Seismic Excitation of Earth Rotation and Gravitational Field Changes

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Introduction

- Model coseismic effect of earthquakes
  - Earth orientation
    - Length of day
    - Polar motion excitation
  - Gravitational field
    - Time series for harmonic degrees 2–5
    - Degree amplitude for harmonic degrees 2–100
  - Great events that occurred during 1950–1989
  - 21600 smaller events that occurred during 1977–2004
    - All earthquakes having magnitude $m_b \geq 5.5$
- Compare modeled results with observations
  - Observed Earth orientation variations
  - GRACE uncertainties

Earthquake Excitation

- Earthquake source modeled as point dislocation
  - Within SNREI Earth model
    - Isotropic version of PREM
  - Located at observed hypocenter of earthquake
  - Representing observed source properties of earthquake
    - Focal mechanism and moment
    - Harvard centroid-moment tensor (CMT) solution
  - Occurring instantaneously
- Earthquake generates displacement field
  - Global in extent
    - Although only local displacements are measurable

Earthquake Excitation (cont.)

- Compute global displacement field
  - By summing normal modes
    - Normal mode eigenfunctions are basis functions of displacement field
  - In static limit (as time $t \Rightarrow \infty$)
    - Seismic waves generated by earthquake not modeled
- Integrate static displacement field to compute
  - Static change in Earth's inertia tensor
    - Earth orientation changes directly proportional to changes in Earth's inertia tensor
  - Static change in Earth's gravitational field

Polar Motion Excitation

- **Conservation of angular momentum expressed within rotating, body-fixed reference frame**

$$\frac{\partial \mathbf{L}}{\partial t} + \boldsymbol{\omega} \times \mathbf{L} = \boldsymbol{\tau}$$

where the angular momentum vector $\mathbf{L} = \mathbf{I} \cdot \boldsymbol{\omega} + \mathbf{h}$

- **Assume rotation is small perturbation from state of uniform rotation at rate Ω. Keeping terms to first order results in long period Liouville equation**

$$\mathrm{m}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathrm{m}}{\partial t} = \psi(t) = \chi(t) - \frac{i}{\Omega} \frac{\partial \chi}{\partial t}$$

where: $\mathrm{m} \equiv (\omega_1 + i\, \omega_2)/\Omega$ (terrestrial location of rotation pole)

$\psi(t)$, $\chi(t)$ are the polar motion excitation functions

σ_{cw} is complex-valued frequency of Chandler wobble

- **Written in terms of reported polar motion parameters:**

$$\mathrm{p}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathrm{p}}{\partial t} = \chi(t) = \frac{1.61}{\Omega\,(C-A)} \left[\mathrm{h}(t) + \frac{\Omega\, \mathrm{c}(t)}{1.44} \right]$$

where $\mathrm{p}(t) = x_p(t) - i\, y_p(t)$

$\mathrm{c}(t) = c_{13}(t) + i\, c_{23}(t)$

Length of Day

- **Conservation of angular momentum expressed within rotating, body-fixed reference frame**

$$\frac{\partial \mathbf{L}}{\partial t} + \omega \times \mathbf{L} = \tau$$

where the angular momentum vector $\mathbf{L} = \mathbf{I} \cdot \mathbf{w} + \mathbf{h}$

- **Assume rotation is small perturbation from state of uniform rotation at rate Ω. Keeping terms to first order yields:**

$$\dot{m}_3(t) = \dot{\psi}_3(t) = -\dot{\chi}_3(t)$$

where:

$$m_3(t) \equiv (\omega_3(t) - \Omega)/\Omega = \frac{\mathrm{d(UT1-TAI)}}{\mathrm{d}t} = -\frac{\Delta\Lambda(t)}{\Lambda_o}$$

$$\chi_3(t) = \frac{1}{C_m \ \Omega}\left[h_3(t) + 0.756 \ \Omega \ c_{33}(t)\right]$$

- **Changes in length-of-day $\Delta\Lambda$ (t) are caused by changes in axial component of relative angular momentum $h_3(t)$ and by changes in 3,3 component of the Earth's inertia tensor $c_{33}(t)$:**

$$\Delta\Lambda(t) = \frac{\Lambda_o}{C_m \ \Omega}\left[h_3(t) + 0.756 \ \Omega \ c_{33}(t)\right]$$

where: Λ_o **is the nominal length of day (86400 seconds)**

C_m **is the greatest principal moment of inertia of the Earth's crust and mantle**

0.756 accounts for yielding of the crust and mantle

Earthquake Excitation

- **Earthquakes do not load the Earth:**

$$\psi_x + i\,\psi_y = \frac{1.61}{I_{zz} - I_{xx}} \left[\Delta I_{xz} + i\,\Delta I_{yz} \right]$$

$$\Delta l.o.d. = \frac{l.o.d.}{I_{zz}} \Delta I_{zz}$$

- **Inertia tensor:**

$$\mathbf{I} = \int \rho(\mathbf{r}) \left[(\mathbf{r}\cdot\mathbf{r})\,\mathbf{1} - \mathbf{r}\mathbf{r} \right] dV$$

- **First order perturbation:**

$$\mathbf{r} \rightarrow \mathbf{r} + \mathbf{u}$$

$$\Delta\mathbf{I} = \int \rho_o(r) \left[2\,(\mathbf{r}\cdot\mathbf{u})\,\mathbf{1} - (\mathbf{u}\mathbf{r} + \mathbf{r}\mathbf{u}) \right] dV$$

Gravitational Field

- **Stokes coefficients C_{lm} and S_{lm}:**

$$U(r_o) = \frac{GM}{r_o} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \left(\frac{a}{r_o}\right)^l \tilde{P}_{lm}(\cos\theta_o) \; (C_{lm} \cos m\phi_o + S_{lm} \sin m\phi_o)$$

$$C_{lm} + i\, S_{lm} = \frac{N_{lm}}{Ma^l} \int_{V_o} r^l \, Y_{lm}(\theta, \phi) \, \rho(\mathbf{r}) \, dV$$

- **Inertia tensor $\mathbf{I}$:**

$$\mathbf{I} = \int \rho(r) \, (r^2 \, \mathbf{l} - r\, r) \, dV$$

- **Relation between inertia tensor and Stokes coefficients**

$$I_{xz} + i\, I_{yz} = -\sqrt{5/3}\, Ma^2 \, (C_{21} + i\, S_{21})$$

$$I_{zz} = \tfrac{1}{3}[\mathrm{Tr}(\mathrm{I}) - 2\sqrt{5}\, Ma^2 \, C_{20}]$$

- **Mass redistribution**
  - Changes the Earth's gravitational field (Stokes coefficients)
    - Including the second-degree harmonics
  - Changes the Earth's inertia tensor
    - Which are related to the second-degree gravitational field harmonics
  - Changes the Earth's rotation

Earthquake-Induced Displacement

- **Equation of motion:**

$$\nabla \cdot \boldsymbol{\tau} + \mathbf{F} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

- **Mode sum:**

$$\mathbf{u}(\mathbf{r}, t) = \sum_k a_k(t)\, \mathbf{u}_k^*(\mathbf{r})$$

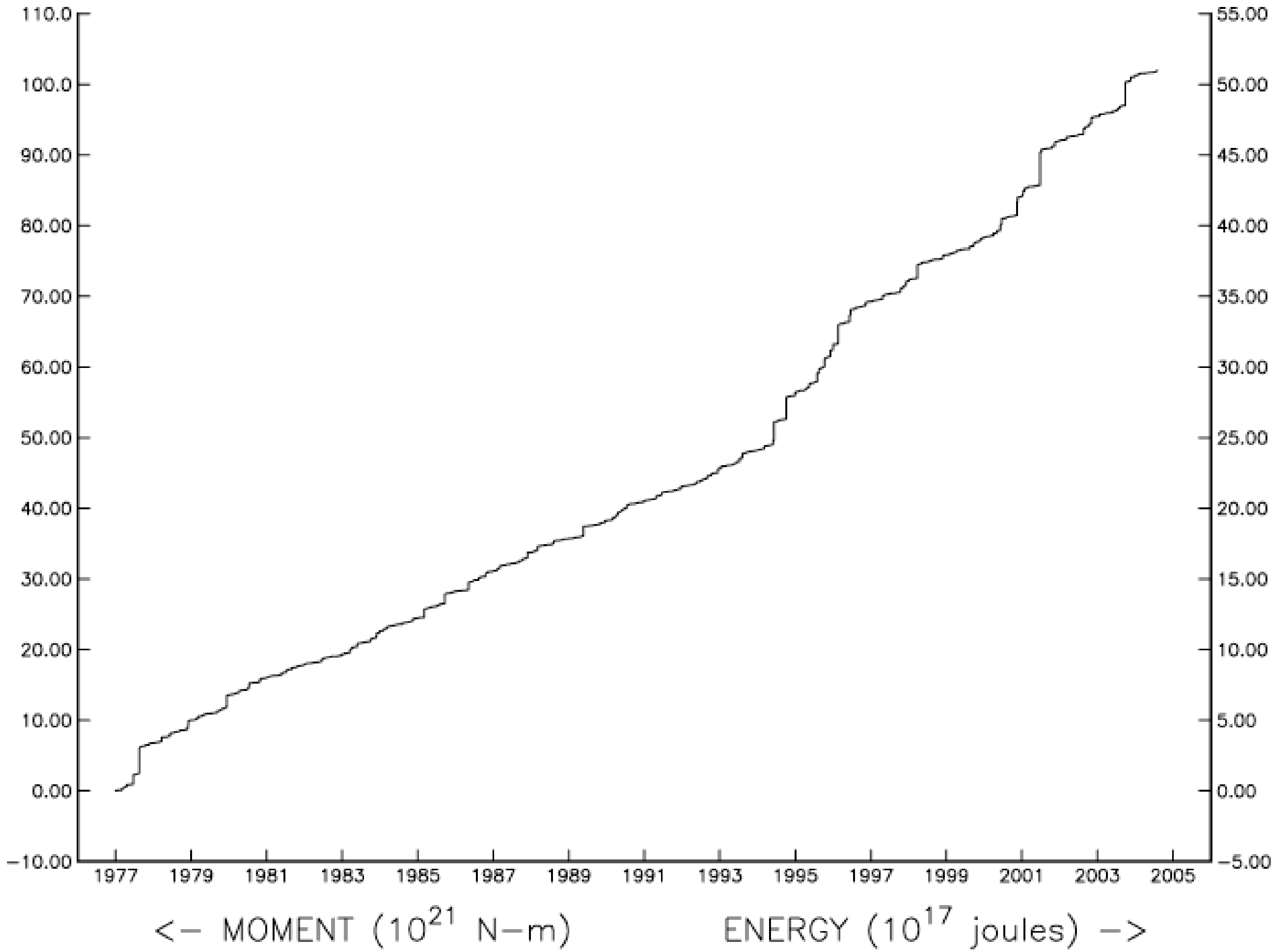
- **Static limit:**

$$a_k(\infty) = \frac{M_o}{\omega_k^2}\, \hat{\mathbf{M}}:\mathbf{E}_k(\mathbf{r}_o)$$

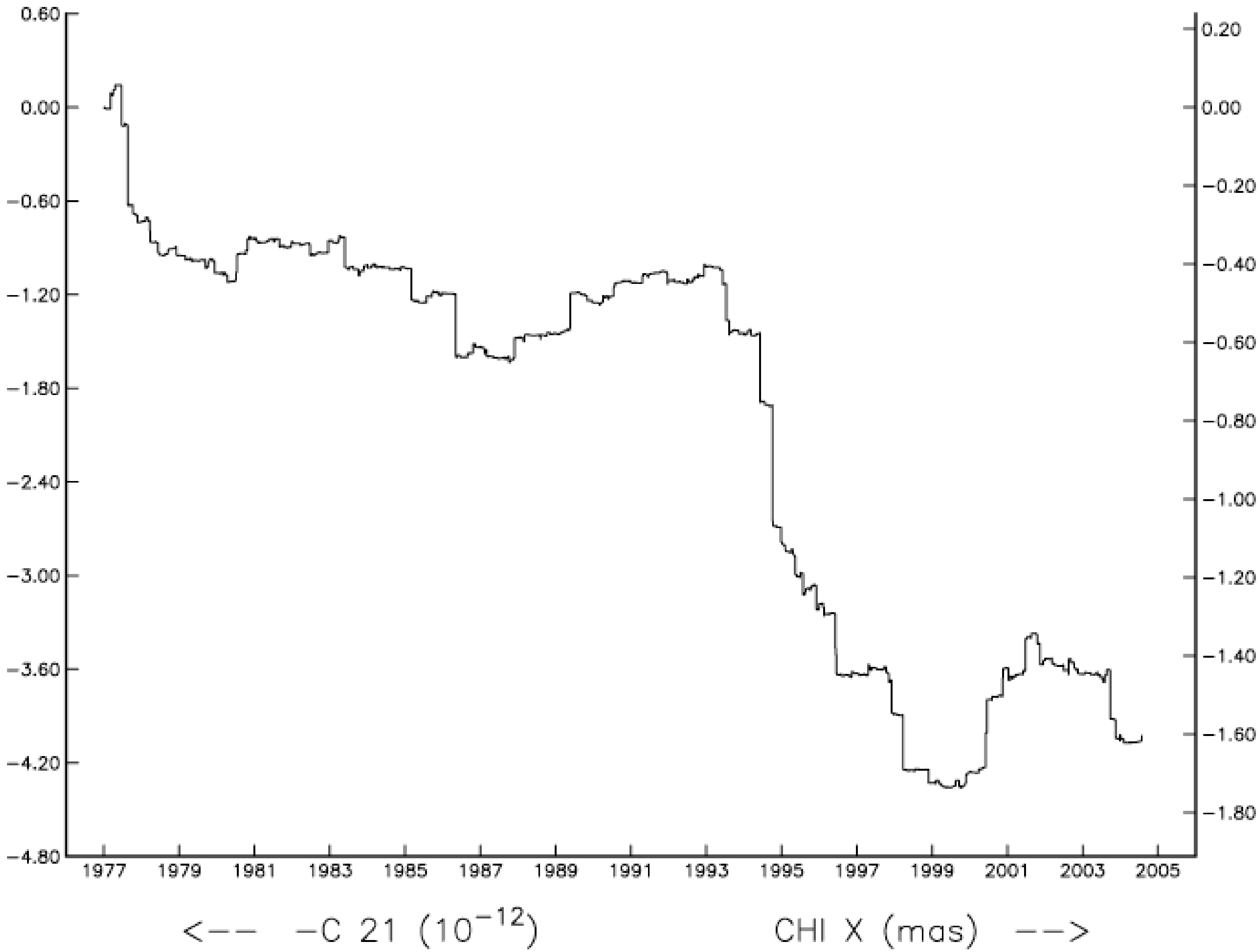
$$\mathbf{u}_k(\mathbf{r}) = {}_nU_l(r)\, Y_{lm}(\theta, \phi)\, \hat{\mathbf{r}} + {}_nV_l(r) \frac{\partial Y_{lm}}{\partial \theta}\, \hat{\boldsymbol{\theta}}$$

$$+ {}_nV_l(r) \frac{1}{\sin\theta} \frac{\partial Y_{lm}}{\partial \phi}\, \hat{\boldsymbol{\phi}}$$

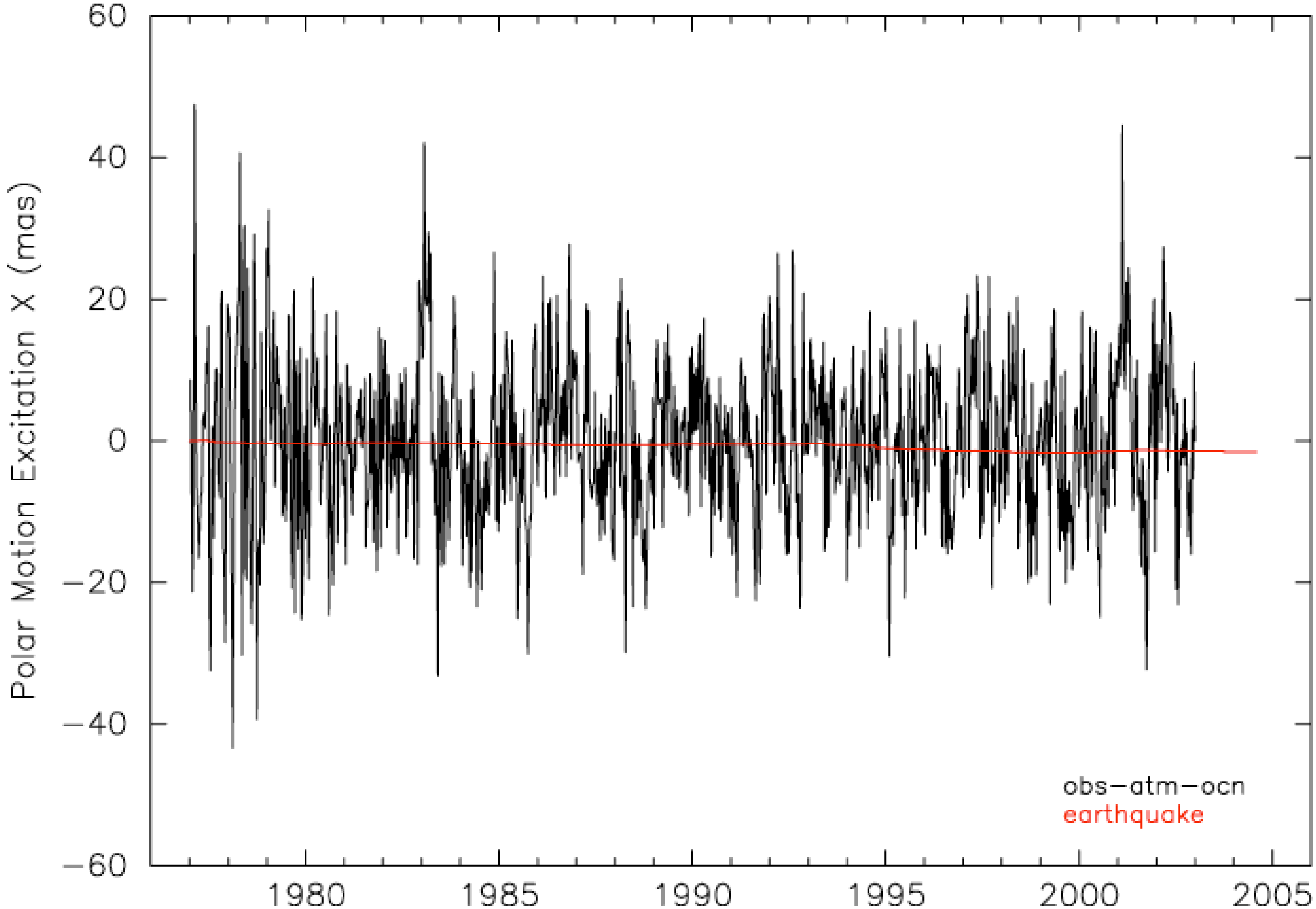
Cumulative Moment Release



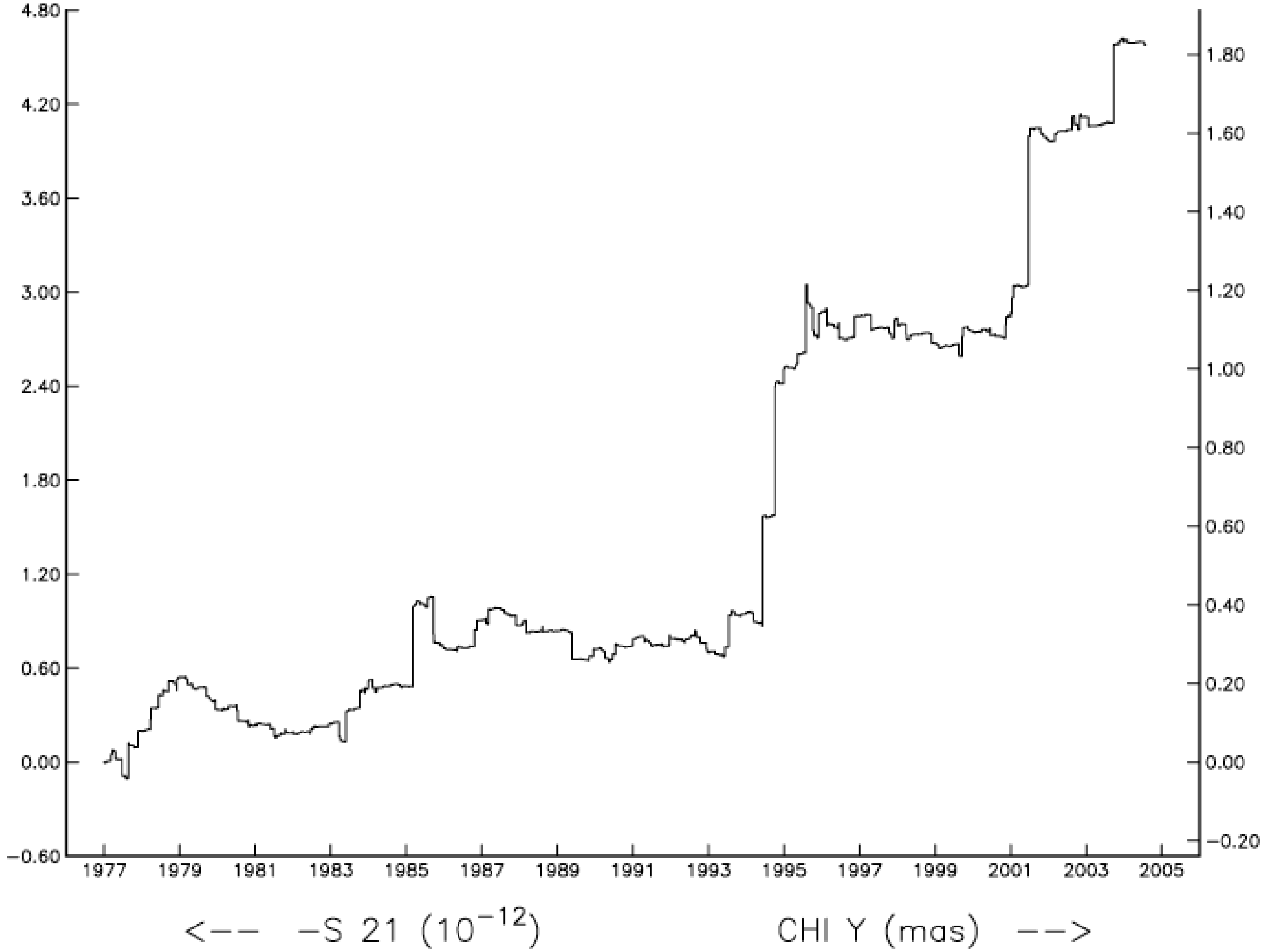
Cumulative Change in C_{21}



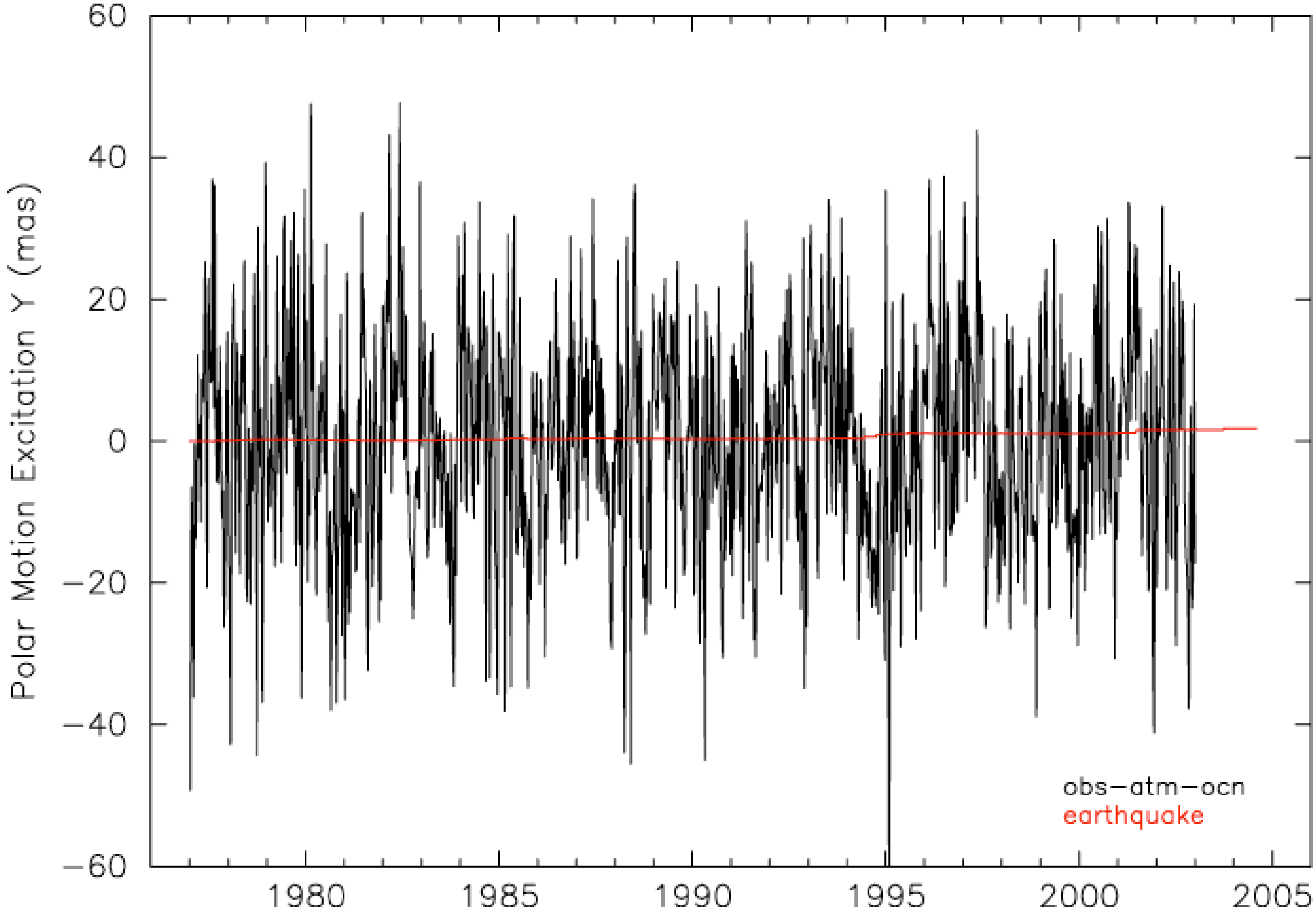
Polar Motion Excitation



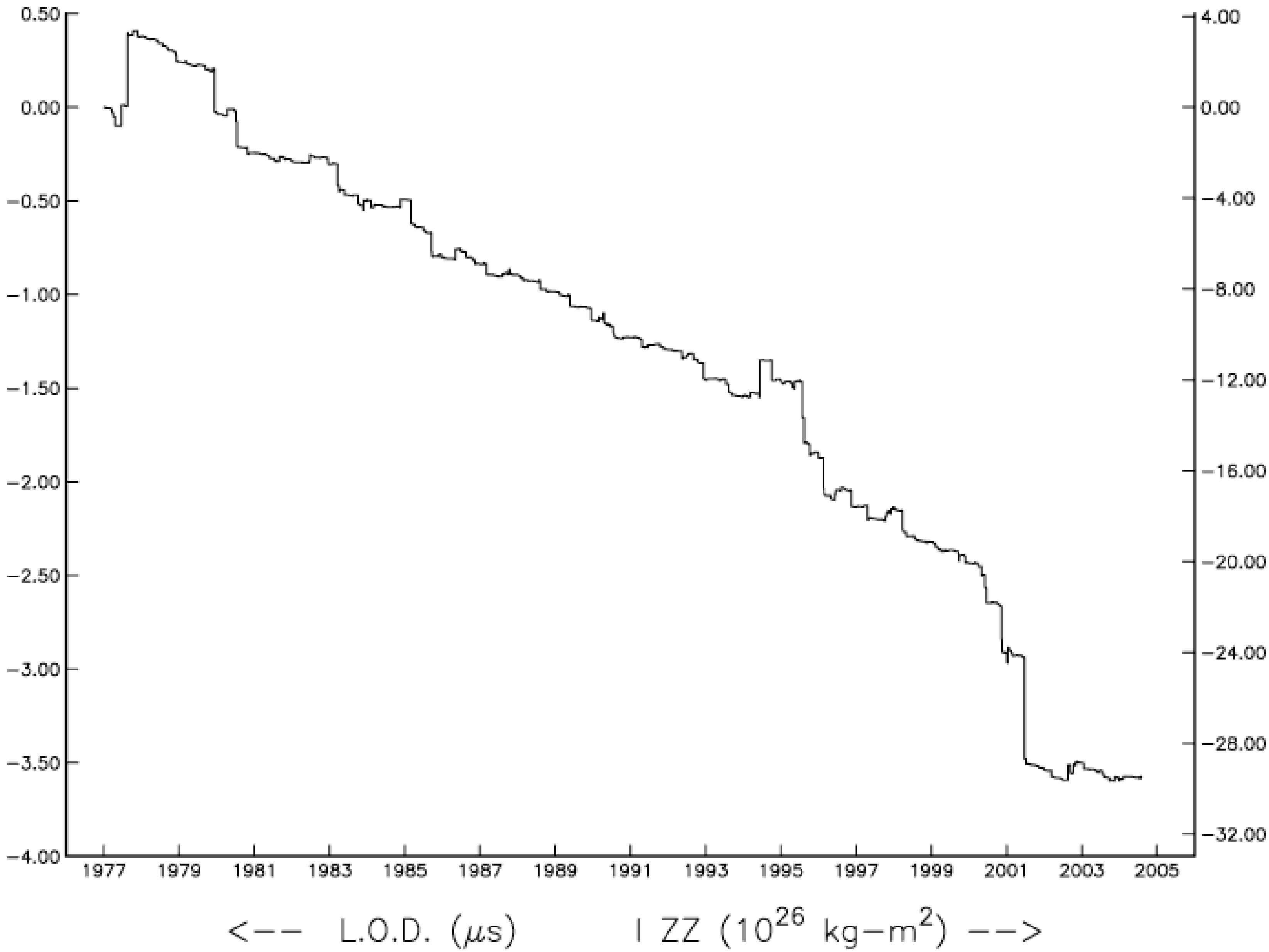
Cumulative Change in S_{21}



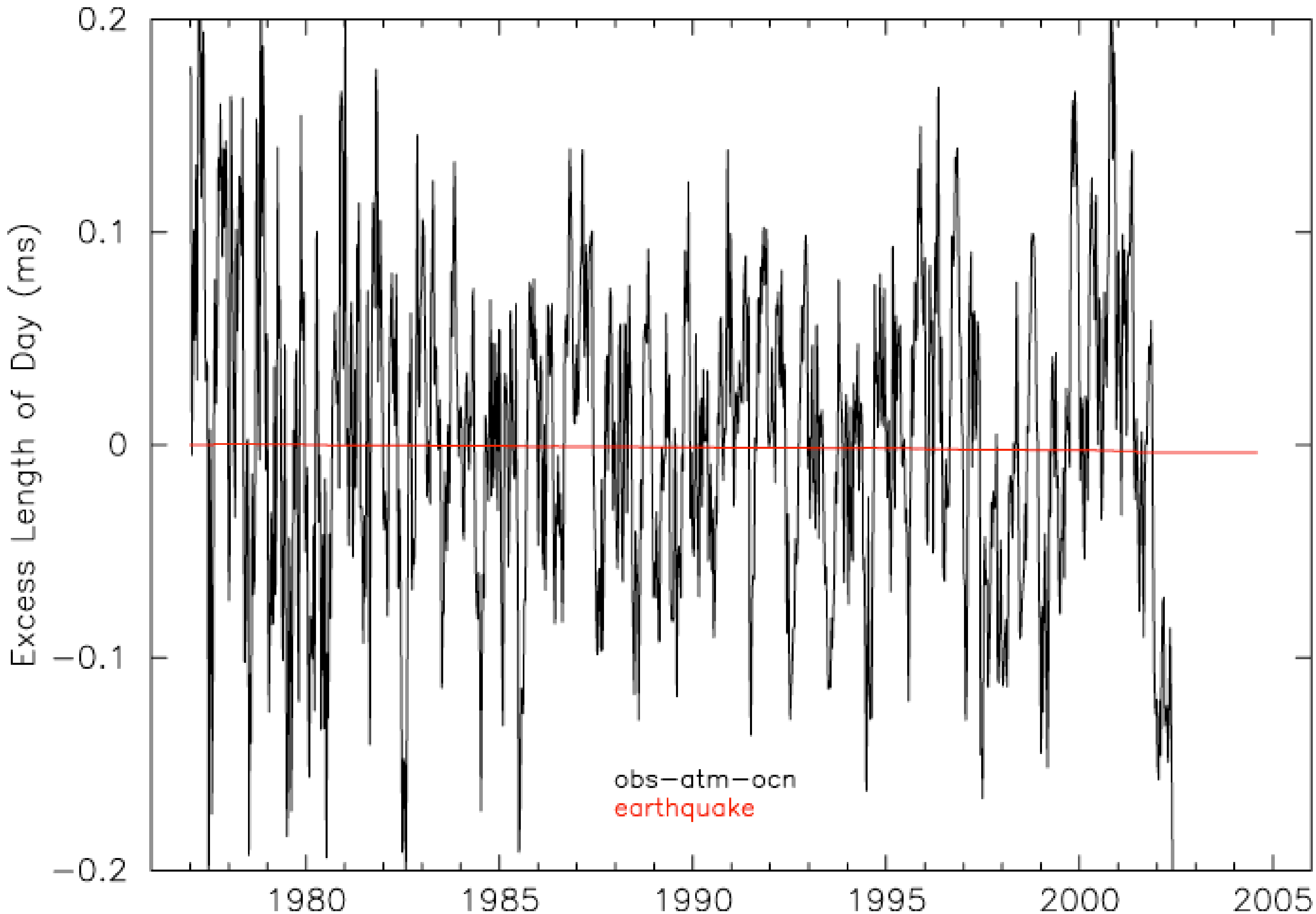
Polar Motion Excitation



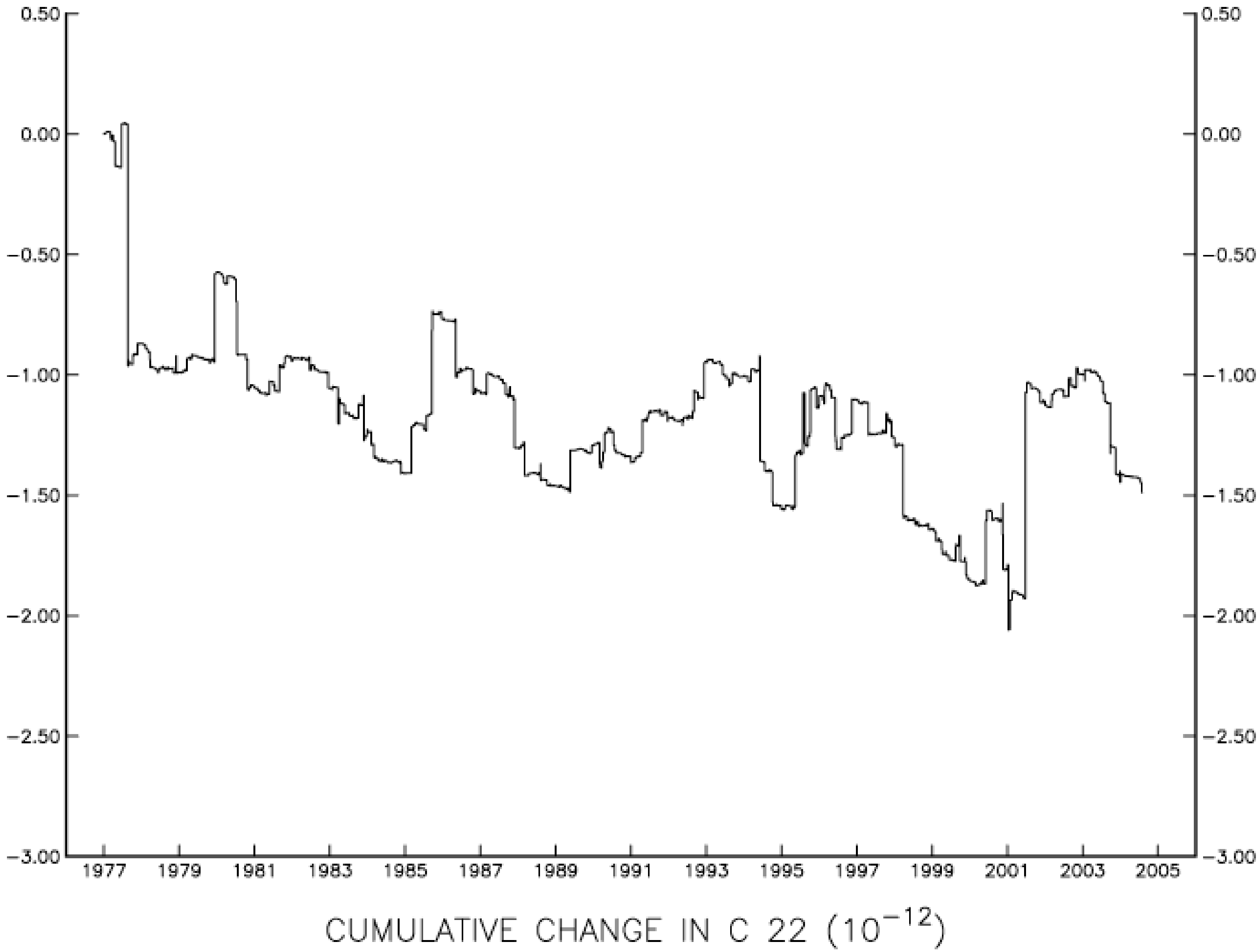
Cumulative Change in LOD



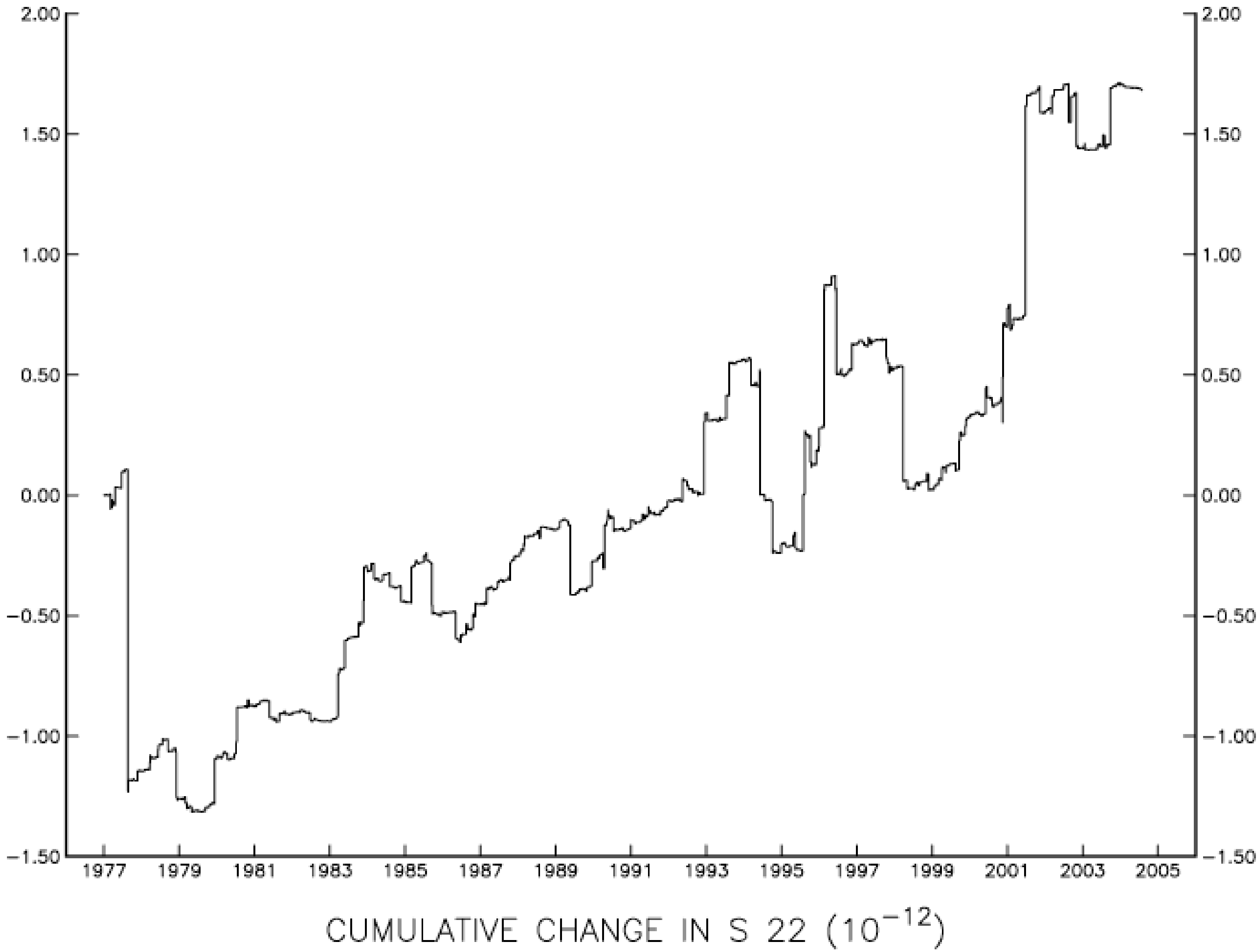
Length of Day



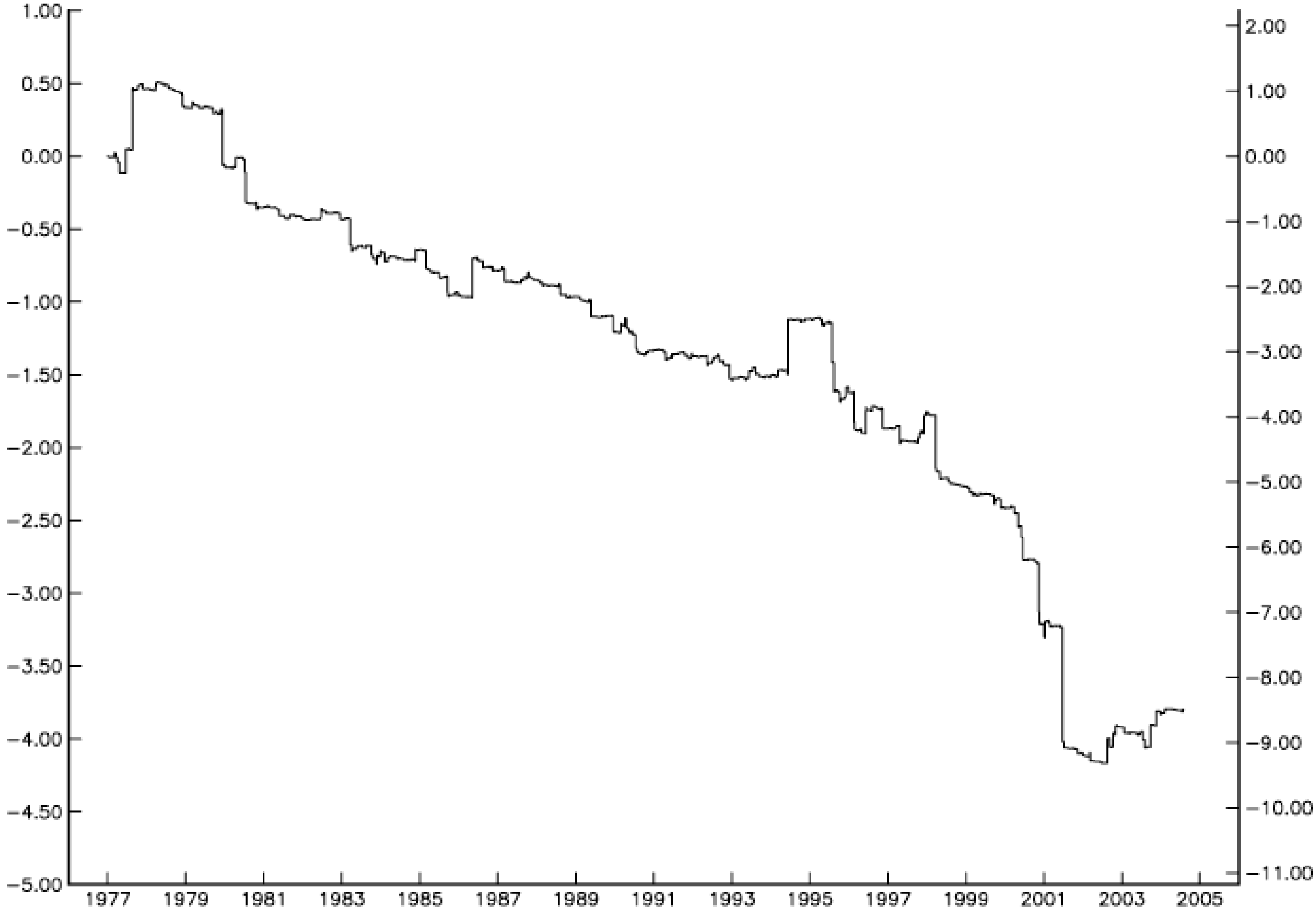
Cumulative Change in C_{22}



Cumulative Change in S_{22}

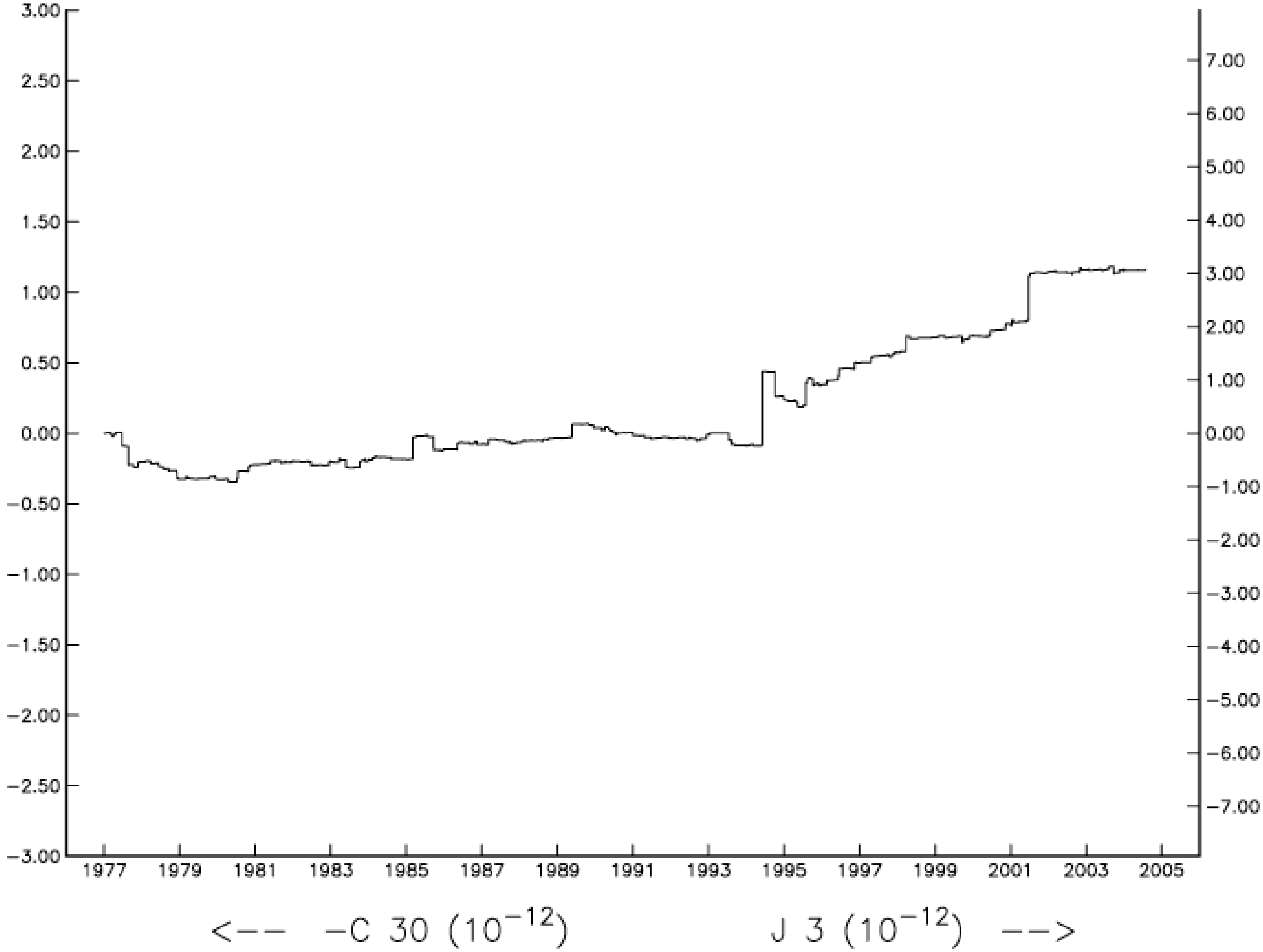


Cumulative Change in C_{20}

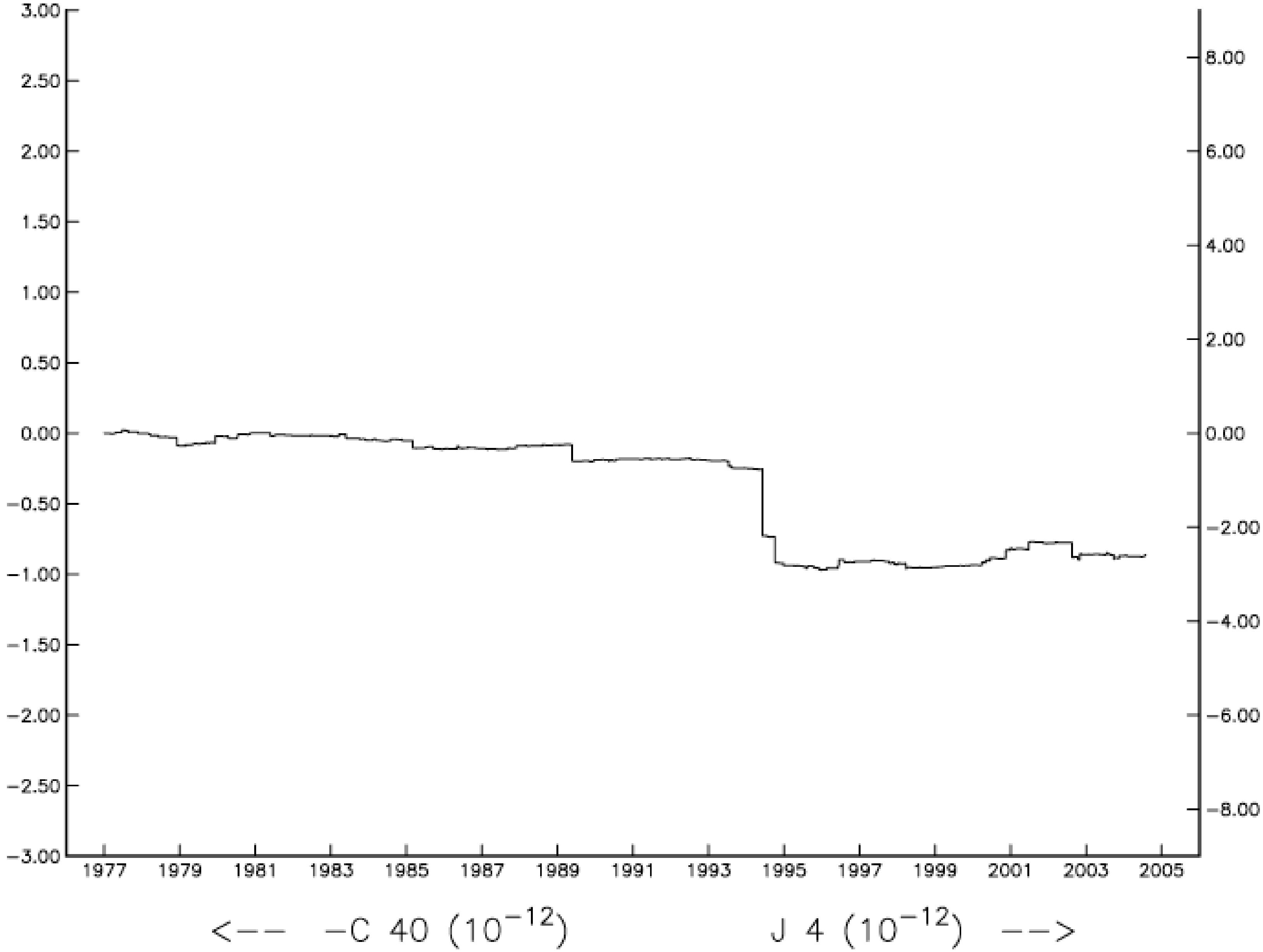


<-- $-C\ 20\ (10^{-12})$ $J\ 2\ (10^{-12})$ -->

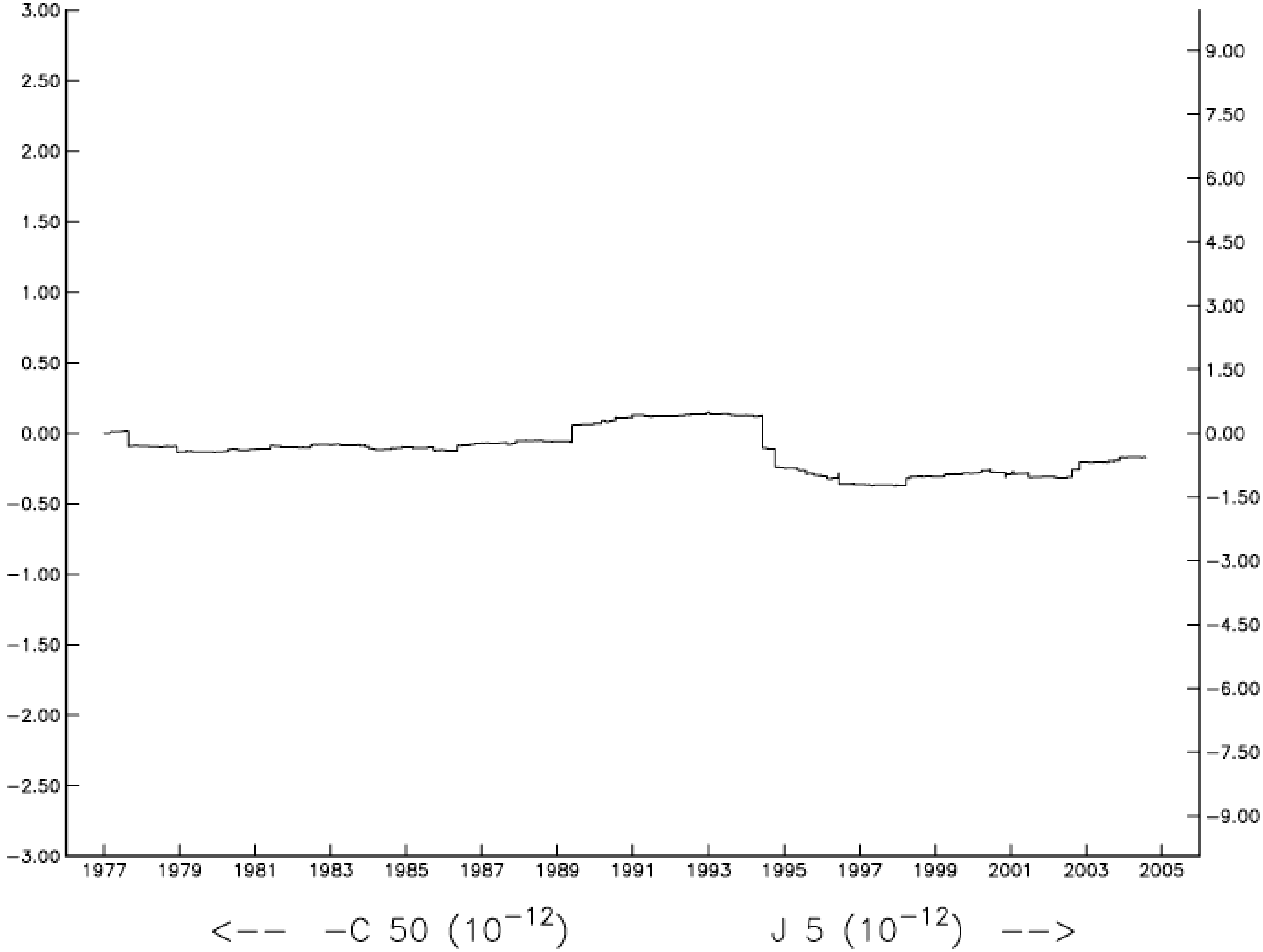
Cumulative Change in C_{30}



Cumulative Change in C_{40}



Cumulative Change in C_{50}



Degree Amplitude

- **Stokes coefficients**

$$\Delta C_{lm} + i\,\Delta S_{lm} = \frac{N_{lm}}{m_e a_e^l}\int_{V_o} \rho(r)\, r^{l-1}\, u\cdot(\hat{r}\, l + \nabla_h)\, Y_{lm}(\theta,\phi)\, dV$$

- **Degree variance spectrum**

$$\sigma_l^2 = \sum_{m=0}^{l}(\Delta C_{lm}^2 + \Delta S_{lm}^2)$$

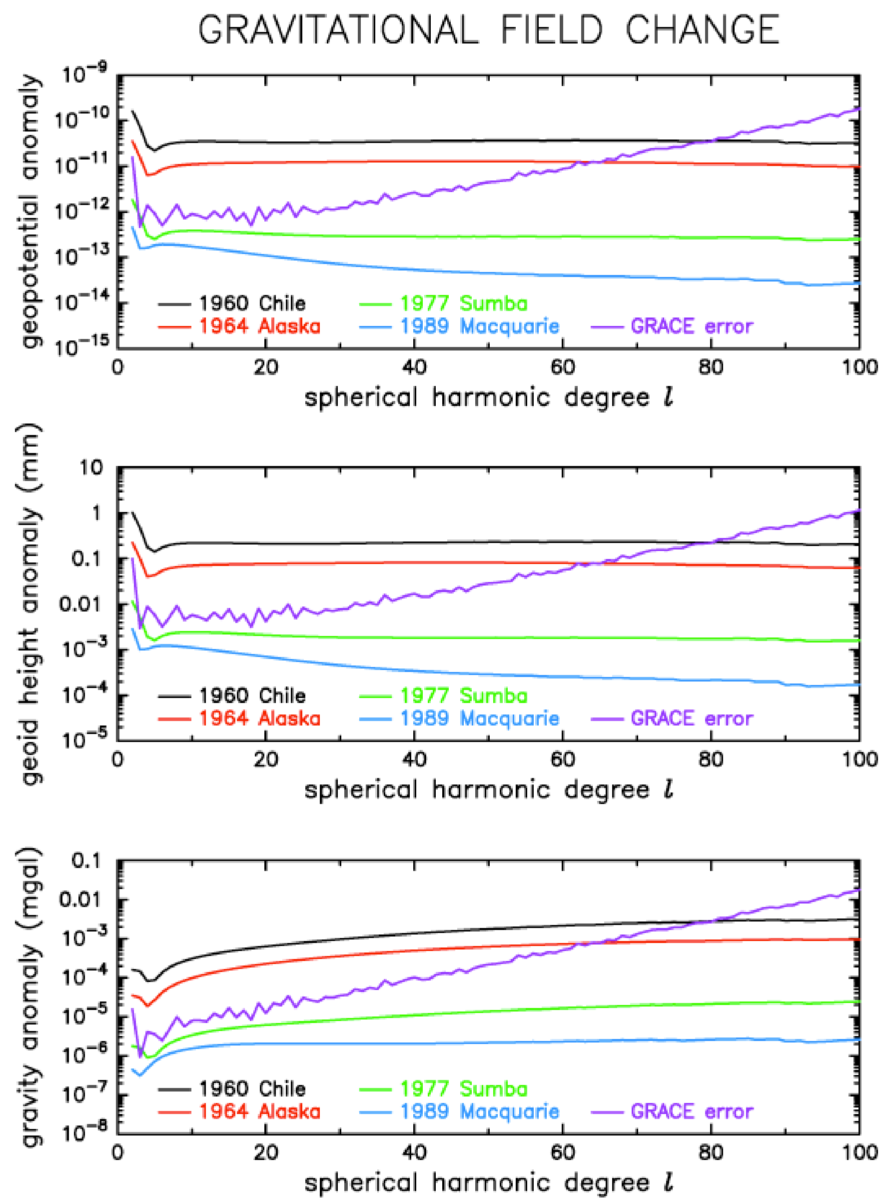
- **Geoid height anomaly amplitude spectrum**

$$\sigma_{geoid} = a_e\,\sigma_l$$

- **Gravity anomaly amplitude spectrum**

$$\sigma_{grav} = \frac{G m_e}{a_e^2}(l-1)\,\sigma_l$$

where : N_{lm} **is a normalization factor**
a_e **is the mean radius of the Earth**
m_e **is the mass of the Earth**
G **is the gravitational constant**



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Great Earthquakes

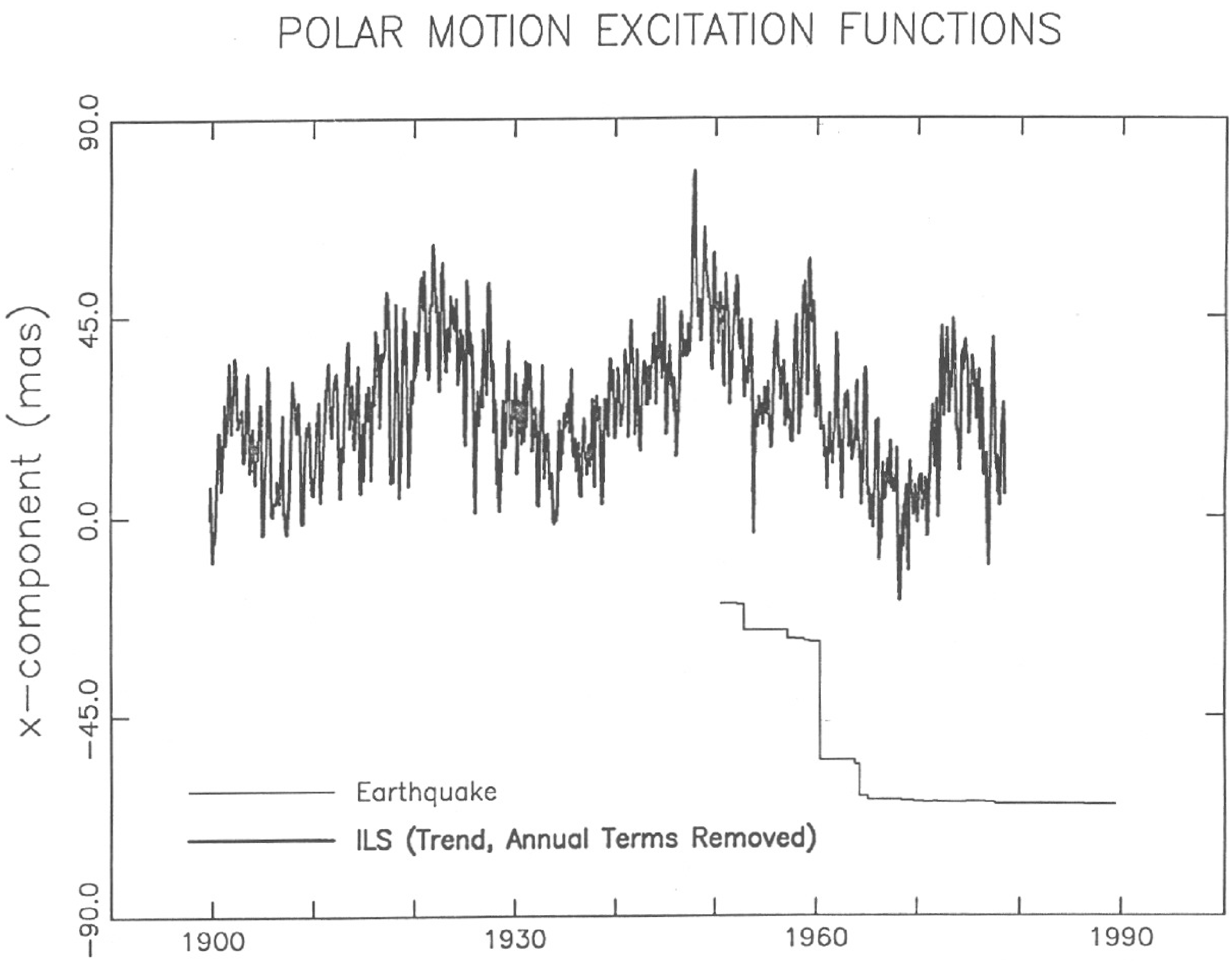
EVENT	M_W	M_O	REFERENCE
1950 ASSAM	8.6	9.5	CHEN & MOLNAR (1977)
1952 TOKACHI–OKI	8.1	1.7	KANAMORI (1977)
1952 KAMCHATKA	9.0	35.	KANAMORI (1976)
1957 ALEUTIAN ISLANDS	8.8	18.	WAHR & WYSS (1980)
1957 MONGOLIA	8.1	1.8	OKAL (1976)
1958 KURILE ISLANDS	8.3	4.0	SCHWARTZ & RUFF (1985)
1959 KAMCHATKA	8.2	2.6	KANAMORI (1977)
1960 CHILE	8.1	2.0	CIFUENTES & SILVER (1989)
1960 CHILE	9.8	550.	CIFUENTES & SILVER (1989)
1963 KURILE ISLANDS	8.5	7.5	KANAMORI (1970a)
1963 BANDA SEA	8.3	3.1	OSADA & ABE (1981)
1964 ALASKA	9.2	75.	KANAMORI (1970b)
1965 ALEUTIAN ISLANDS	8.7	14.	WU & KANAMORI (1973)
1966 PERU	8.1	2.0	ABE (1972)
1968 TOKACHI–OKI	8.2	2.8	KANAMORI (1971)
1969 KURILE ISLANDS	8.2	2.2	ABE (1973)
1970 PERU	7.9	1.0	ABE (1972)
1970 COLUMBIA	8.1	2.1	FURUMOTO & FUKAO (1976)
1971 SOLOMON ISLANDS	8.0	1.2	LAY & KANAMORI (1980)
1971 SOLOMON ISLANDS	8.1	1.8	LAY & KANAMORI (1980)
1971 KAMCHATKA	7.9	1.0	KURITA & ANDO (1974)
1974 PERU	8.1	1.5	STEWART (1977)
1976 MINDANAO	8.1	1.9	STEWART & COHN (1979)
1977 TONGA	8.0	1.4	CMT SOLUTION
1977 SUMBA	8.3	3.6	CMT SOLUTION
1979 ECUADOR	8.1	1.7	CMT SOLUTION
1985 CHILE	7.9	1.0	CMT SOLUTION
1985 MEXICO	8.0	1.1	CMT SOLUTION
1986 ANDREANOF ISLANDS	7.9	1.0	CMT SOLUTION
1989 MACQUARIE RIDGE	8.0	1.4	CMT SOLUTION

moment values (M_O) in units of 10^{28} dyne-cm

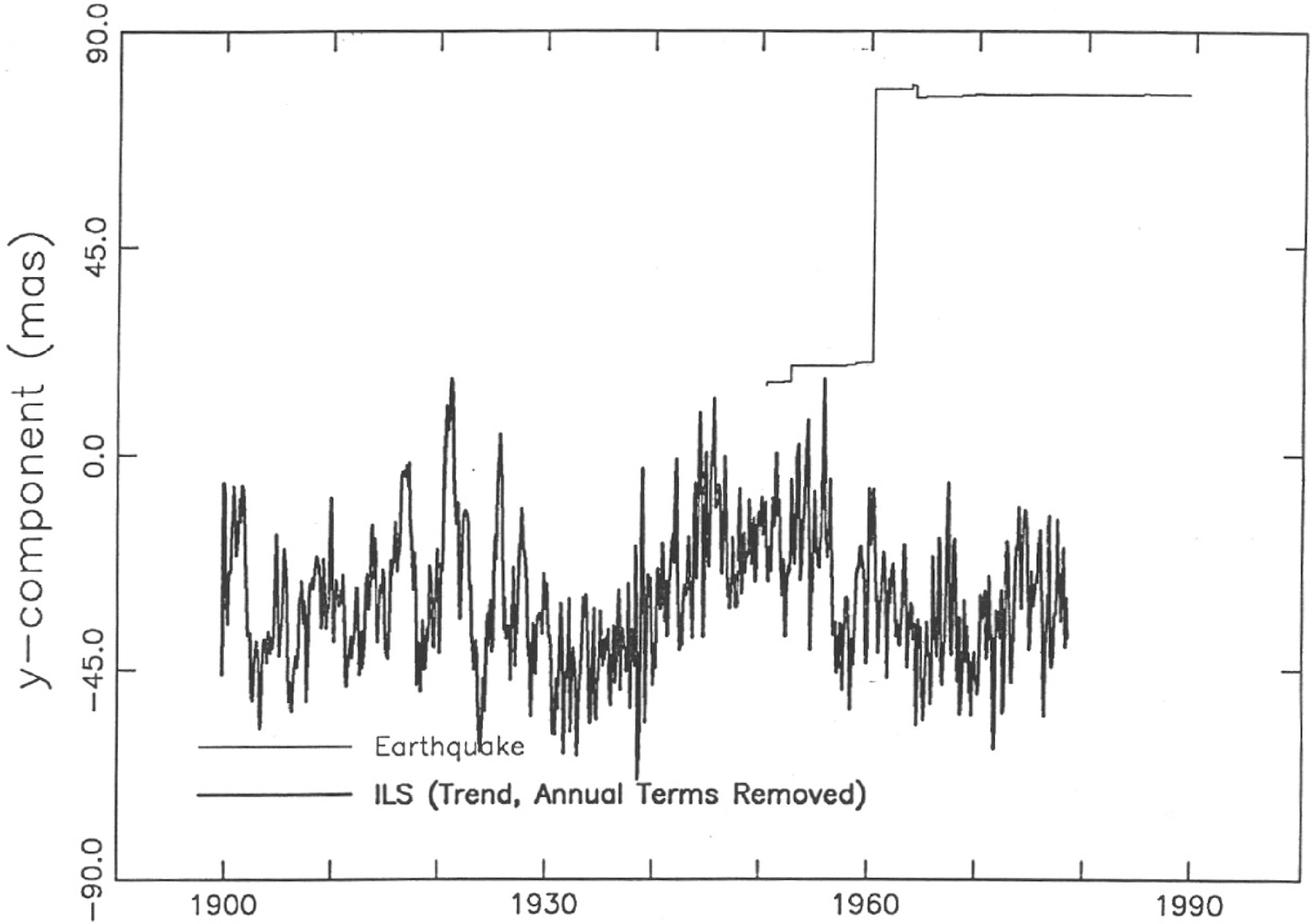
Great Earthquakes

EVENT	M_O	$\lvert\chi\rvert$ (mas)	phase (^{O}E)	Δlod (μsec)
1950 ASSAM	9.5	.63	88.5	–.20
1952 TOKACHI–OKI	1.7	.25	126.	–.015
1952 KAMCHATKA	35.	6.8	149.	.91
1957 ALEUTIAN ISLANDS	18.	1.8	178.	.81
1957 MONGOLIA	1.8	.27	126.	–.0077
1958 KURILE ISLANDS	4.0	.57	134.	–.0064
1959 KAMCHATKA	2.6	.19	137.	.090
1960 CHILE	2.0	.23	114.	–.090
1960 CHILE	550.	63.	114.	–23.
1963 KURILE ISLANDS	7.5	1.2	134.	.017
1963 BANDA SEA	3.1	.23	–119.	–.20
1964 ALASKA	75.	7.5	–162.	6.8
1965 ALEUTIAN ISLANDS	14.	.83	169.	.56
1966 PERU	2.0	.085	73.5	–.23
1968 TOKACHI–OKI	2.8	.30	138.	–.099
1969 KURILE ISLANDS	2.2	.27	132.	–.012
1970 PERU	1.0	.088	–110.	.24
1970 COLUMBIA	2.1	.098	179.	.36
1971 SOLOMON ISLANDS	1.2	.071	–63.9	–.32
1971 SOLOMON ISLANDS	1.8	.055	26.6	–.17
1971 KAMCHATKA	1.0	.068	147.	.030
1974 PERU	1.5	.089	78.5	–.20
1976 MINDANAO	1.9	.093	169.	–.32
1977 TONGA	1.4	.10	–156.	.10
1977 SUMBA	3.6	.21	160.	.33
1979 ECUADOR	1.7	.037	–139.	–.24
1985 CHILE	1.0	.18	110.	–.10
1985 MEXICO	1.1	.084	–83.0	–.089
1986 ANDREANOF ISLANDS	1.0	.13	174.	.049
1989 MACQUARIE RIDGE	1.4	.11	–37.1	–.059

moment (M_O) in units of 10^{28} dyne-cm


POLAR MOTION EXCITATION FUNCTIONS
x-component (mas)
90.0
45.0
0.0
-45.0
-90.0
1900
1930
1960
1990
Earthquake
ILS (Trend, Annual Terms Removed)

POLAR MOTION EXCITATION FUNCTIONS



Summary

- Earthquakes redistribute the Earth's mass on a global scale
  - Change the Earth's rotation and gravitational field
- Greatest earthquakes have greatest effect
  - 1960 Chile
    - 63 mas change in polar motion excitation
    - 23 μs change in length of day
- Current observing systems are accurate enough to detect changes caused by next great earthquake
  - Polar motion accuracy about 0.05 mas
  - LOD accuracy about 20 μs