Clues for detecting the inner core wobble: polar motion, geopotential and gravity variation

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Mathews et al. (1991a,b) named the inner core wobble (ICW) in theoretical investigation. It is a free rotation mode of the inner core relative to the mantle in which the figure axis of the inner core is inclined to that of the mantle, and is doing a precession relative to the mantle with fixed obliquity. The period, i.e. the time required to accomplish one cycle is about 6.6 years for PREM, and 5.0 years for 1066A. The period of the ICW is also related to the super rotation of the inner core if it exists (*Guo and Ning* 2002). The influence of the ICW on polar motion is formulated in *Mathews et al.* (1991a,b). It's influence on the variation of the gravity at Earth's surface as well as on the variation of the Legendre coefficients of Earth's gravitational potential are formulated in this work. The magnitude of variation of these quantities are given for various values of the amplitude of the ICW, i.e. the obliquity of the inner core. The aim of this work is to explore the possibility of identifying the ICW using polar motion, variation of gravity observed using the very precise super-conducting gravimeters worldwide in the frame of the Global Geodynamic Project (GGP), and variation of gravitational potential determined using CHAMP and GRACE.

The orientation of the inner core for the ICW is shown in Figure 1. The centers of inner core and mantle are assumed to coincide and is denoted with O. We use $Oxyz$ to denote the mantle fixed frame. x is in the equator. z is along the mantle axis. y is determined according to the right-handed role. It is not drawn out in the figure for clarity. The direction of the figure axis of the inner core is denoted with colatitude θ_s and longitude λ_s. In the ICW, the figure axis of the inner core, z_s, is inclined an angle θ_s to z. The ICW is a regular precession of the inner core with respect to the frame $Oxyz$, i.e. the longitude of the inner core axis λ_s varies steadily with time while keeping the obliquity θ_s constant. The period of the ICW is about 6.6 years for PREM, and 5.0 years for 1066A, i.e. the figure axis of the inner core draw a closed circle about every 6.6 years for PREM, and 5.0 years for 1066A. Thus the value of λ_s varies with angular rate ω_p which is equal to $360^\circ/6.6 = 54.5^\circ$ yr-1 or 0.95 yr^{-1} for PREM, and $360^\circ/5.0 = 72^\circ$ yr^{-1} or 1.3 yr^{-1} for 1066A. For convenience, assuming $\lambda_s = \lambda_0$ when $t = 0$, we can then express λ_s as function of time t

$$\lambda_s = \omega_p t + \lambda_0 \,. \tag{1}$$

The polar motion related to the ICW was formulated in *Mathews et al.* (1991a,b). In Figure 1, $\mathbf{\Omega}$ is Earth's angular velocity, and m is the amplitude of polar motion. We see that $\mathbf{\Omega}$ at the opposite side of z_s relative to the mantle axis. This implies that the phase difference between polar motion and inner core tilt is 180°. According to *Mathews et al.* (1991a,b), θ_s and m are proportional, that is

$$m = \gamma\theta_s \,, \tag{2}$$

where $\gamma = 1.123 \times 10^{-4}$ for PREM, and $\gamma = 1.681 \times 10^{-4}$ for 1066A. We omitted the minus sign of *Mathews et al.* (1991a,b) here as we treat explicitly the difference in phase.

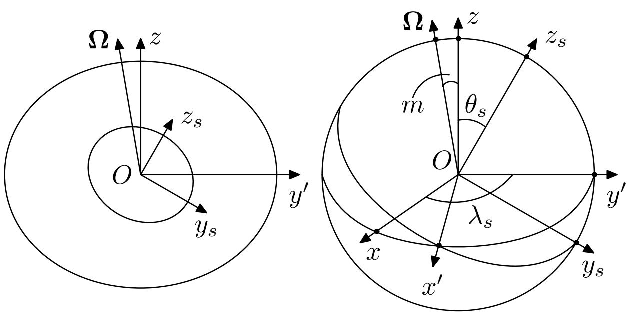

Figure 1. Orientation of the inner core.

The varying part of gravity at Earth's surface related to the ICW can be shown to be

$$\delta g = -9\, g\, \delta J\, \theta_s \sin\theta \cos\theta \cos(\lambda - \lambda_s)\,, \tag{3}$$

$$\delta J = \frac{(C_s - A_s) - (C_s' - A_s')}{M a^2} \tag{4}$$

where g is the gravity at Earth's surface; A_s and C_s are the inner core principle moments of inertia around the equatorial and polar axes respectively; A_s' and C_s' are the inner core principle moments of inertia around the equatorial and polar axes if the inner core density were equal to the outer core density at the vicinity of the inner core boundary (ICB); a is the semi-major axis of the Earth; and M is the mann of the Earth.

The geopotential coefficients C_2^1 and S_2^1 are most sensitive to the ICW. The varying part of them can be shown to be

$$\delta C_2^1 = -\sqrt{\frac{3}{5}}\delta J\, \theta_s \cos\lambda_s\,, \qquad \delta S_2^1 = -\sqrt{\frac{3}{5}}\delta J\, \theta_s \sin\lambda_s\,. \tag{5}$$

For computing the per year variation of δC_2^1 and δS_2^1, we take the derivative of them with respect to time in making use of (1):

$$\frac{d\,\delta C_2^1}{dt} = \sqrt{\frac{3}{5}}\delta J\, \theta_s \omega_p \sin\lambda_s\,, \qquad \frac{d\,\delta S_2^1}{dt} = -\sqrt{\frac{3}{5}}\delta J\, \theta_s \omega_p \cos\lambda_s\,, \tag{6}$$

The polar motion with a period of around 6 years, if exists, would be the order of $4 \sim 5$ mas. Assuming it is the ICW, we can obtain that the amplitude of the gravity signal would be $0.024 \sim 0.03$ μgal, and the variation of $d\,\delta C_2^1/dt$ and $d\,\delta S_2^1/dt$ would be $4.0 \times 10^{-12} \sim 5.0 \times 10^{-12}$ yr^{-1}. Thus we may conclude that it is most probable to identify the ICW from polar motion data.

REFERENCES

Greiner-Mai, H., Jochmann,H., Barthelmes, F., Influence of possible inner-core motions on the polar motion and the gravity field, *Phys. Earth planet. Inter., 117*, 81-93, 2000.

Greiner-Mai, H., Barthelmes, F., Relative wobble of the Earth's inner core derived from polar motion and associated gravity variation, *Geophys. J. Int., 144*, 27-36, 2001.

Guo, J. Y., *Introduction to physical geodesy* (in Chinese), Press of the Wuhan Technical University of Surveying and Mapping, Wuhan, China, 1994.

Guo, J. Y. and J. S. Ning, Influence of inner core rotation and obliquity on the inner core wobble and the free inner core nutation, *Geophys. Res. Lett., 29(8)*, 10.1029/2001GL014058, 2002.

Heiskanen, W. A. and H. Moritz, *Physical geodesy*, W. H. Freeman, San Francisco, 1967.

Mathews, P.M., Buffett, B.A., Herring, T.A., Shapiro, I.I., Forced Nutations of the Earth: Influence of inner core Dynamics 1. Theory, *J. geophys. Res., 96*, 8219-8242, 1991a.

Mathews, P.M., Buffett, B.A., Herring, T.A., Shapiro, I.I., Forced Nutations of the Earth: Influence of inner core Dynamics 2. Numerical results and comparisons, *J. geophys. Res., 96*, 8243-8257, 1991b.

Mathews, P. M., Herring T. A. and Buffett B. A., Modeling of nutation-precession: New nutation series for nonrigid Earth, and Insights into the Earth's Interior, *J. geophys. Res., 107(B4)*, 10.1029/2001JB000390, 2002.