Estimating Aquifer-Storage Change from Temporal Changes in Gravity

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Accurate measurements of aquifer-storage change are difficult to obtain. The water budget of an aquifer can be expressed as:

$$Q_{in} - Q_{out} = \Delta\text{Storage}$$

where Q_{in} is flow into the aquifer (recharge and ground-water underflow from up gradient areas), Q_{out} is flow out of the aquifer (discharge from wells and ground-water underflow to down gradient areas), and ΔStorage is the change in water storage of the aquifer. If inflow and outflow could be accurately measured, then the change in storage would be accurately known. In discussing the Lower Cañada del Oro basin in Arizona, Pool (1999) stated:

> Accurate estimates of recharge, ground-water underflow, and aquifer-storage change are difficult to obtain using traditional methods. Natural ground-water recharge occurs across a large area and cannot be directly measured. Ground-water underflow, into and out of the basin, can be estimated; however, a high degree of uncertainty is inherent in the estimated values. Aquifer-storage change can be indirectly estimated assuming water-level change is measured and specific yield, which is the amount of water that the aquifer yields per unit change in water level [per unit surface area], is known. Indirect estimates of aquifer-storage change using water-level change and specific yield also have a high degree of uncertainty because specific yield is poorly known.

The availability of a simple method that can accurately measure aquifer-storage change would be a significant improvement.

Aquifer-storage change can be estimated using the equation:

$$M = \frac{1}{2\pi G} \cdot \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \Delta g(x, y)\,dxdy$$

where M is the mass flux, G is the universal gravitational constant, and Δg is the temporal change in gravity for the surface area element dxdy (Cole, 1991). If the gravity anomaly is contained within the area of summation, then this integral can be approximated as:

$$M \approx \frac{1}{2\pi G} \cdot \sum \left(\Delta g_M \cdot \Delta A\right)$$

where Δg_M is the change in gravity that is caused by the change in mass for the area element ΔA. The change in gravity that is caused by a change in mass can be written as:

$$\Delta g_M = \Delta g - \gamma \cdot \Delta h$$

where Δg is the change in gravity, γ is the vertical gravity gradient, and Δh is the change in elevation. Inserting this equation into the approximation and dividing by the density of the mass gives the volume:

$$V \approx \frac{1}{2\pi G} \cdot \frac{1}{\rho} \cdot \sum \left((\Delta g - \gamma \cdot \Delta h) \cdot \Delta A \right)$$

If each area element is of equal size, then this can be expressed as:

$$V \approx \frac{1}{2\pi G} \cdot \frac{\Delta A \cdot n}{\rho} \cdot \frac{\sum (\Delta g - \gamma \cdot \Delta h)}{n}$$

where n is the number of area elements. Finally, the volume can be approximated as:

$$V \approx 23.9 \cdot \frac{A}{\rho} \cdot \left(\overline{\Delta g} - \overline{\gamma \cdot \Delta h} \right) \quad [m^3]$$

where A is the study area (m^2), ρ is the density (kg/m^3), $\overline{\Delta g}$ is the measured average change in gravity (μGal), and $\overline{\gamma \cdot \Delta h}$ is the measured average change in gravity (μGal) caused by the change in elevation Δh.

The uncertainty budget of this method needs to be analyzed and can be considered in 5 parts: (1) the edge problem, (2) the density of the mass, (3) the size of the study area, (4) the average change of gravity, and (5) the average change of gravity caused by an elevation change. The edge problem refers to the impracticality of integrating to infinity (Cole, 1991; Schmerge, 2003); it is possible to compensate for this problem by applying a correction (Grant and West, 1965). The density of the mass (assumed to be fresh water) is about 1000 ± 5 kg/m^3; this assumption is valid if other mass movement (such as magma, mining, and construction) is not significant. The size of the study area should be able to be measured to more than 3 significant figures, so its uncertainty is assumed to be insignificant.

The uncertainty of the average change in gravity can be significant. Cole (1991) concluded that it would be necessary to measure absolute gravity for this method to yield accurate results. Nevertheless, without measuring absolute gravity, Pool and Schmidt (1997) used a relative gravimeter to estimate aquifer-storage change near Rillito Creek in Arizona; they simply assumed that gravity on bedrock was constant, but they did not consider the effect of this assumption on the uncertainty of the aquifer-storage change. The effect can be seen by considering 2 cases in which absolute gravity was not measured. In case 1, Pool (1999) estimated an aquifer-storage loss of about 1.4×10^7 m^3 between July 1997 and September 1998 in an area of about 1.2×10^8 m^2 in the Lower Cañada del Oro basin in Arizona. In case 2, Schmerge (2003) estimated an aquifer-storage loss of about 1.3×10^8 m^3 in the Lower Cañada del Oro basin (area of 1.4×10^8 m^2) between October 1998 and October 2002. Absolute gravity was measured at 3 sites on bedrock within about 20 km of the Lower Cañada del Oro basin between 1998 and

2005; the average change in gravity at these sites between time t_i and time t_j (for $t_j > t_i$) is about 0.0 ± 5.5 μGal. If these 3 sites are representative of all bedrock in the area, then the uncertainty in the assumption that gravity is constant on bedrock is 5.5 μGal, which causes an uncertainty of 1.6×10^7 m^3 for the aquifer-storage change for case 1, and 1.9×10^7 m^3 for case 2. For case 1, the assumption that gravity is constant on bedrock introduced an uncertainty that is larger than the estimated aquifer-storage change. For case 2, the estimated aquifer-storage loss is about 7 times larger than the estimated uncertainty.

The uncertainty associated with the change in elevation may also be significant. Land-surface deformation (both permanent and reversible) commonly occurs in alluvial basins due to ground-water withdrawal and recharge; reversible deformation commonly results in land surface displacements of several centimeters in the vertical direction (Amelung and others, 1999; Galloway and others, 1999; Bawden and others, 2001). The vertical gravity gradient is about -3 μGal/cm; therefore, unmeasured deformation may cause a significant error in the estimation of aquifer-storage change. For both previously mentioned cases, surface deformation was assumed to be insignificant due to the results of GPS measurements of a few sites in the study area (Pool, 1999; Schmerge, 2003). However, these few sites were not necessarily representative of the entire area. Also, the GPS measurements were not conducted simultaneously with the gravity measurements; therefore, even if there was no significant change in elevation between the GPS surveys, there may have been a significant change in elevation between the gravity surveys. Multiple GPS surveys of all the sites in the Lower Cañada del Oro basin were recently conducted. Between February 2004 and January 2005 the measured average deformation was -1.6 cm, and between January 2005 and September 2005 the measured change was -0.7 cm (It is presently not clear whether this deformation is permanent, reversible, or some combination of both). Based upon these considerations, it is assumed possible that some significant deformation may have occurred within the time periods of both cases 1 and 2. For case 1, an uncertainty of 1 cm for the average change in elevation results in an uncertainty of about 0.9×10^7 m^3 for the aquifer-storage change; this one error is more than half the size of the estimate. For case 2, an elevation uncertainty of 1 cm results in an uncertainty of about 1.0×10^7 m^3 for the aquifer-storage change.

It may be possible to use changes in gravity to accurately measure aquifer-storage change, but using just a relative gravimeter can clearly result in large uncertainties. For case 1, the total uncertainty was larger than the estimated aquifer-storage change. However, case 2 demonstrates that it may be possible to accurately estimate aquifer-storage change using temporal changes in gravity. The significant difference between these cases is that the estimated aquifer-storage loss of case 2 is about 9 times larger than that of case 1, but the estimated uncertainties for both cases are about the same. In this analysis, it was assumed that the observed changes in gravity on bedrock were typical, but this may not be correct; this assumption should be an improvement over the assumption that changes in gravity on bedrock are insignificant, but the uncertainty can be reduced by simultaneously measuring relative gravity and absolute gravity. Also, it was assumed that the deformation uncertainty was 1 cm, but if undetected permanent land subsidence was occurring then the error could have been much larger than this. Large errors caused by deformation can be reduced by simultaneously measuring gravity and elevation.

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