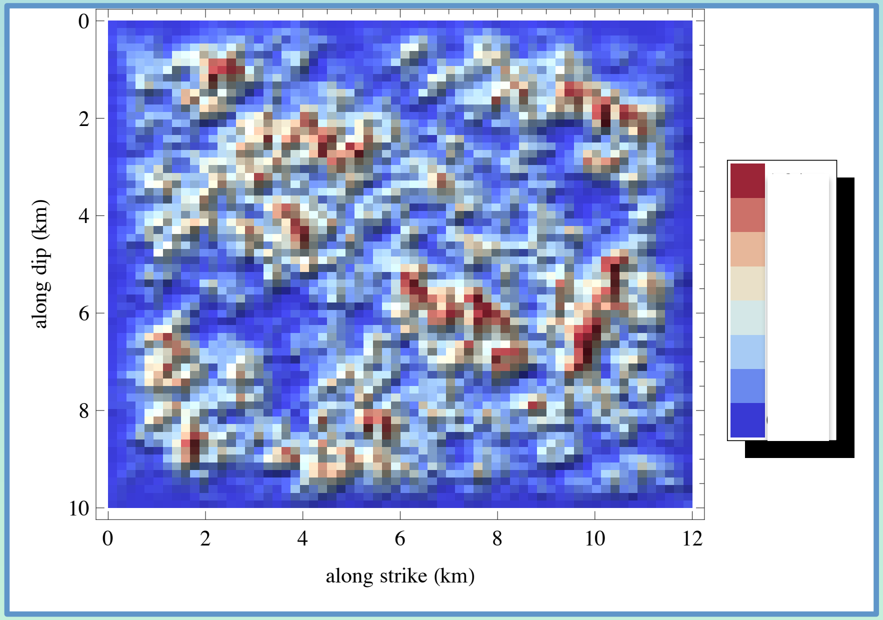
$S = \tau^y - \tau^o \,/\, \tau^o - \tau^f$ =Strength Excess/Stress Drop

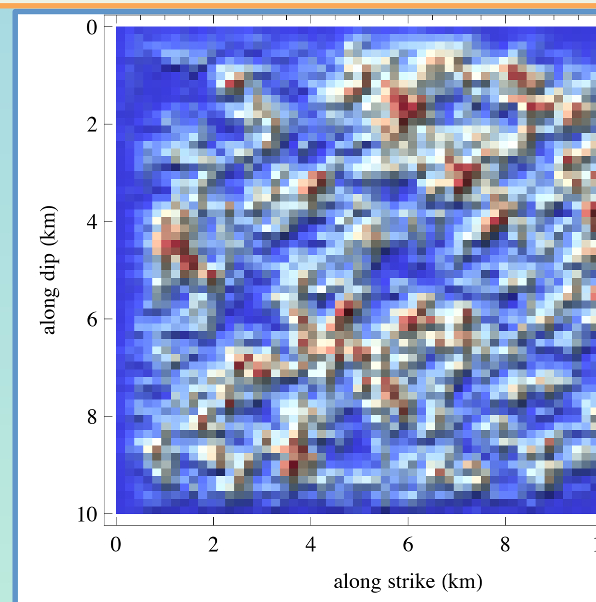
Stress and Dynamics



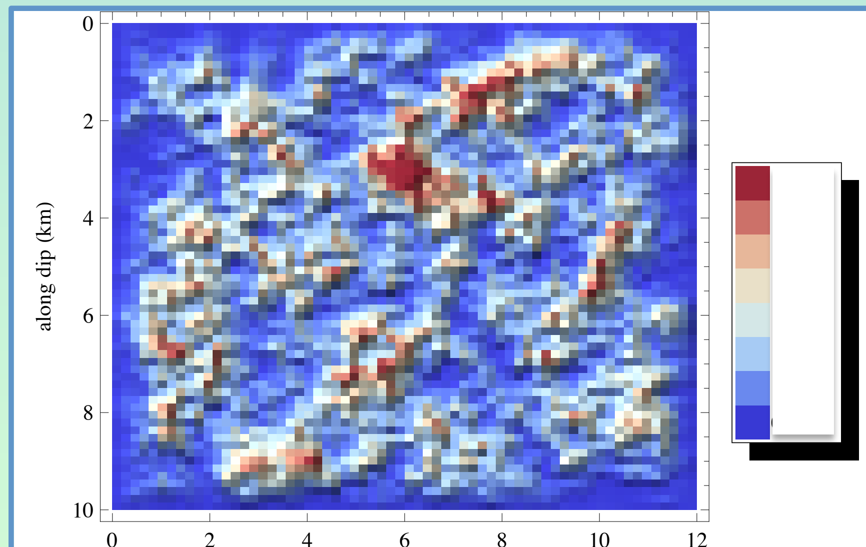
Stress and Dynamics

Initial Stress



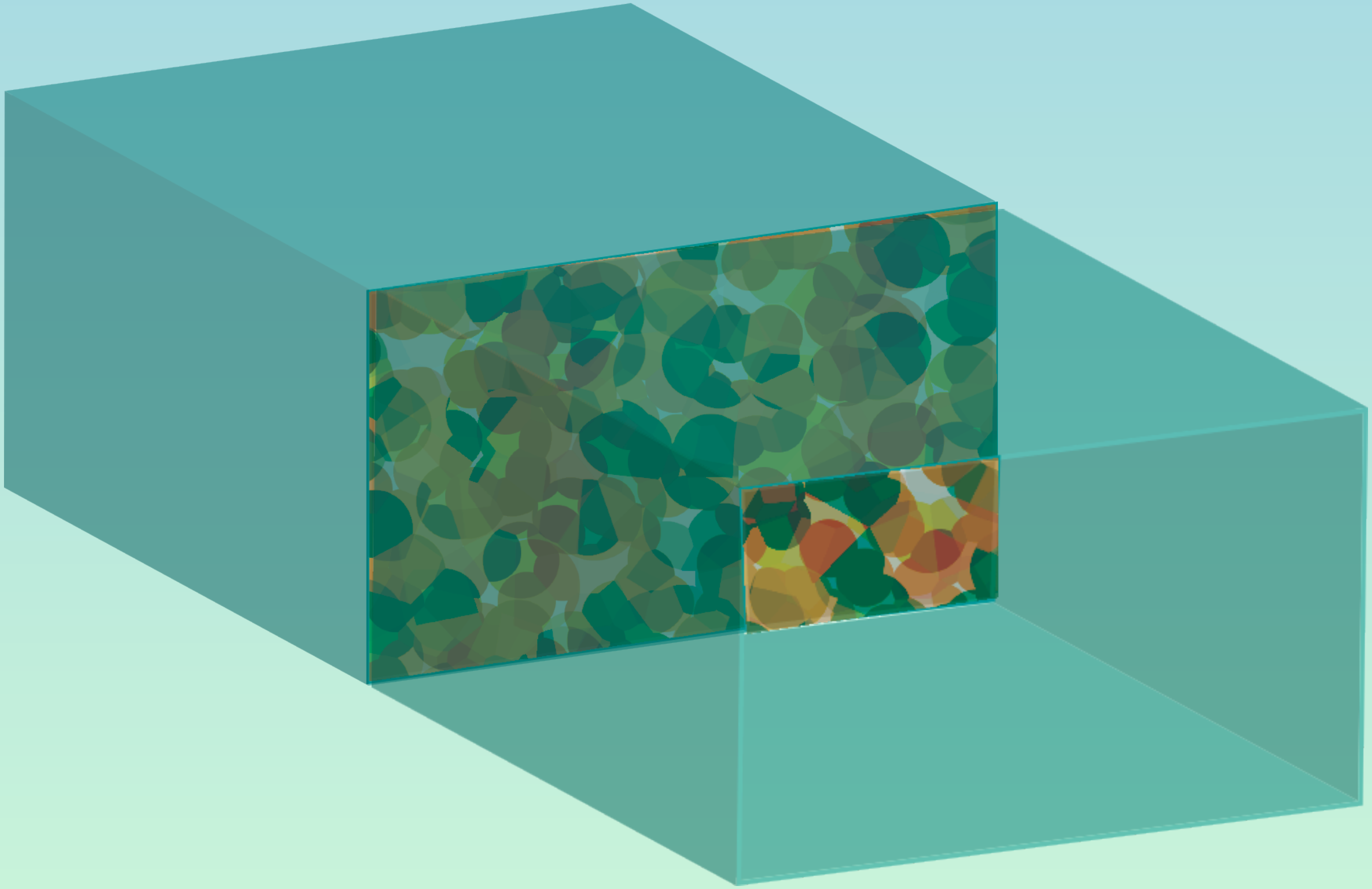


Peak Stress

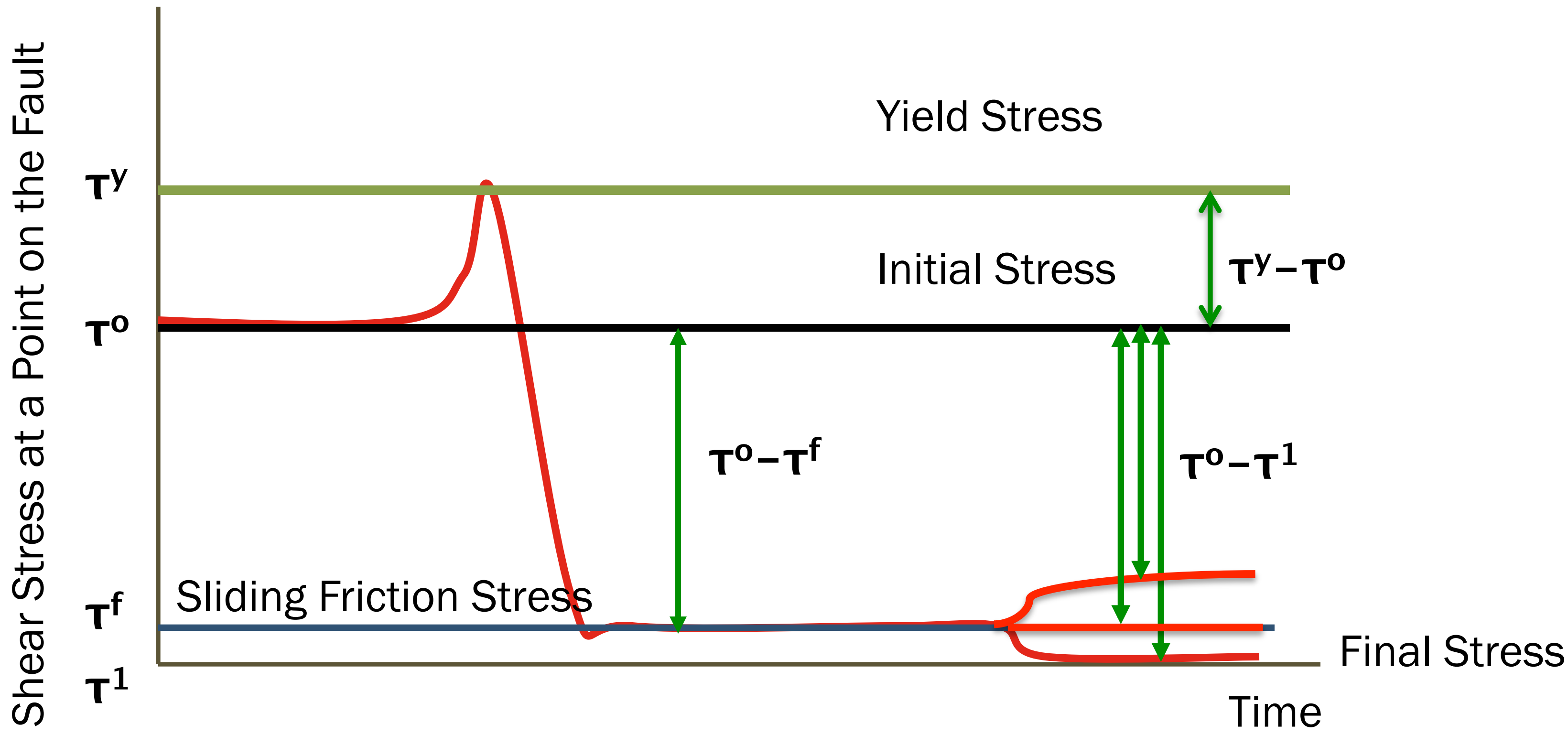


Sliding Friction Stress

Fault in Heterogeneous Medium

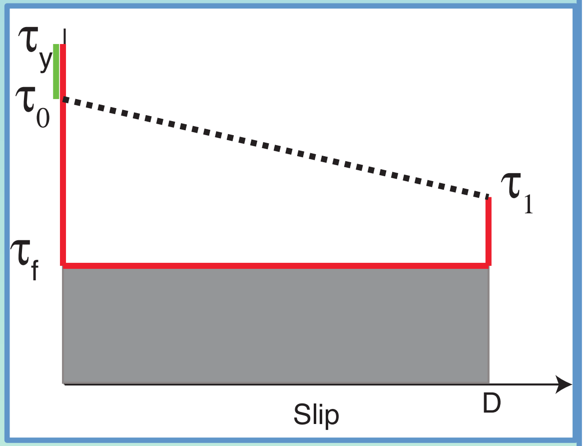


Stress at a Point on the Fault



$S = \tau^y - \tau^o \,/\, \tau^o - \tau^f$ = Strength Excess/Stress Drop

Static Stress Drop, Stress Drop, Apparent Stress



$$\Delta W = \frac{\tau_0+\tau_1}{2} DA = \frac{1}{\mu}\frac{\tau_0+\tau_1}{2}\mu DA = \frac{1}{\mu}\frac{\tau_0+\tau_1}{2} M_0 = \frac{1}{\mu}\bar{\tau} M_0$$

$$\bar{\tau} = \frac{\mu \Delta W}{M_0}$$

If frictional work is zero, the change in potential energy ΔW goes entirely into radiated energy E_R. When the radiation efficiency is not equal to 1, then

$$\bar{\tau} = \frac{\mu \Delta W}{\eta M_0} = \frac{\tau_a}{\eta}$$ where η is the radiation efficiency: $E_R = \eta \Delta W$.

We could also write

$\Delta W = E_R + E_f$ where $E_f = \tau_f DA$ the energy lost to friction with τ_f the stress at sliding friction.

$$\bar{\tau} = \frac{\mu}{M_0}\left(E_R + \tau_f DA\right) = \tau_a + \tau_f$$

$$\textit{Since}\ \ \bar{\tau} = \frac{1}{2}\left(\tau_0+\tau_1\right) = \frac{1}{2}\left(\tau_0+\tau_0-\Delta\tau_S\right) = \tau_0 - \frac{\Delta\tau_S}{2} \qquad \textit{where}\ \Delta\tau_S = \tau_0 - \tau_1$$

$$\tau_a = \bar{\tau} - \tau_f = \tau_0 - \tau_f - \frac{\Delta\tau_S}{2}$$

$$\boxed{\tau_a = \Delta\tau_d - \frac{\Delta\tau_S}{2}}$$

where $\Delta\tau_d$ is the dynamic stress drop $\Delta\tau_d = \tau_0 - \tau_f$ and τ_a is the apparent stress.

Kanamori and Heaton, 2000

Scaling Between f_C and r

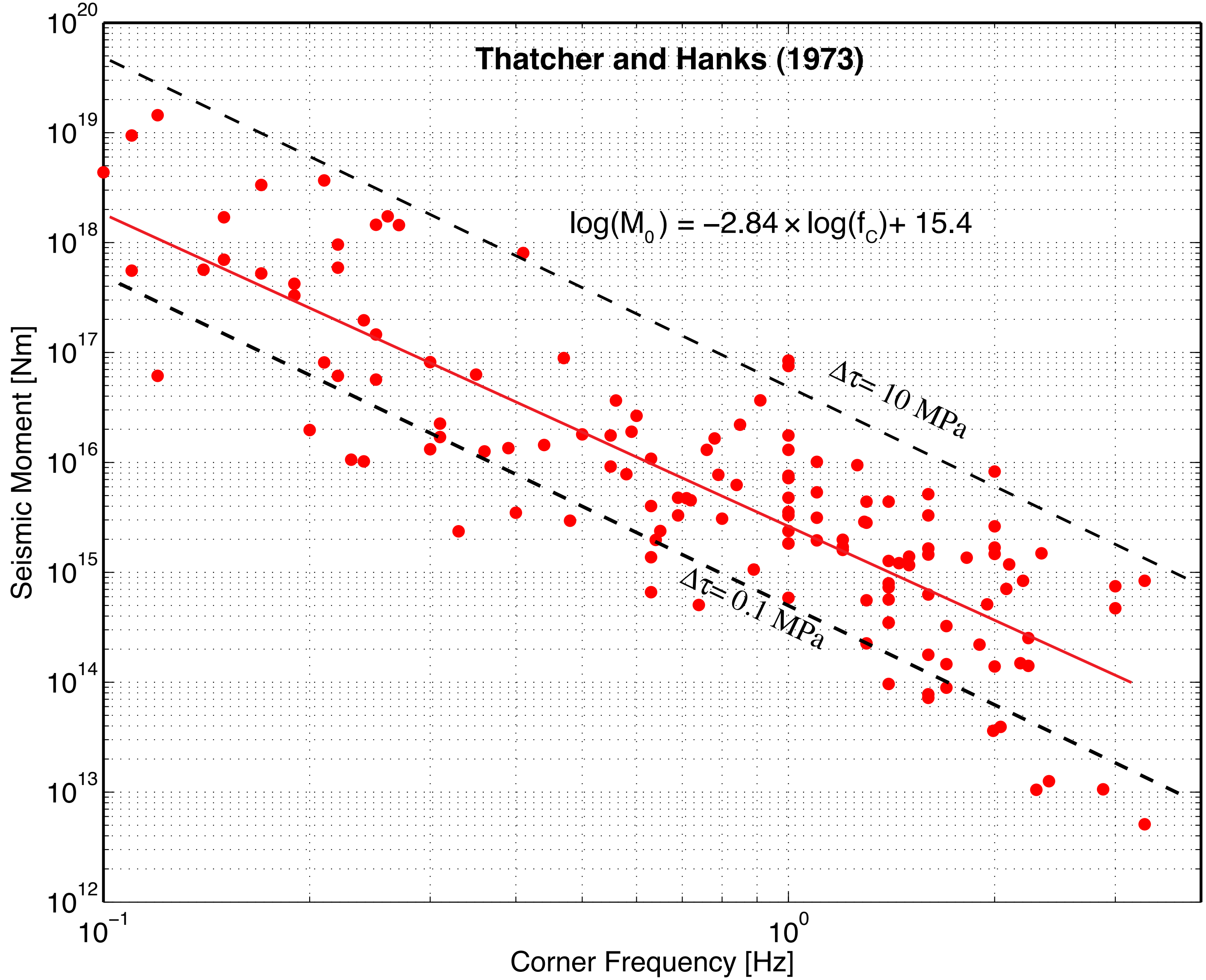
$$f_C = kV_S / r$$

$$\Delta\tau = 7M_0 / 16r^3 = 7M_0 f_C^3 / 16(kV_S)^3$$

	V/Vs	C_P	C_S	$k=(C_S/2\pi)$	Multiplier on stress drop
Andrews*	0.90	1.55	2.86	0.46	0.52
Snoke*	0.90	1.73	2.83	0.45	0.56
Silver*	0.90	2.79	3.65	0.58	0.26
Sato & Hirasawa (1973)	0.90	1.6	1.99	0.32	1.55
Madariaga (1976)	0.90	$(3/2)C_S$—Use V_S	1.32	0.21	5.47
Kaneko & Shearer (2012)	0.90	0.38—Use V_S	1.63	0.26	2.88
Brune (1970, 1971)			2.32	0.37	1.0

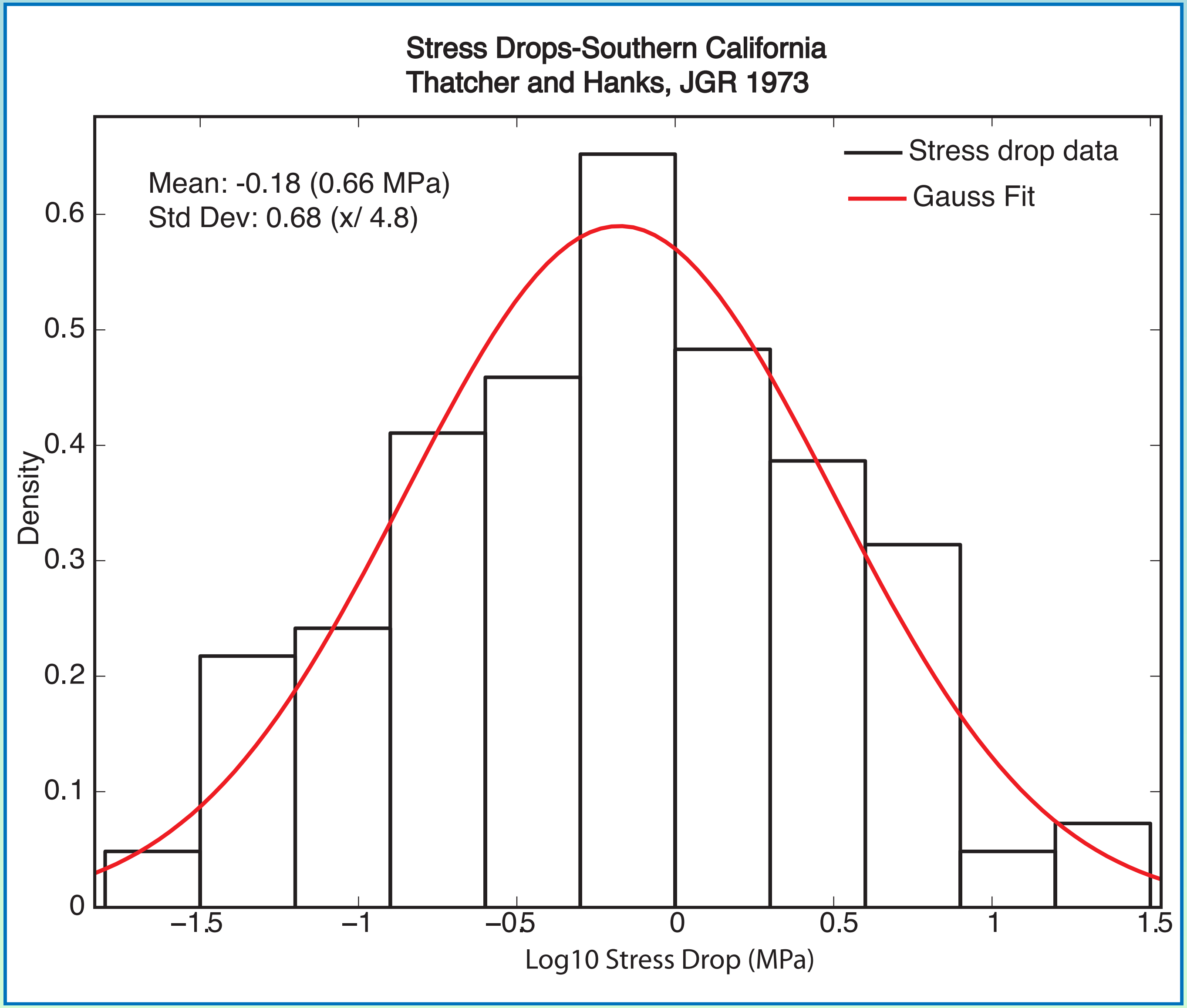
* Dong and Papageorgiou, BSSA 2002

Southern California: M_0 vs f_c



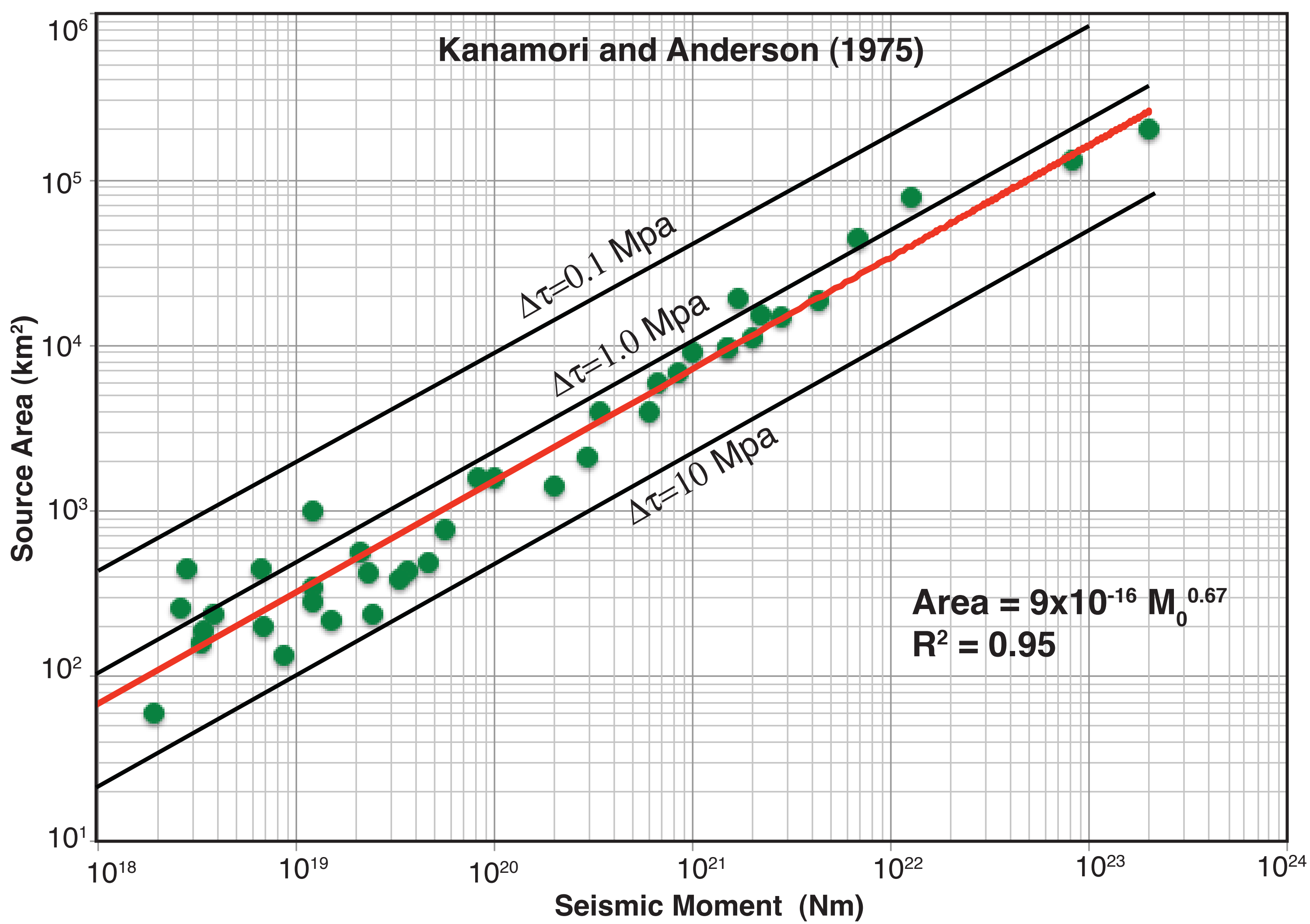
Thatcher and Hanks, 1973 JGR

Brune Stress Drops: Southern California



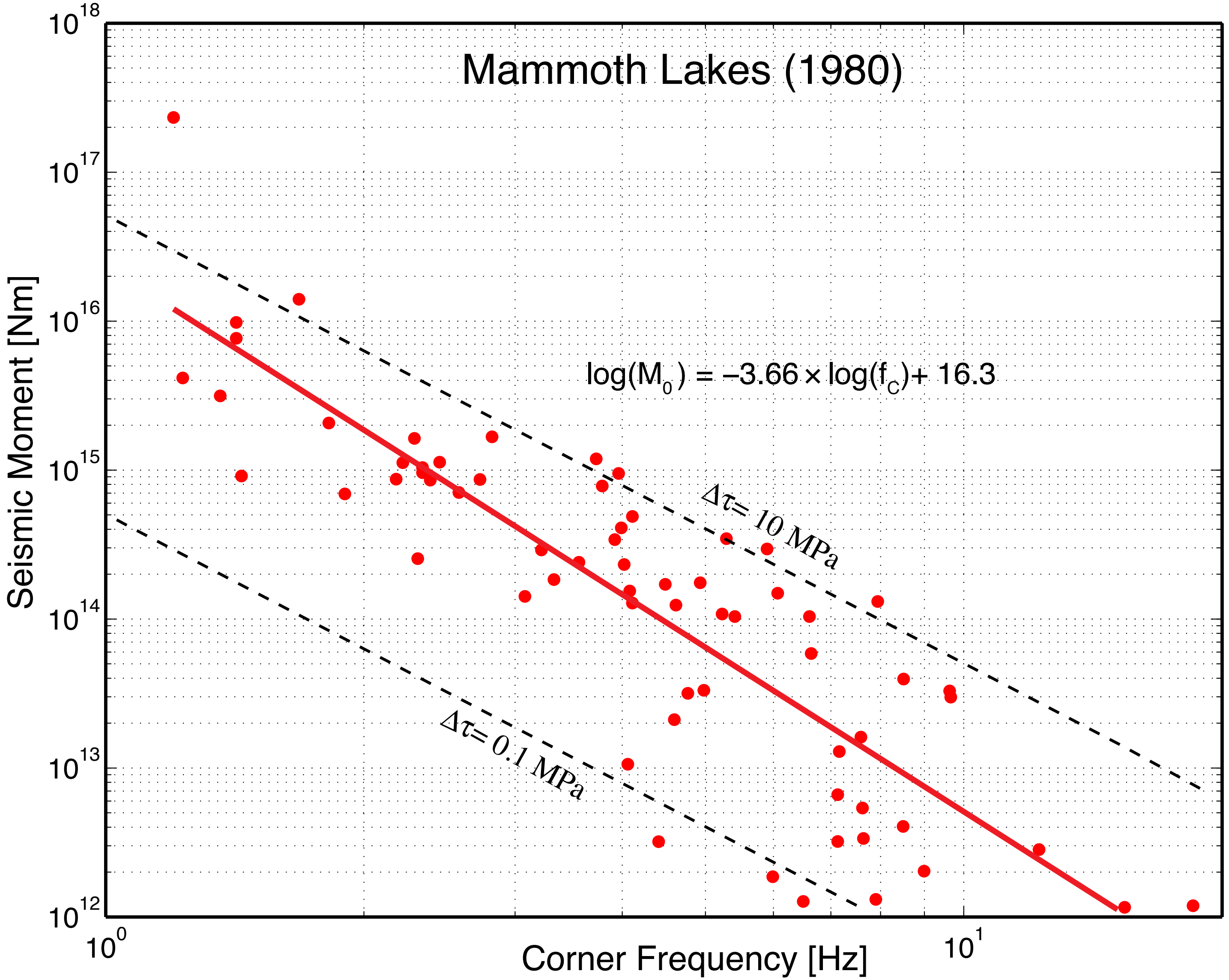
Thatcher and Hanks, 1973 JGR

Stress Drop Based on Area-Moment



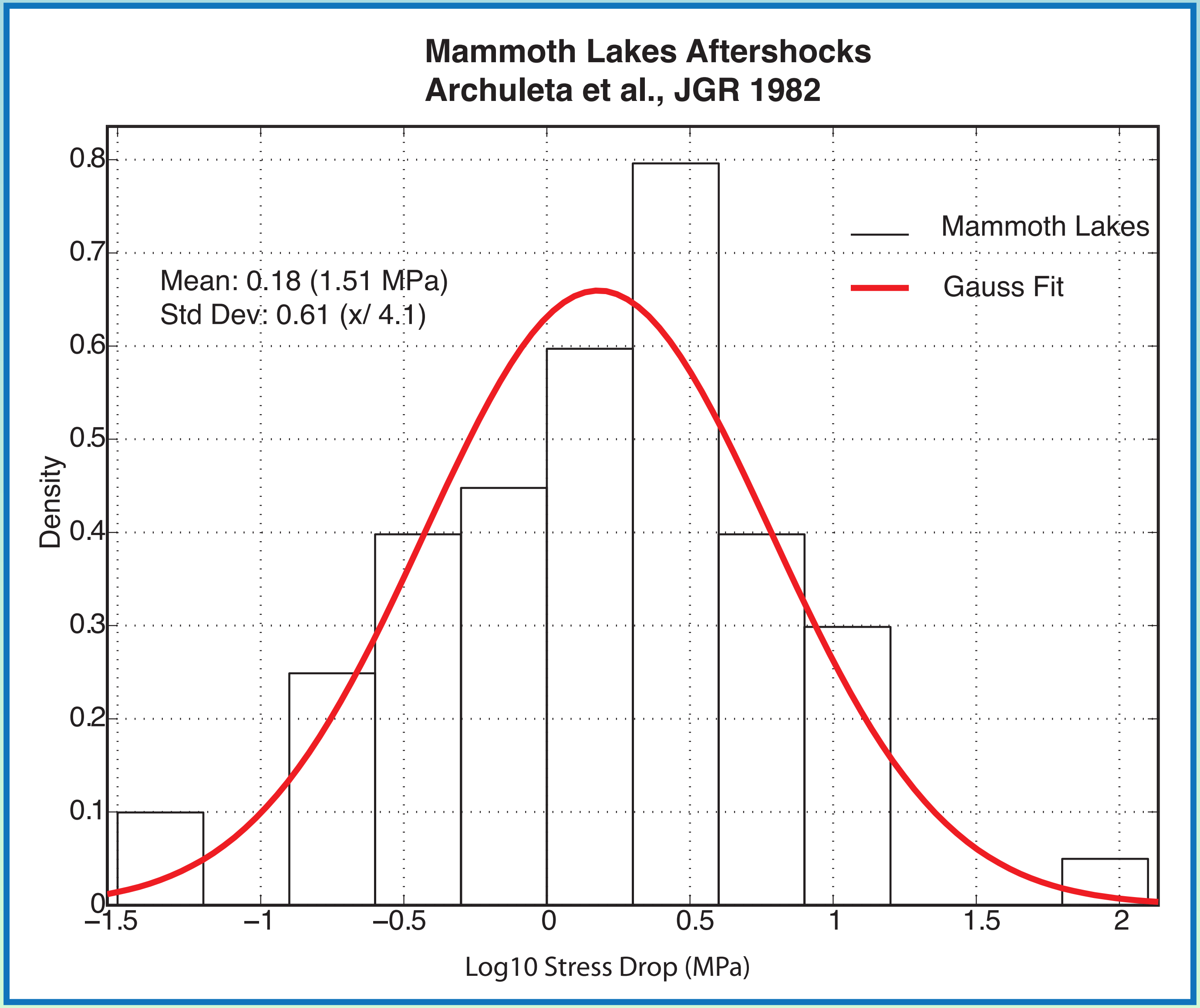
Kanamori and Anderson, 1975 BSSA

Mammoth Lakes: M_0 vs f_c



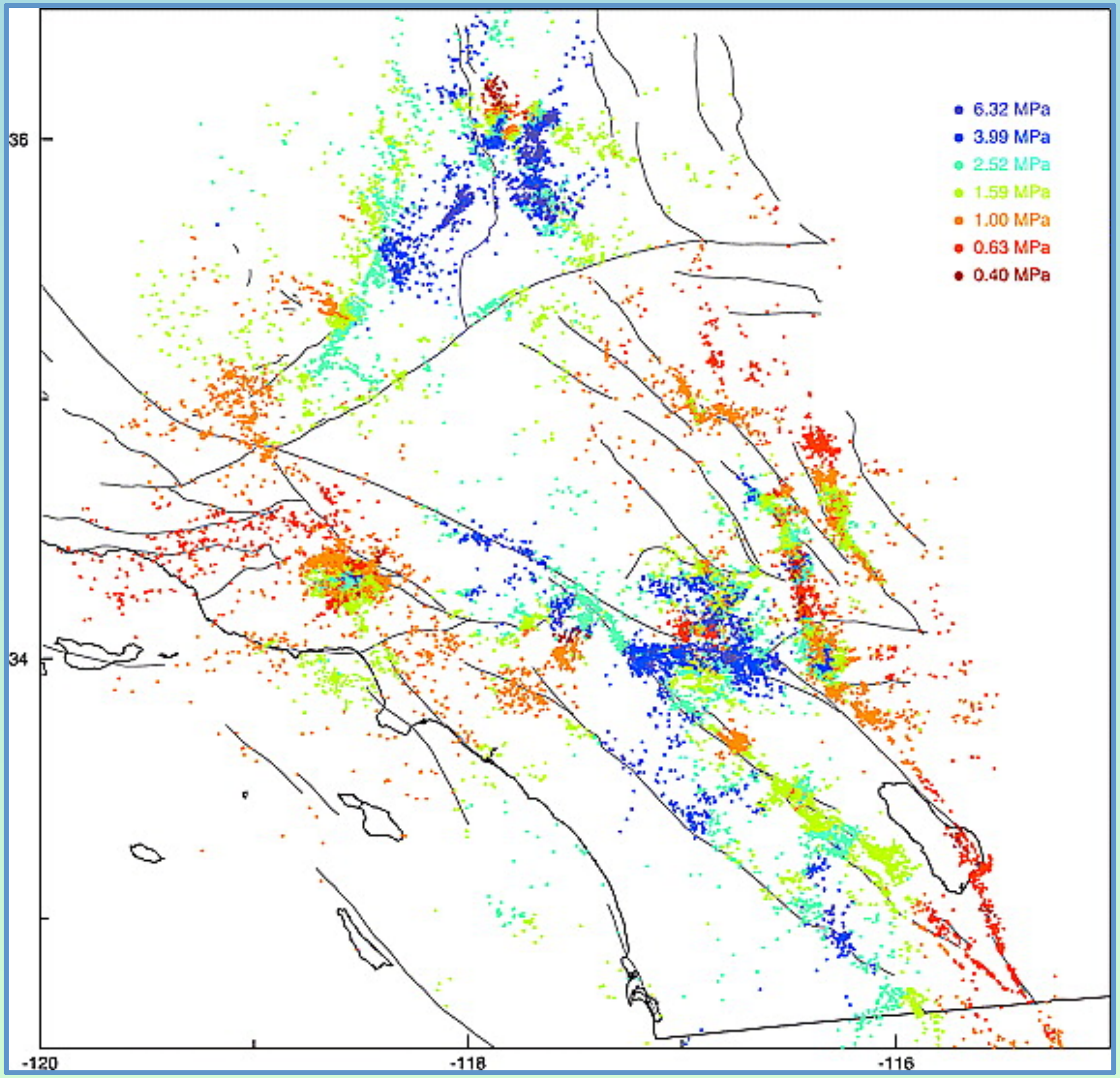
Archuleta, Cranswick, Mueller, Spudich, 1982 JGR

Brune Stress Drops: Mammoth Lakes



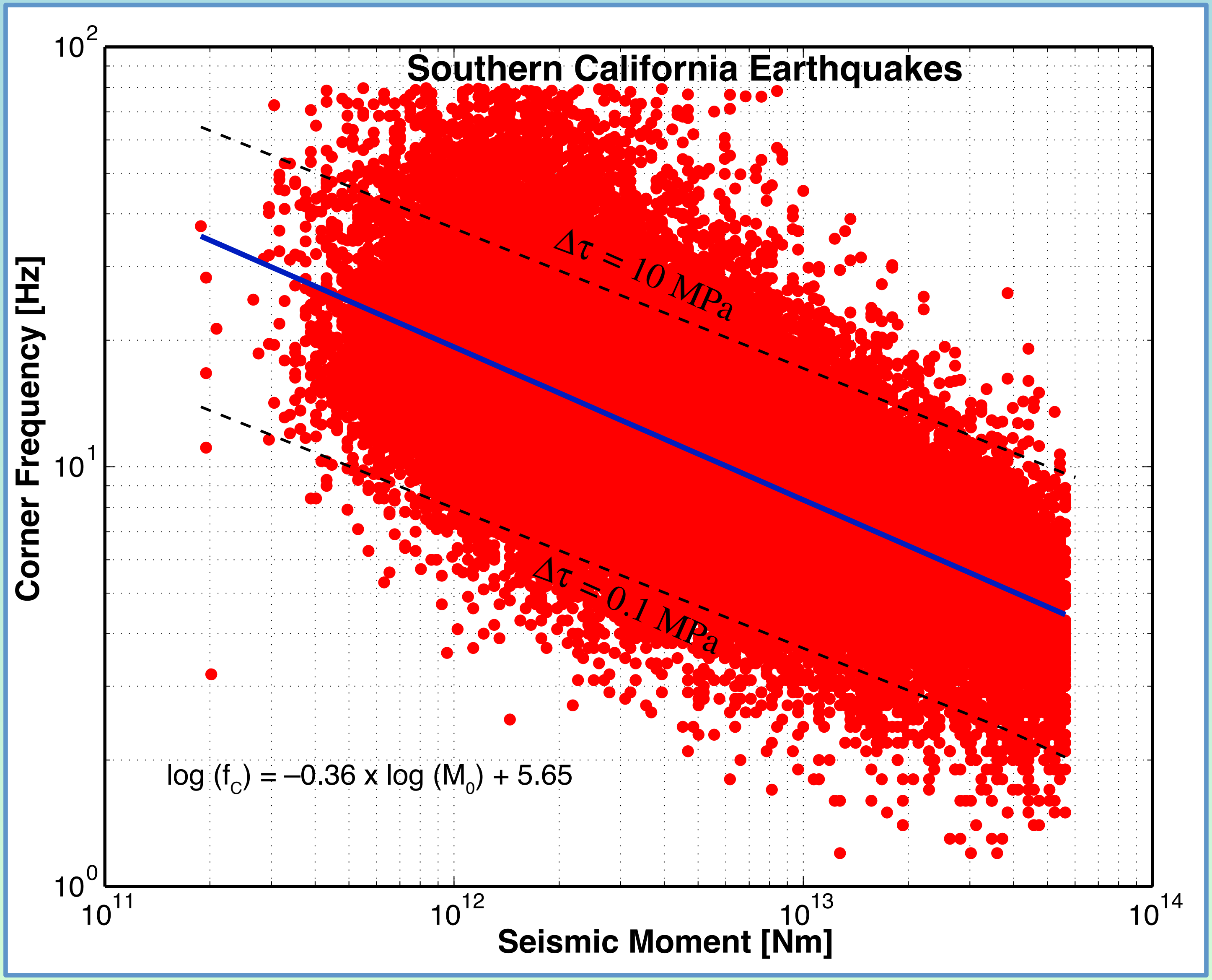
Archuleta, Cranswick, Mueller, Spudich, 1982 JGR

Southern California Stress Drops



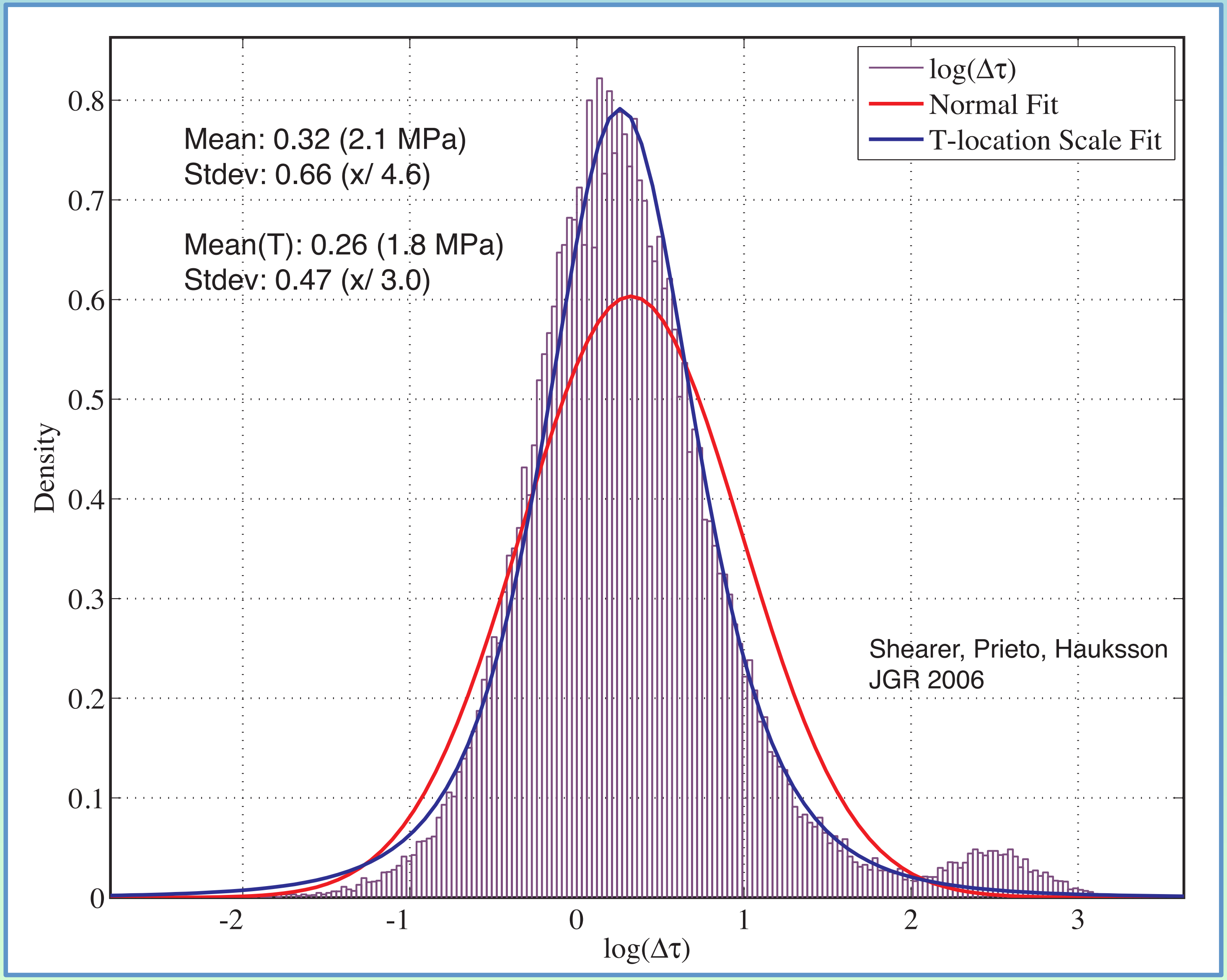
Shearer, Prieto, Hauksson, 2006 JGR

Southern California: M_0 vs f_c



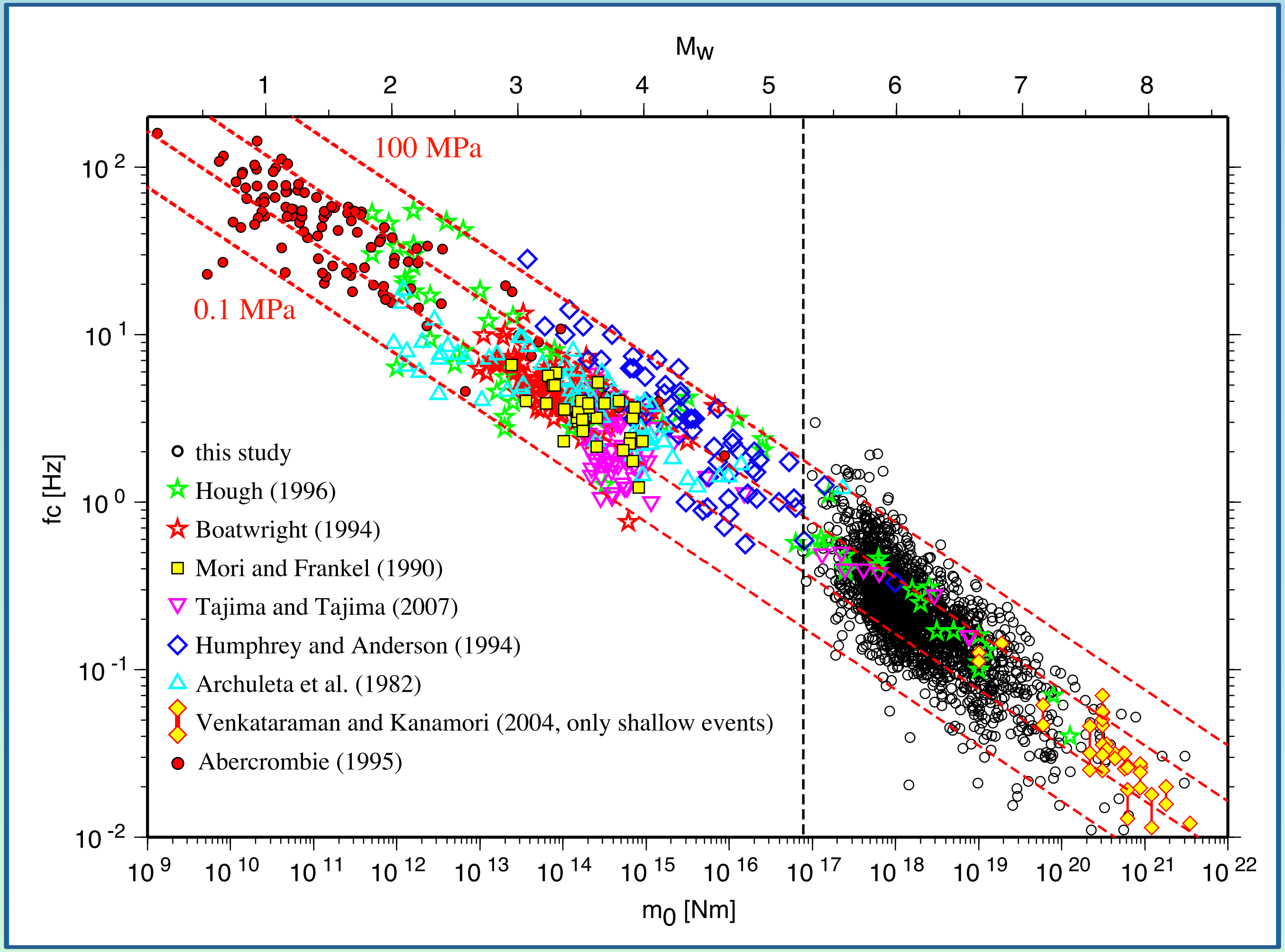
Shearer, Prieto, Hauksson, 2006 JGR

Southern California: >60,000 with M_L = 1.5 to 3.1



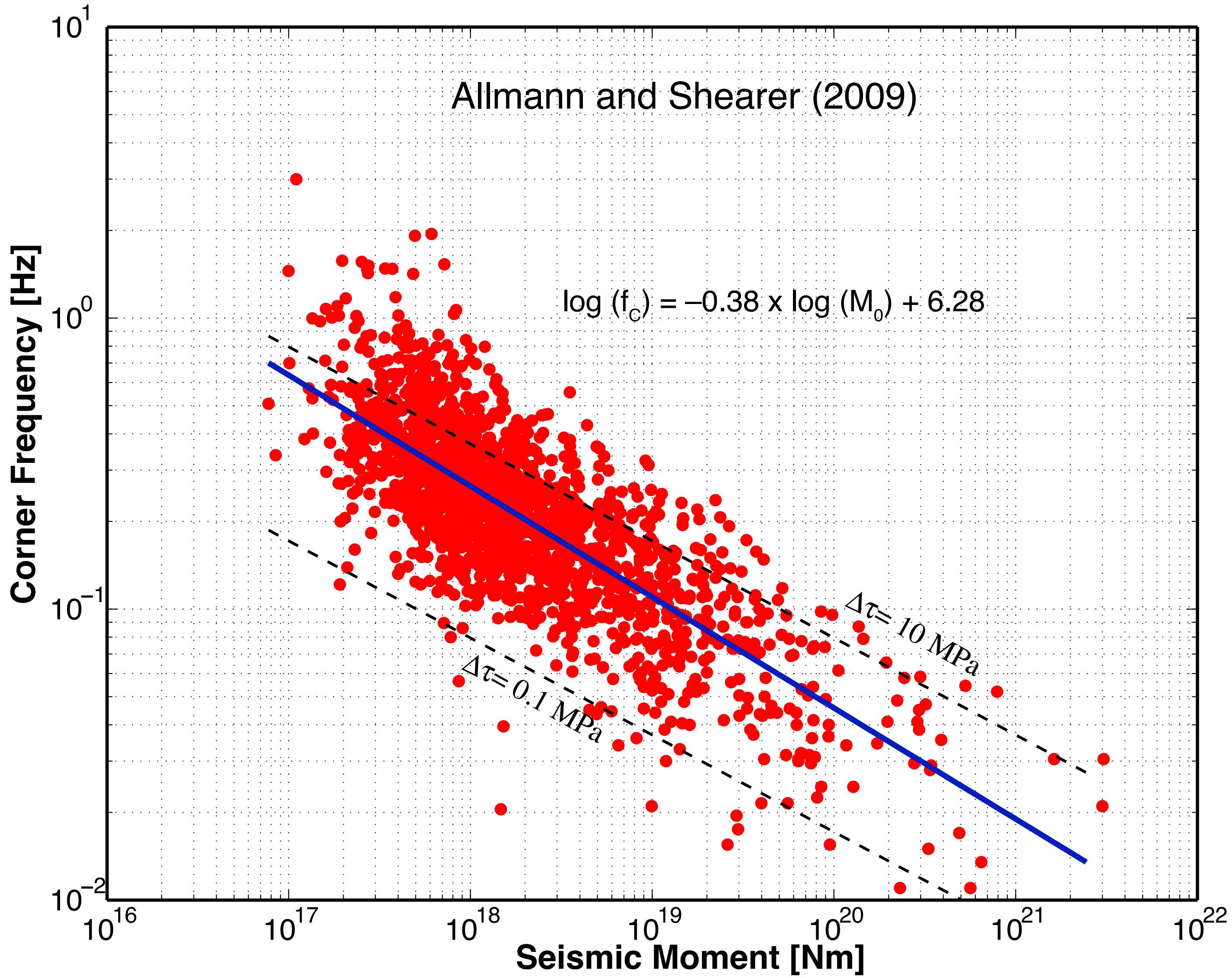
Shearer, Prieto, Hauksson, 2006 JGR

Stress Drop: $10^9 \leq M_0 \leq 10^{22}$ Nm



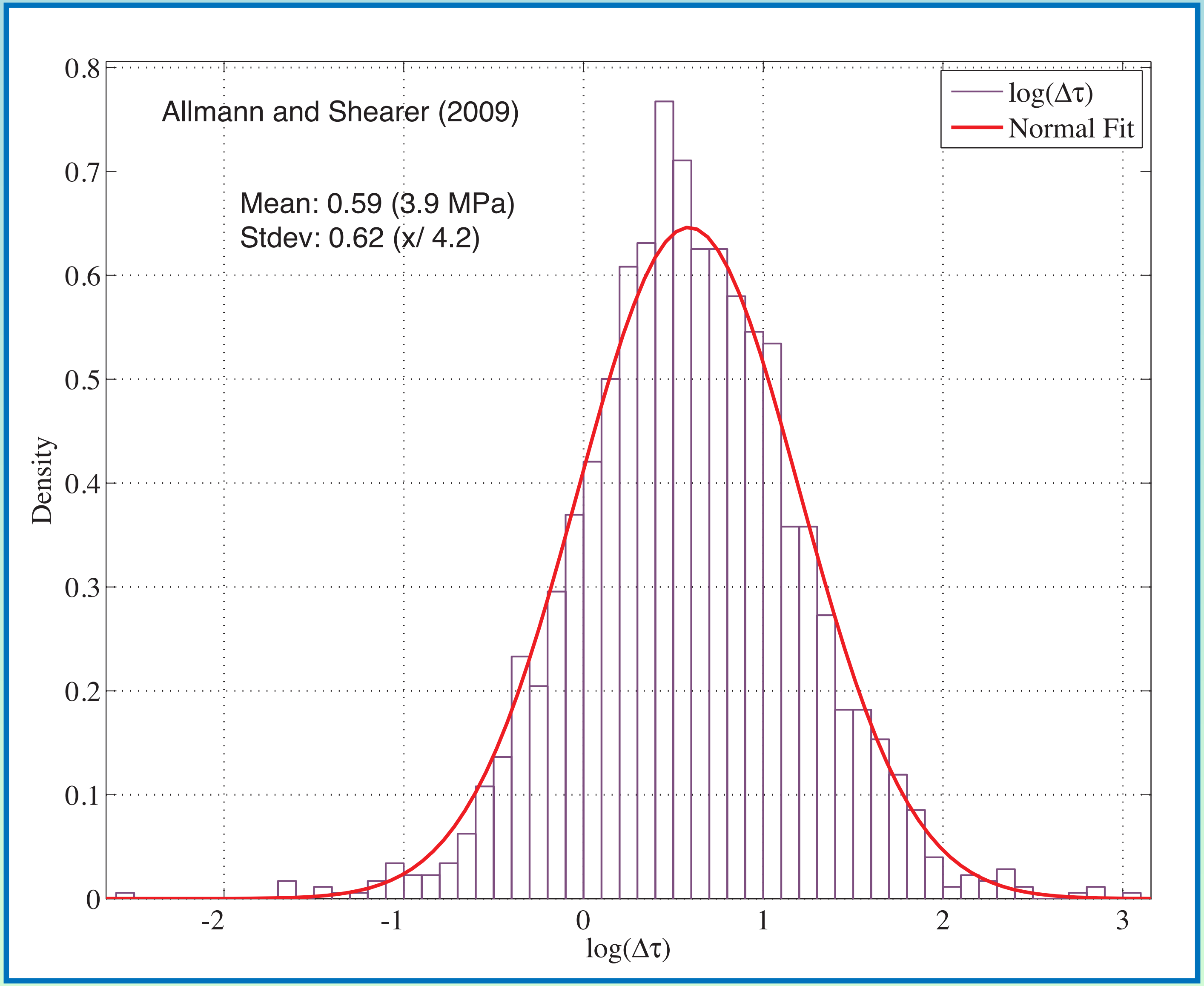
Allmann and Shearer, 2009 JGR

Global Earthquakes: M_0 vs f_c



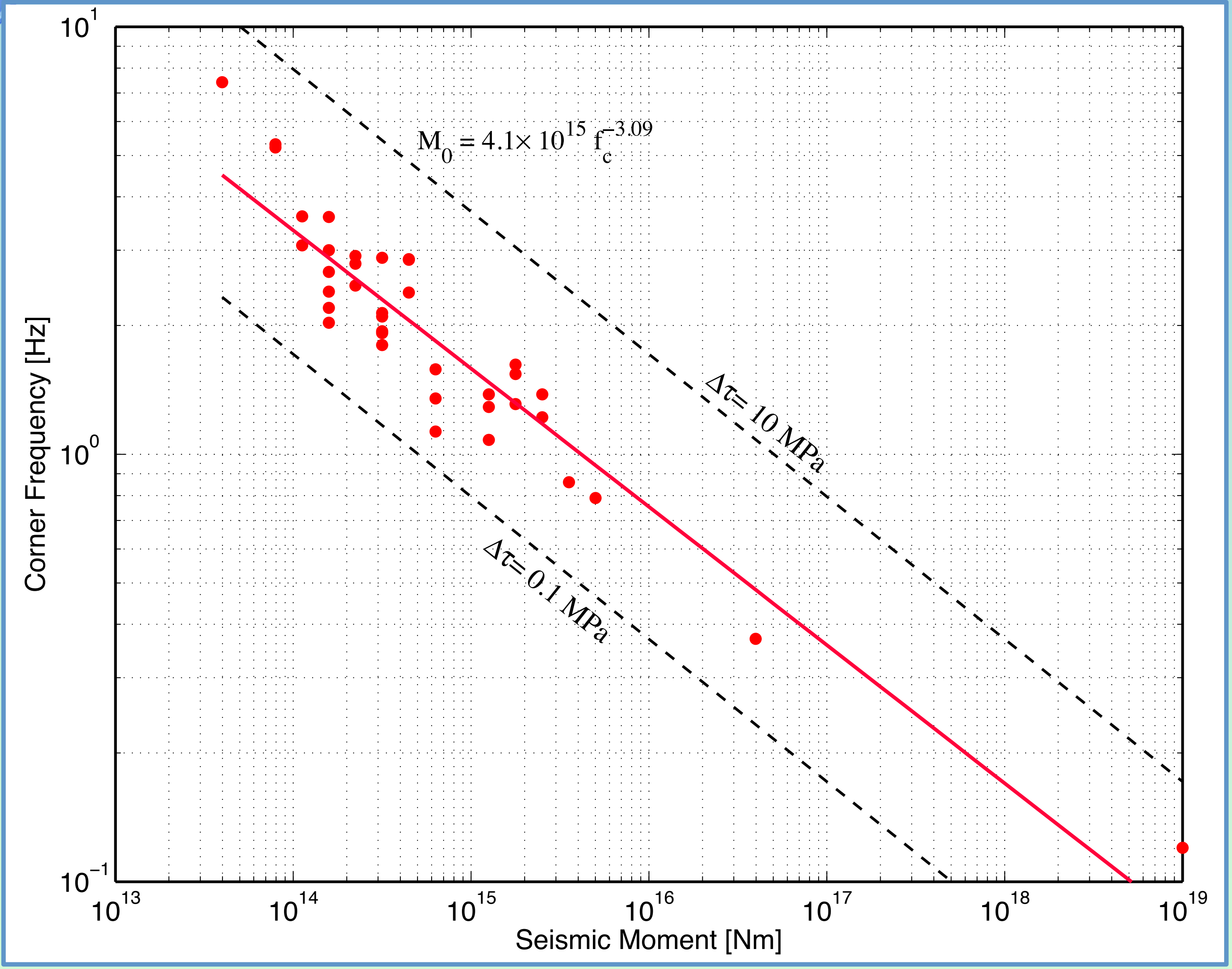
Shearer, Prieto, Hauksson, 2006 JGR

Distribution of Stress Drop



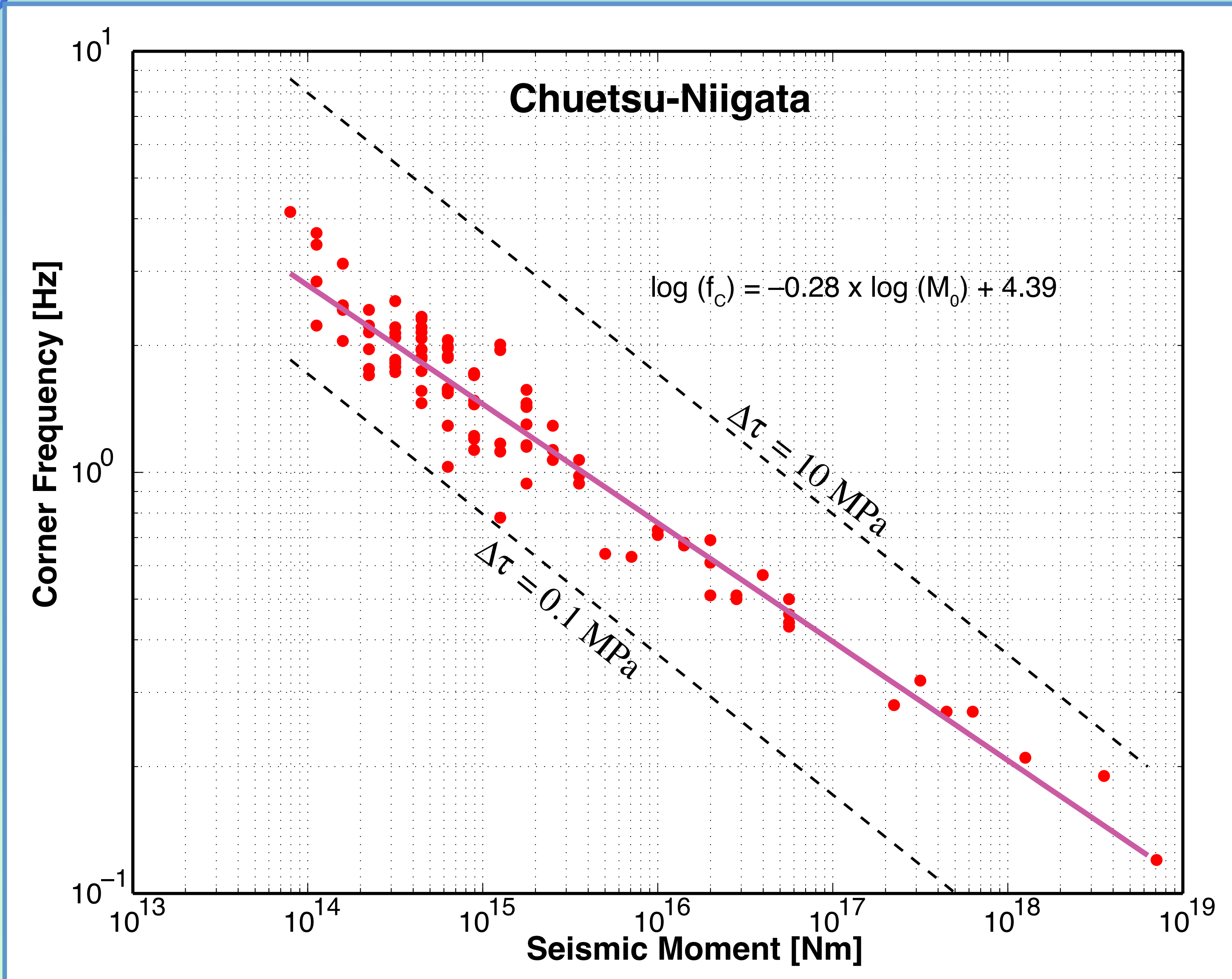
Allmann and Shearer, 2009 JGR

Tottori: M_0 vs f_c



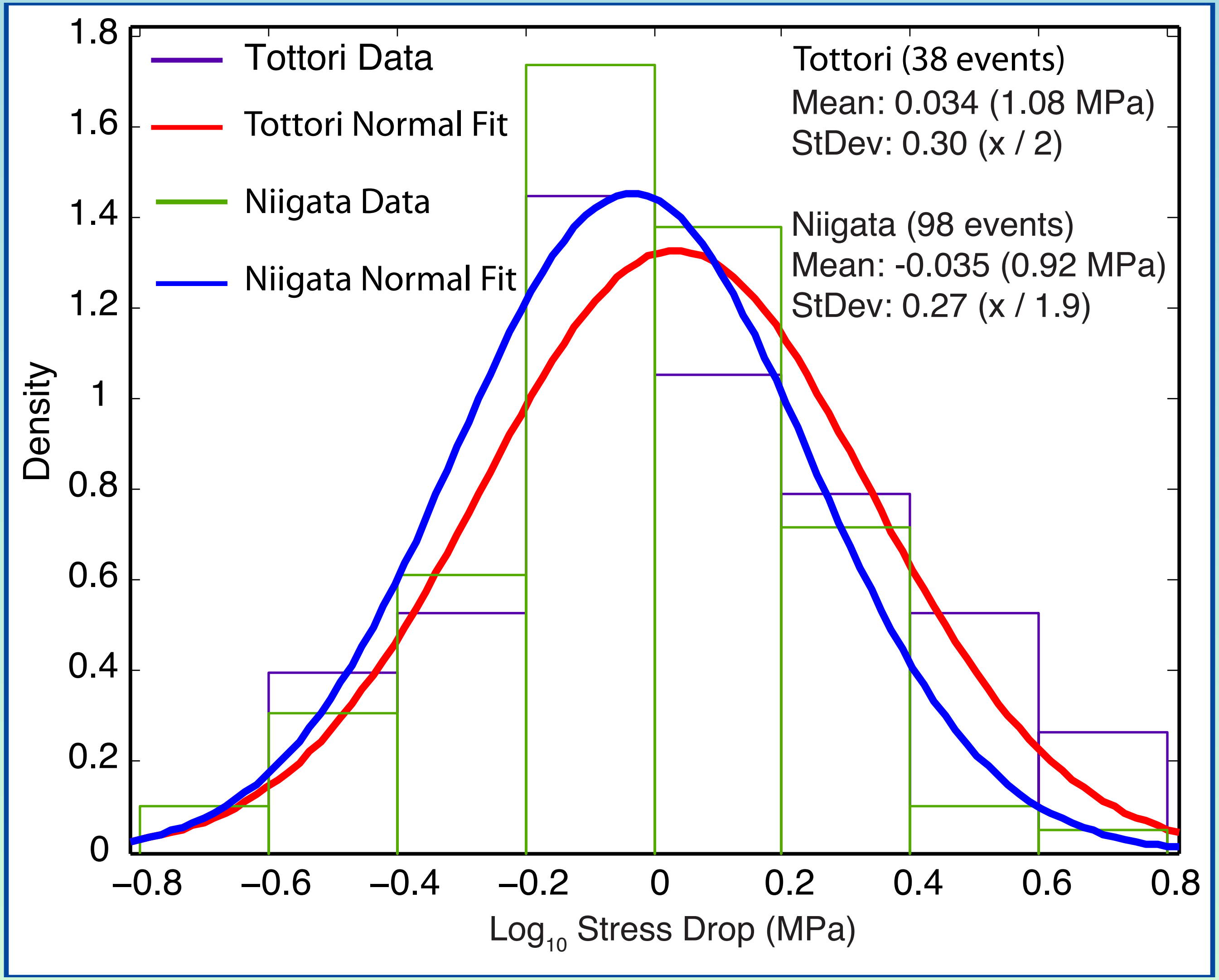
Adrien Oth, Dino Bindi, Stefano Parolai and Domenico Di Giacomo, 2010 GRL

Chuetsu-Niigata: M_0 vs f_c



Data from Oth et al., 2010 GRL

Normal Distribution: Chuetsu-Niigata and Tottori



Adrien Oth, Dino Bindi, Stefano Parolai and Domenico Di Giacomo, 2010 GRL

Stress Drop and Variability from Spectral Studies (M_0,f_c)

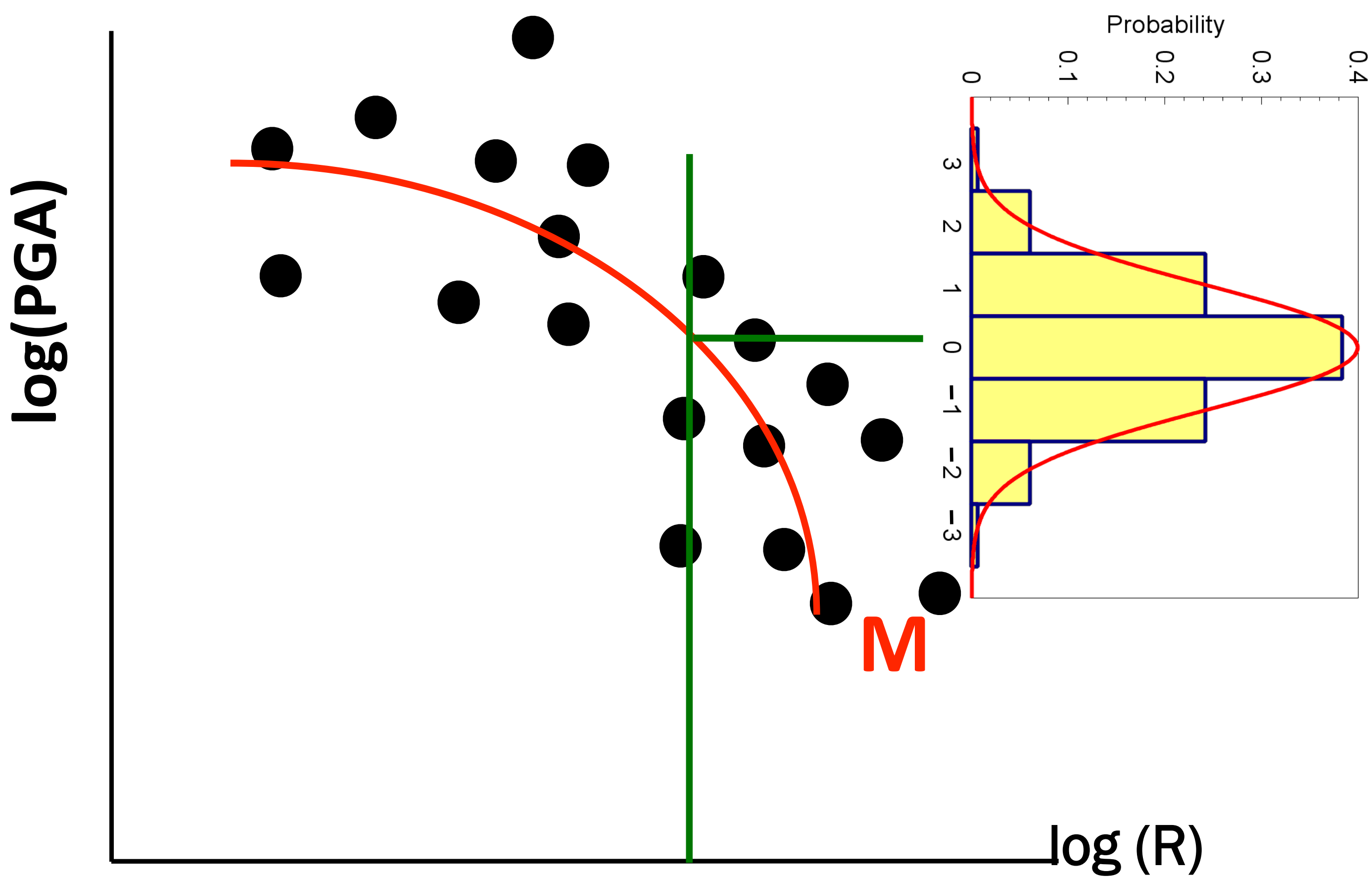
Source study	Region	Mean Brune stress-drop (MPa)	Stress-drop variability (Natural log)	No. earthquakes
Allmann and Shearer, 2009	Interplate $5.5 \leq M_W \leq 8$	0.84*	1.67	799
Allmann and Shearer, 2009	Intraplate $5.5 \leq M_W \leq 8$	1.50*	1.46	61
Oth et al, 2010	Japan (crustal) $2.7 \leq M_{JMA} \leq 8$	1.1	1.38	1951
Rietbrock et al., 2012	UK	1.8	1.38	273
Edwards and Fah, 2012	Switzerland (foreland)	0.2	1.83	161
Edwards and Fah, 2012	Switzerland (alpine)	0.12	1.43	351
Shearer et al., 2006	Southern California $1.6 \leq M_L \leq 3.1$	0.52*	1.52	64800
Margaris and Hatzidimitriou, 2002	Greece $5.2 \leq M_W \leq 6.9$	6.3	0.57	18
Johnston et al., 1994	Intraplate	10	0.7	?

*Published results are divided by 3.95 to take into account the difference between a Madariaga (1976) corner frequency/source radius compared to that of Brune (1970,1971) and the difference in shear wave velocity.

Cotton, Archuleta, Causse, SRL 2012

Variability in GMPE's

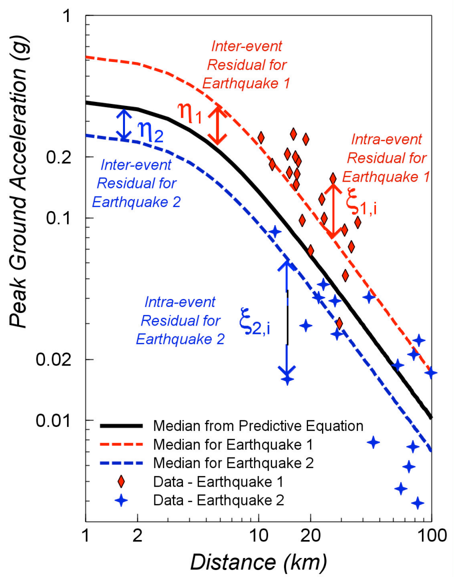
The logarithmic residuals are generally found to conform to a normal distribution with mean 0 and standard deviation σ_T



Cotton, Archuleta, Causse, 15WCEE 2012

Variability in GMPE's

Method 2 : analyze the observed ground-motion variability



Strasser *et al*. (2009)

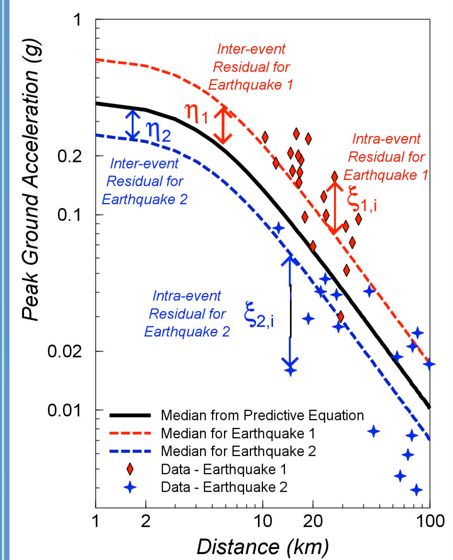
$$\sigma_T^2 = \sqrt{\sigma_b^2 + \sigma_w^2}$$

σ_T is the total variability

σ_b is the between-event (earthquake-to-earthquake) variability

σ_w is the within-event (record-to-record) variability

Method 2 : analyze the observed ground-motion variability



Strasser *et al*. (2009)

The between-event (earthquake-to-earthquake) variability (σ_b) is «on the first order» controlled by the stress-drop variability ($\sigma_{\ln(\Delta\tau)}$)

Inferred Stress Drop Variability from Recent GMPE's

GMPE	Database	Observed Between Event Variability (Natural log)	Inferred Stress Drop Variability (Natural log)
Rodriguez-Marek et al., 2011	Japan	0.49	0.59
Abrahamson and Silva, 2008 (M=5)	NGA	0.42	0.50
Abrahamson and Silva, 2008 (M=6)	NGA	0.36	0.43
Abrahamson and Silva, 2008 (M=7)	NGA	0.35	0.42
Akkar and Bommer, 2010	Europe	0.23	0.28
Boore and Atkinson, 2008	NGA	0.26	0.31
Campbell and Bozorgnia, 2008	NGA	0.22	0.26
Chiou and Youngs, 2008 (M=5)	NGA	0.34	0.41
Chiou and Youngs, 2008 (M=6)	NGA	0.30	0.36
Chiou and Youngs, 2008 (M=7)	NGA	0.26	0.31
Zhao et al., 2006	Japan	0.40	0.48

Cotton, Archuleta, Causse, SRL 2012

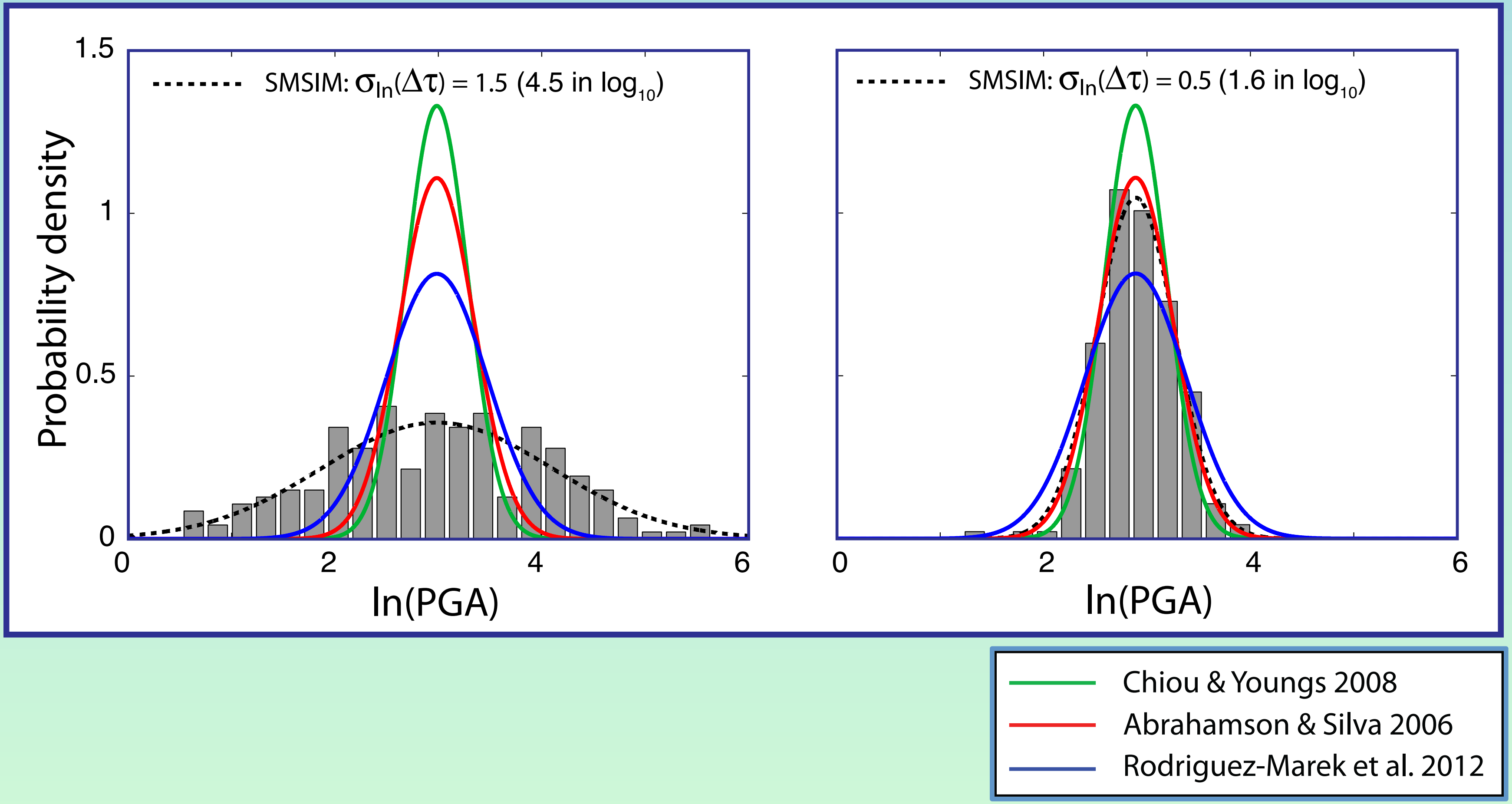
Comparing Variability from Source Studies and Inferred from GMPE's

Generally a factor ~3-4

Source Studies Stress Drop Variability (Log10)		GMPE Inferred Stress Drop Variability (Log 10)
3.85	No one-to-one comparison	1.36
3.36		1.15
3.18		0.99
3.18		0.97
4.21		0.64
3.29		0.71
3.50		0.60
1.31		0.94
1.61		0.83
		0.71
		1.11

Cotton, Archuleta, Causse, SRL 2012

Sigma: Its Effect on Ground Motion



Cotton, Archuleta, Causse, SRL 2012

Error in Corner Frequency Drives Variability in Stress Drop

$$\Delta\tau = 7M_0/16r^3$$

$$\Delta\tau = (7/16)\,M_0 f_C^3/(kV_S)^3$$

$$\log\Delta\tau = \log\left[7/16(kV_S)^3\right] + \log M_0 + 3\log f_C$$

$$\zeta = \log\Delta\tau$$

$$\sigma_\zeta^2 = \sigma_\chi^2 + 9\sigma_\lambda^2$$

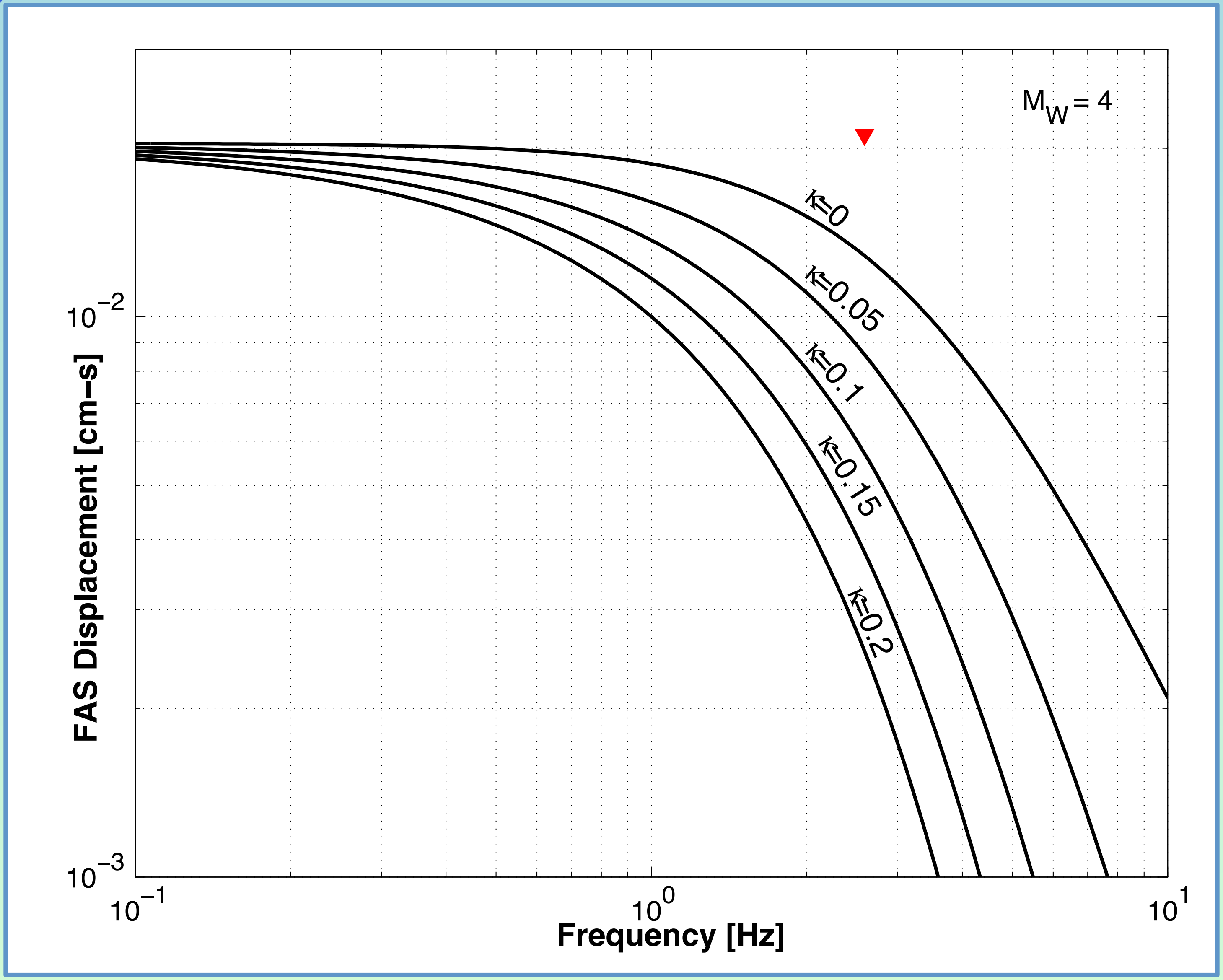
$$\sigma_{\log(\Delta\tau)}^2 = \sigma_{M_0}^2 + 9\sigma_{f_C}^2$$

$$\sigma_\zeta^2 = (0.3)_\chi^2 + 9(0.3)_\lambda^2$$

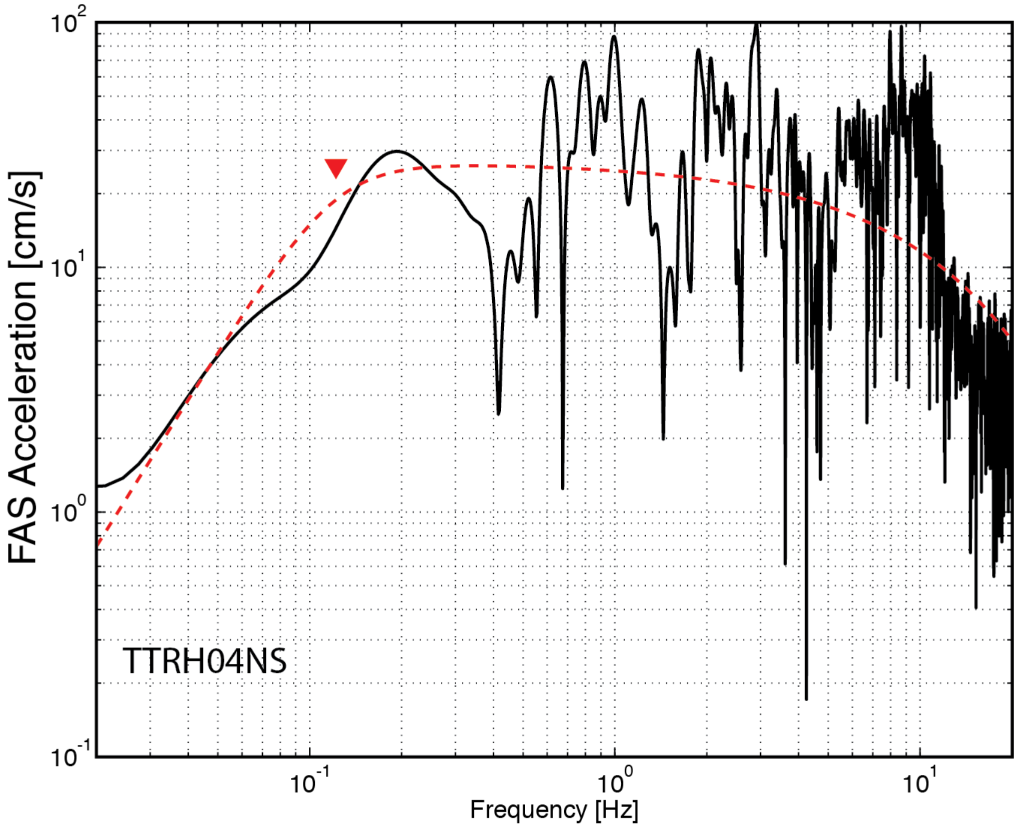
$$\sigma_\zeta^2 = 0.09 + 0.81$$

$$\sigma_\zeta = \sigma_{Log\Delta\tau} = 0.95$$

Effect of Attenuation



Stress Drop Based on Root-Mean-Square Acceleration



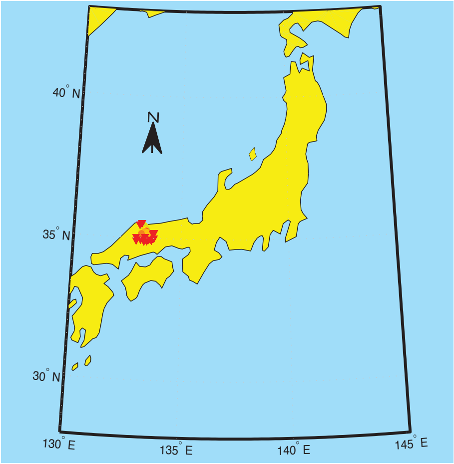
$$a_{rms} = \frac{(2\pi)^2 \Delta\tau}{106\rho R}\sqrt{\frac{f_{max}}{f_C}}$$

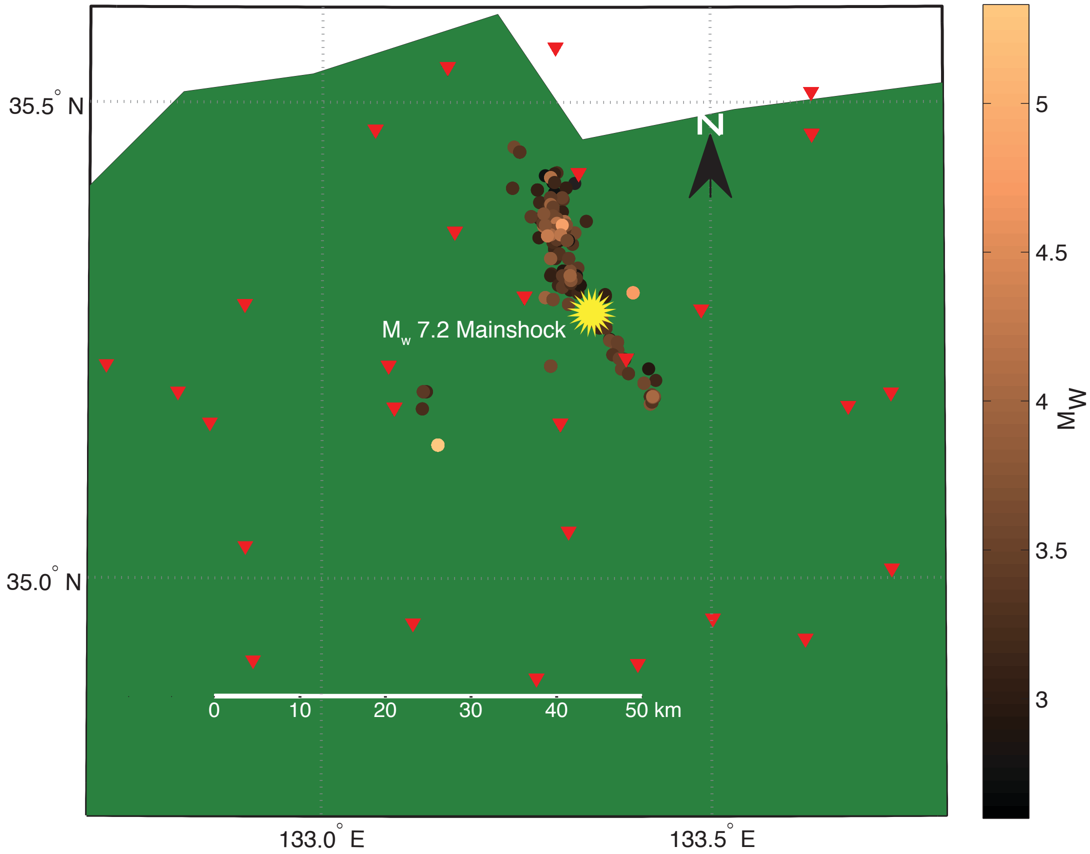
$$f_{max} = QV_S / \pi R$$

$$\Delta\tau = \frac{a_{rms} 106\rho R}{(2\pi)^2}\sqrt{\frac{f_C}{f_{max}}}$$

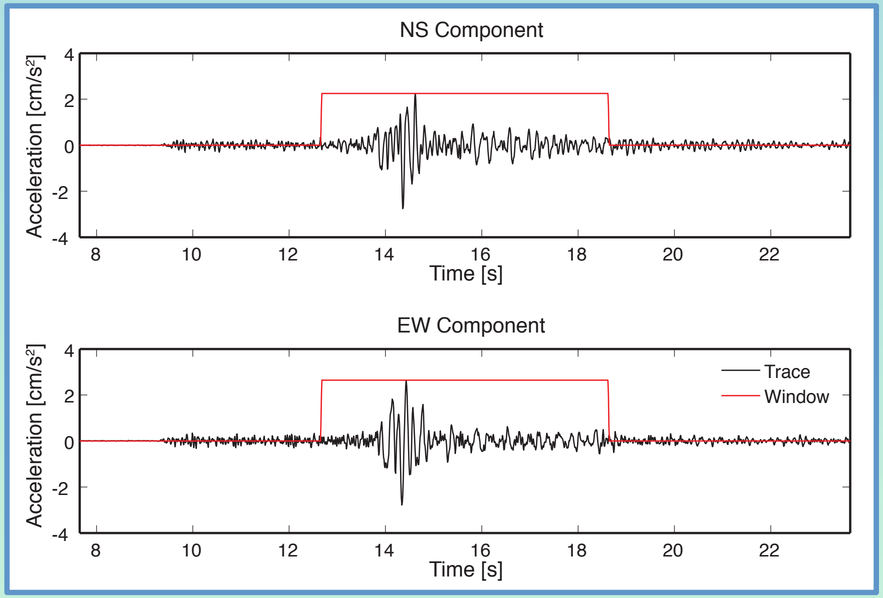
Hanks, 1979 JGR; McGuire and Hanks, 1980 BSSA

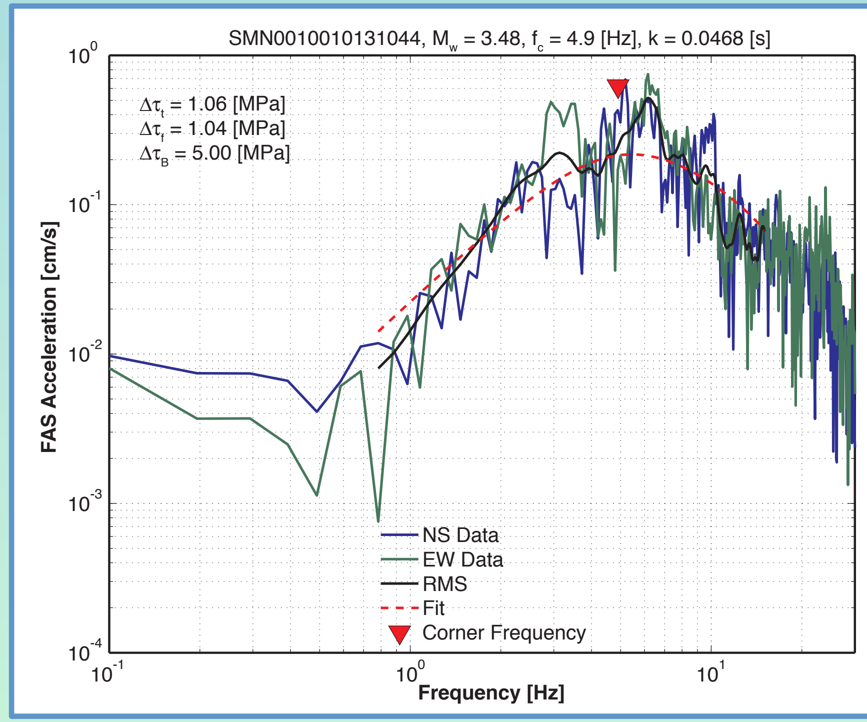
Map of Tottori Earthquake



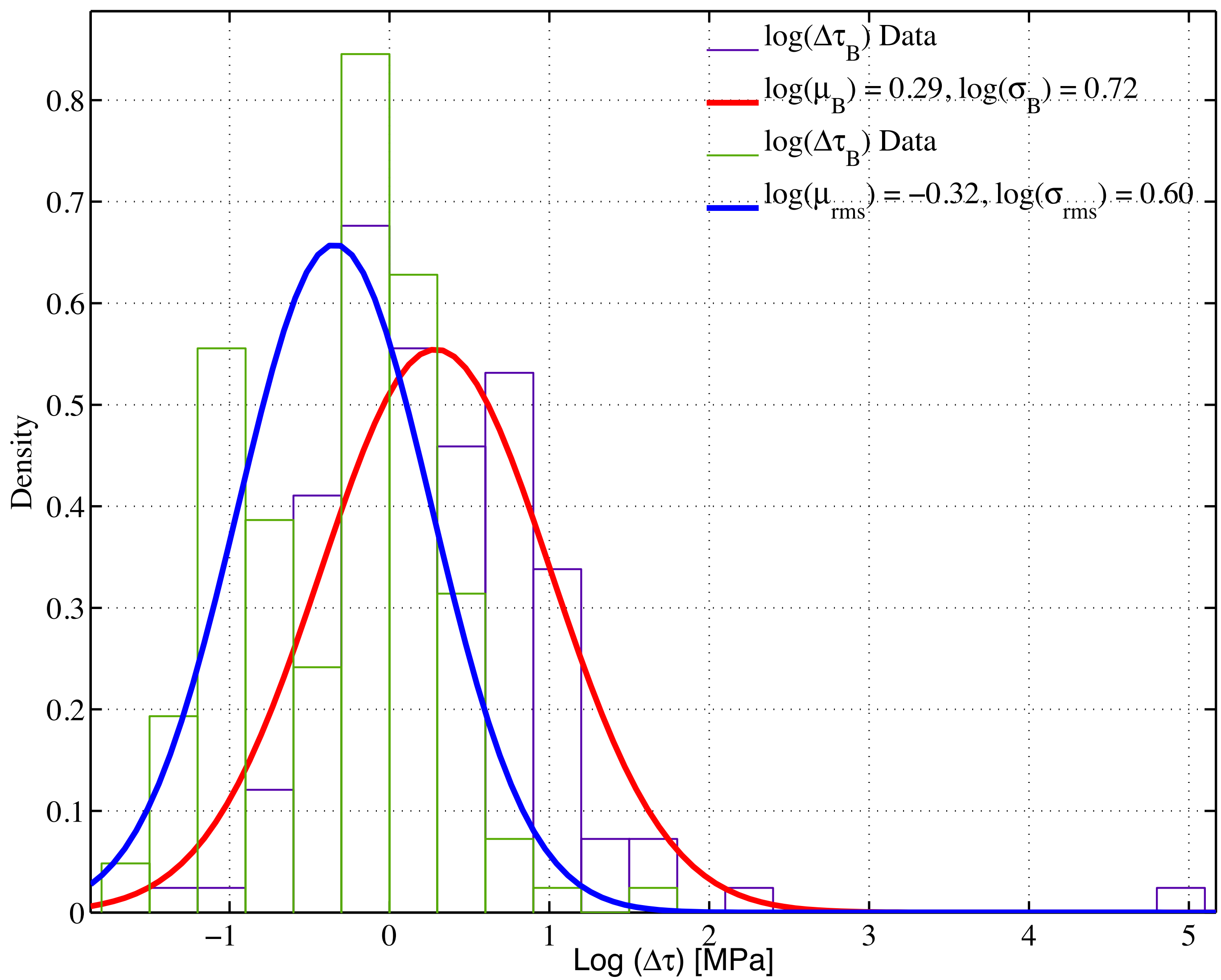


Root-Mean-Square Acceleration





Tottori: Stress Drop Distribution



Crempien and Archuleta, SCEC 2012

Summary

- The real variability in stress drop is less than that found using the standard Brune analysis of the seismic spectrum, i.e., corner frequency and spectral level at "0" frequency.
- Variability of the stress drop is real. There is a regional dependence of the mean and could be in the variability.
- In theory, using root-mean-square acceleration should provide a more reliable estimate of the variability in stress drop.
- Attenuation (path and site) plays a significant role in both the standard Brune analysis as well as analysis of the root-mean-square spectrum.

The End

Variability of Stress Drop Derived from Variability of GMPE

$$PGA = a_{rms}\sqrt{2\ln\left(2 f_{\max} T_D\right)} = a_{rms}\sqrt{2\ln\left(\frac{2 f_{\max}}{f_C}\right)}$$

$$T_D = 1/\, f_C$$

$$PGA = \frac{(2\pi)^2 \Delta\tau}{106\rho R}\sqrt{\frac{f_{\max}}{f_C}}\sqrt{2\ln\left(\frac{2 f_{\max}}{f_C}\right)}$$

$$\Delta\tau = (7/16)\, M_0 f_C^3 \Big/ \left(kV_S\right)^3$$

shows

$$\sqrt{1/\, f_C} \propto 1/\Delta\tau^{1/6}$$

Thus

$$PGA \propto \Delta\tau^{5/6}$$

$$\sigma_{\Delta\tau} = (6/5)\sigma_{PGA}$$

Cotton, Archuleta, Causse, SRL 2012