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Spatial Correlation of Strong Ground Motion and Uncertainty in Earthquake Loss Estimation

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Extended Abstract

A key element of seismic hazard assessment (SHA) and seismic loss estimation is a consideration of uncertainties, which are classified as epistemic and aleatory. The epistemic uncertainty reflects the incomplete knowledge of the nature of all inputs to the assessment and variability of interpretation of available data. Epistemic uncertainty can be incorporated into SHA using the logic tree method.

Aleatory uncertainty, which is related to the inevitable unpredictability of nature of the ground motion parameters, is mainly quantified in SHA through the use of the standard deviation of the scatter of the data about the ground motion prediction equations. It has become common practice to separate the total aleatory variability into two independent components. The components represent (1) the earthquake-to-earthquake (*inter-event* or *between-earthquake*) variability; (2) the site-to-site (*intra-event* or *within-earthquake*) variability.

The between-earthquake variability emphasizes that earthquake ground motion at different sites caused by the same earthquake must have something in common. The within-earthquake variability considers that earthquake ground motion for a given event at different sites must vary to some extent. The within-earthquake variability is determined mostly by peculiarities of propagation path and local site conditions, while the between-earthquake component depends on variations of earthquake source characteristics.

The spatial correlation of earthquake ground motion is determined by relation between the components of variability. The ground motion parameter a at n locations during m earthquakes is represented by

$$\log a_{i,j} = f(e_i, s_{i,j}, \beta) + \eta_i + \varepsilon_{i,j} \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n; \tag{1}$$

where e_i denotes variables that are properties of the earthquake source, $s_{i,j}$ are the properties of site location j during earthquake i; β is the vector of parameter estimates; f is a suitable function. The error random variables η_i and $\varepsilon_{i,j}$ represents the between-earthquake and within-earthquake components of variability (independent and normally distributed with variances ${\sigma_\eta}^2$ and σ_ε^2). The value of η_i is common to all sites during particular earthquake i, and the value of $\varepsilon_{i.j}$ depends on the site. Assuming independence of the two random terms, the total aleatory variance ${\sigma_T}^2$ is given by ${\sigma_T}^2 = \sigma_\eta^2 + \sigma_\varepsilon^2$.

Let us consider error random variables (normally distributed with zero mean and standard deviations σ_η and σ_ε) at two sites $x = \eta + \varepsilon_x$ and $y = \eta + \varepsilon_y$. The joint probability density function follows bivariate normal distribution with zero means, standard deviation σ_T and correlation coefficient $\rho = \frac{\sigma^2{}_\eta}{\sigma_\eta^2 + \sigma_\varepsilon^2}$. Two close sites may exhibit correlation of ground motion during an earthquake due to commonality of wave paths (*within-earthquake site-to-site* correlation), which depends on the sites separation distance. For earthquake i and site j the total correlation in $\varepsilon_{i,j}$ values is

$$\rho_T(\Delta) = \frac{\sigma_\eta^2 + \rho_\varepsilon(\Delta)\sigma_\varepsilon^2}{\sigma_T^2} = \rho_\eta + \rho_\varepsilon(\Delta)\left(\frac{\sigma_\varepsilon^2}{\sigma_T^2}\right) \tag{2}$$

where $\rho_\varepsilon(\Delta) = \rho_{\varepsilon i,j,1;\varepsilon i,j,2}(\Delta)$ is the empirical correlation coefficient calculated for within-earthquake $\varepsilon_{i,j}$ values separated by a distance Δ; $\rho_\eta = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\varepsilon^2}$ is the between-earthquake correlation coefficient.

The within-earthquake site-to-site correlation may be evaluated using the data from a dense network. The correlation usually is expressed in exponential form $\rho_\varepsilon(\Delta) = \exp(-a\Delta^b)$. So-called "correlation distance" may be considered as a characteristic of the correlation. Correlation distance shows site-to-site distance, for which the correlation coefficient $\rho_\varepsilon(\Delta)$ reduces up to $1/e = 0.368$ (Figure 1).

In ground motion models, the parameter of motion Y generated by earthquake i at a site j is estimated as a lognormally distributed random variable $\ln Y_{ij} = N(\ln \overline{Y}_{ij}, \sigma^2)$, where $\ln \overline{Y}_{ij} = f(M, R, site)$. Besides mean value of ground motion $\overline{Y}_{ij}$, we need to generate standard normal variates (errors) η_i and $\varepsilon_{i,j}$.

For generation of k-sites random field of ground motion values that are spatially correlated, it is necessary to generate a Gaussian vector of correlated, standard normal variables (total residual term) X = [X_1, X_2, …., X_k] with a symmetric correlation matrix Σ, or

$$\Sigma = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1k} \\ \rho_{21} & 1 & \vdots & \rho_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{k1} & \rho_{k2} & \cdots & 1 \end{pmatrix} \quad (3)$$

where ρ_{ij} is the empirical correlation coefficient (Equation 2) calculated for the sites separated by a distance Δ. These X_j values are added to the mean ground motion term to obtain realization of spatially correlated ground motions.

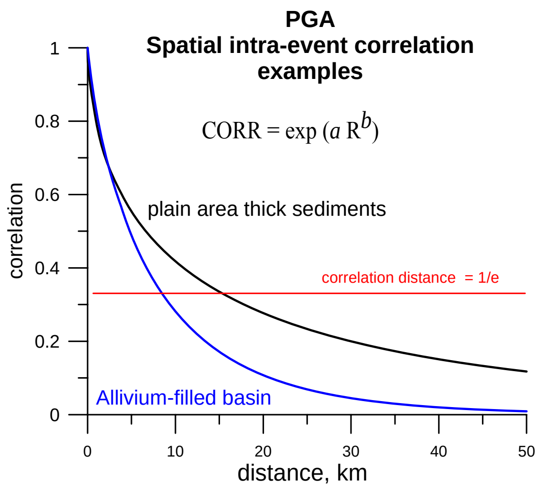

Figure 1 Example of within-earthquake correlation functions estimated using the data from Taiwan.

The correlation is very important when calculating ground motion parameters along a wide area, for example, in estimation of seismic hazard and loss for building assets and spatially distributed systems (lifelines). In this study we analyzed influence of correlation (between-earthquake and site-to-site within-earthquake) on estimations of aggregated loss for a portfolio (widely located constructions of several types) for the case of a particular single event, so called "scenario" earthquake.

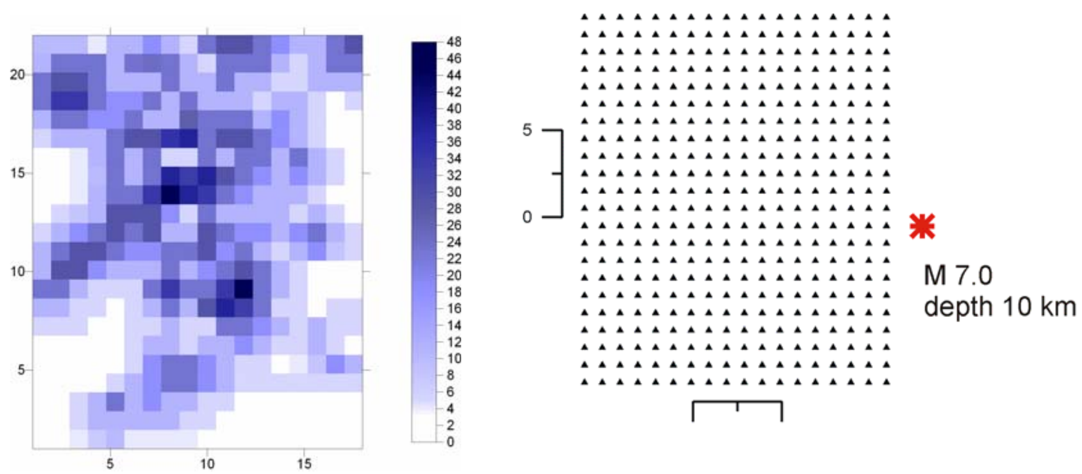

Figure 2. Distribution of total number of buildings within the cells 1 km x 1 km (left) and location of the scenario earthquake (right).

The portfolio has been constructed based on three types of buildings, namely: (1) concrete 1-3 stories; (2) steel frame 4-7 stories; (3) frame with shear wall more than 8 stories. The whole area of 22 km x 18 km has been divided into the cells of 1 km x 1 km, and different number of buildings has been assigned, more or less randomly, to every cell. The total number of buildings is 5478; and the total replacement cost is 5077 Mln \$. Figure 2 shows distribution of buildings along the considered territory and location of the scenario earthquake.

The Monte-Carlo technique was used for generation of correlated PGA values (10000 generations) for every cell. We consider various values of the between-earthquake correlation as well as different correlation distances in the site-to-site correlation. Examples of strong motion distribution estimated using some considered parameters of correlation are shown in Figure 3. For a given ground motion amplitude generated for the cell, a single loss value was estimated for every building using the building-specific fragility curves. We did not consider uncertainty related to monetary parameters of loss. These loss values estimated for all buildings within particular cell were summarized to obtain the cell-specific (CS) loss. Then a single total loss amount for given generation of the ground motion distribution was obtained as the sum of CS losses. The generated set of total loss values (10000 generations) is used for estimation of Probability Density Function (PDF) and Cumulative Probability Function (CPF) and analysis of parameters of loss distribution.

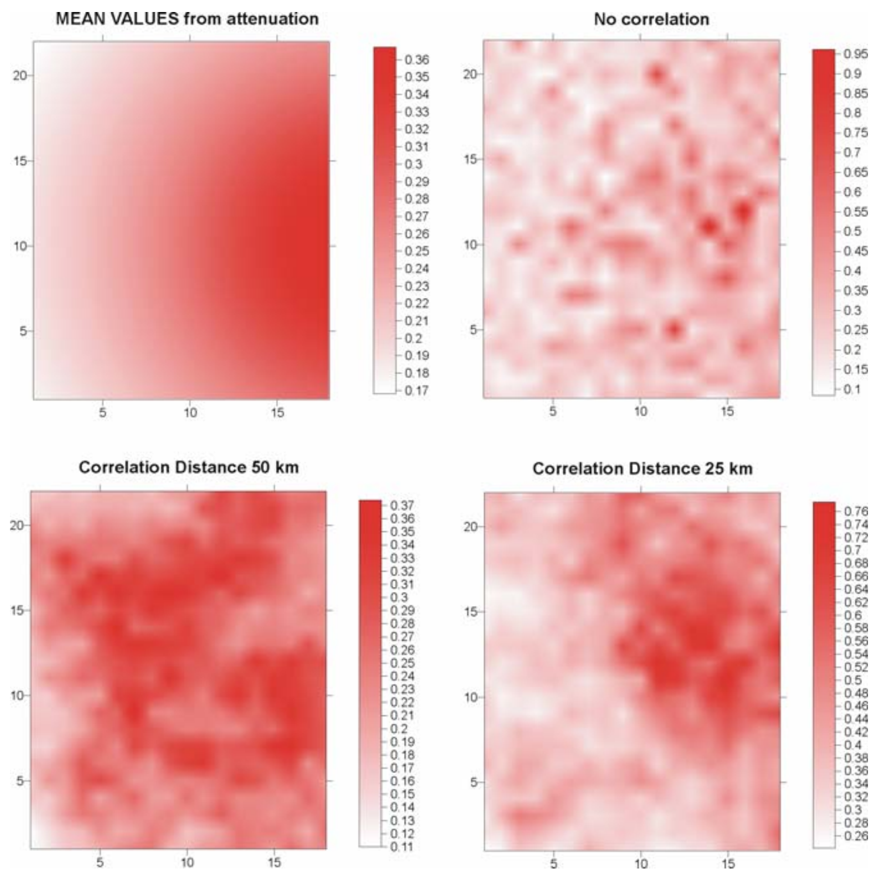

Figure 3. Examples of strong motion distribution along considered territory estimated using various parameters of correlation. Total variance $\sigma_T^{\,2} = 0.16$ units of natural logarithm; ratio $\sigma_\eta / \sigma_\varepsilon = 0.3$ (between-earthquake variation / within-earthquake variation).

We considered the following parameters of loss distribution, namely: mean value of loss $L_{MEAN} = \sum_{i=1}^{N} L_i \Big/ N$, where L_i is the total loss value for i simulation, N is the total number of simulations; standard deviation of the loss distribution σ_{LD}; coefficient of variation $CV = \sigma_{LD} / L_{MEAN}$; median value, for which the cumulative probability function (CPF) equals to 0.5; particular values of loss with certain probability of not being exceeded, e.g. 90% (P_{90}) or 95% (P_{95}).

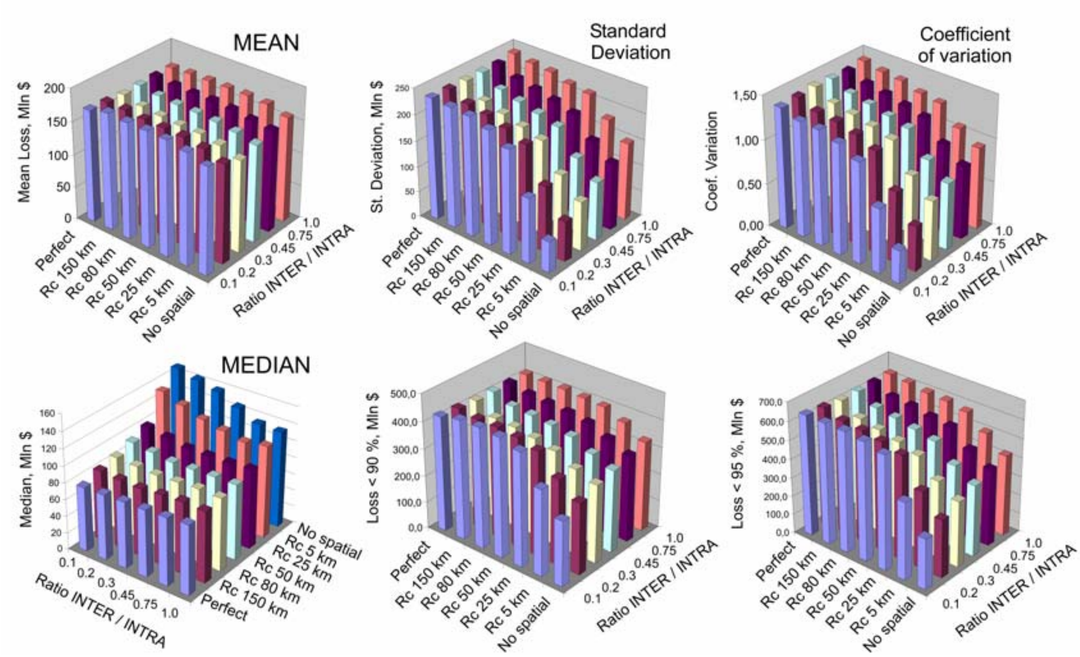

Figure 4. Parameters of loss distribution estimated for various correlation distances *Rc* and ratios $\sigma_\eta / \sigma_\varepsilon$ (between-earthquake or inter-event variation / within-earthquake or intra-event variation).

Analysis of the results of the loss modeling (Figure 4) leads to the following conclusion

1. The proper consideration of (a) ratio $\sigma_\eta / \sigma_\varepsilon$ between inter-event and intra-event components of uncertainty and (b) parameters of spatial correlation (correlation distance) is very important when estimating seismic losses for distributed portfolios.
2. The increase of contribution of inter-event (or between-earthquake) variability leads to the larger variation of possible loss.
3. The increase of contribution of intra-event (or within-earthquake) variability leads to the larger mean and median values of possible loss.
4. Large correlation distances in spatial correlation leads to the increase of variability of possible loss.
5. Loss estimations, which were obtained without consideration of ground motion variability and correlation, e.g. using mean values of ground motion attenuation, could not be considered as the mean or the median estimations.