Gravity anomaly determination in mountainous areas – General aspects revisited

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Abstract

The geophysical meaning of gravity anomalies has been debated again in several geophysical and geodetic journals. This paper picks up the approach of composing synthetically all gravity effects contributing to the gravity observed at the earth's surface, as this aspect enables to determine exactly the physical meaning of Bouguer anomalies obtained under certain assumptions (e.g. reference surface). The paper also tries to clarify the errors introduced when calculating Bouguer anomalies with special respect to the situation in high mountainous areas.

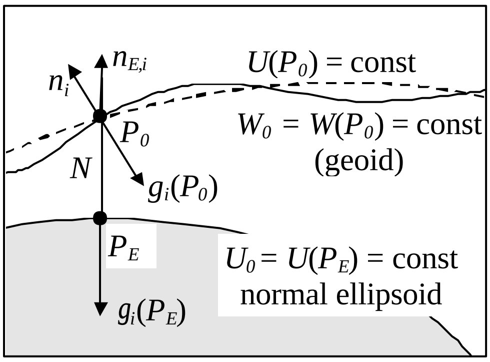
Introduction

Recently in several journals the discussion on the physical meaning of gravity anomalies in geophysics has been revived (e.g. Hackney and Featherstone 2003). Often this is done in terms of gravity anomaly or gravity disturbance as defined in physical geodesy. In fact, the interpretation of what the Bouguer anomaly really means is an old problem. Geophysicists dealing with potential theory do not simply regard the Bouguer anomaly as gravity effect of density inhomogeneities on a reference level. They do this as an approximation at best. It is true, that even in modern textbooks this fact is not accordingly emphasized, but there are a lot of papers having addressed this problem early in the past (e.g. Naudy and Neumann 1965, Tsuboi 1965). E.g. Tsuboi (1965) distinguished between SCBA (station Bouguer anomaly) and TCBA (true Bouguer anomaly) in order to clarify, that the Bouguer gravity is still related to the observation surface and not to any reference surface. Another topic in this discussion addresses the question which height system should be used when calculating the normal gravity at a gravity station. It has also been shown by several authors, that ellipsoidal heights rather than orthometric heights should be used (e.g. Vogel 1982, Meurers 1992).

Therefore it is a more appropriate approach to discuss the terminus "gravity anomaly" in geophysics not in terms of reduction or correction, but to use a synthetic concept in order to clarify the physical meaning of what is called Bouguer anomaly (e.g. Vogel 1982, Meurers 1992). It is essential to know the errors introduced when applying simplifications and assumptions.

Definition of gravity anomalies in geodesy and geophysics

In physical geodesy one of the most important goals is the determination of the geoid derived from gravity observations at the earth surface. The geoid is closely related to the disturbing potential T. This leads to the fundamental equation of physical geodesy describing the general boundary value problem where Dg is the scalar representation of the gravity anomaly Dg_i in its classical sense.

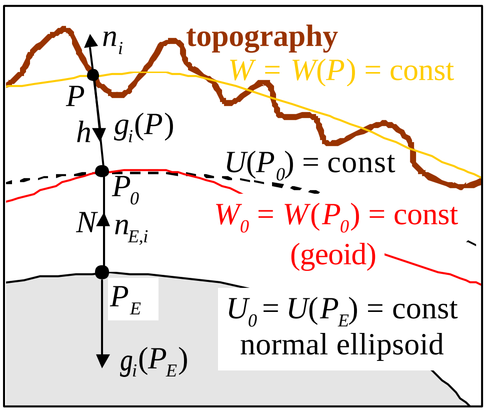


$T(P_0) = W(P_0) - U(P_0)$ disturbing potential

$\Delta g_i = g_i(P_0) - \gamma_i(P_E)$ gravity anomaly

$$\Delta g = -\left(\frac{\partial T}{\partial n}\right)_{P_0} + \frac{1}{\gamma(P_E)}\left(\frac{\partial \gamma}{\partial n_E}\right)_{P_E} T(P_0)$$

In geophysics we are looking for the geometry and the density contrast of anomalous sources referred to any reference model. The definition of the Bouguer anomaly (1) implies to use the gravity disturbance *dg* that again is a scalar quantity:



$$dg = g(P) - \gamma(P)$$

$$BA(P) = g(P) - \gamma(P_E) - \int_0^z \frac{\partial \gamma}{\partial z} dz - dg_T(P) \qquad (1)$$

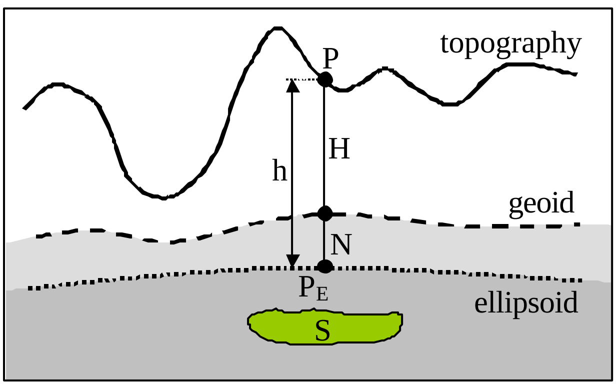
dg_T *complete topographical mass effect*

Its physical meaning depends on the height system applied.

Synthetic composition of observed gravity

If we compose the gravity g observed at any point P on topography synthetically, we can consider $g(P)$ as

$$g(P) = \gamma(P_E) + \int_0^N \frac{\partial \gamma}{\partial z} dz + \int_N^{N+H} \frac{\partial \gamma}{\partial z} dz + g_S(P_E) + \int_0^N \frac{\partial g_S}{\partial z} dz + \int_N^{N+H} \frac{\partial g_S}{\partial z} dz + dg_T(P) + dg_G(P) \qquad (2)$$



dg_T gravity effect of mass distributed between topography and geoid

dg_G gravity effect of mass distributed between geoid and normal ellipsoid

h ellipsoidal height

H orthometric height

N geoid undulation

At this stage this implies, that we already ignored the misalignment of all vectors contributing to the gravity vector observed at *P*. The last two terms in (2) describe the gravity effect of topographic masses between topography and geoid or between geoid and ellipsoid respectively. At this moment let us assume to know the density distribution exactly.

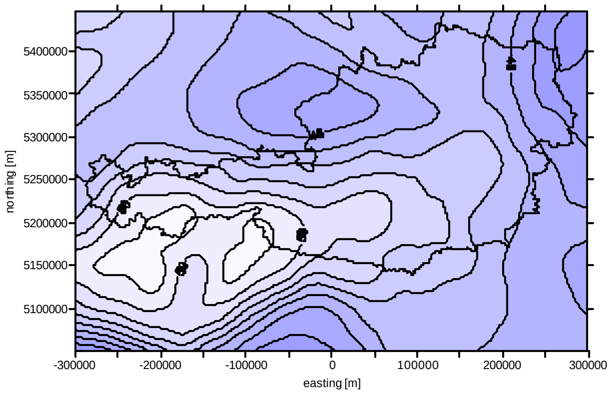
1. *Bouguer anomaly determination based on orthometric heights:*

$$BA(P) = g(P) - g(P_E) - \int_N^{N+H} \frac{\partial g}{\partial z} dz - dg_T(P) \tag{3}$$

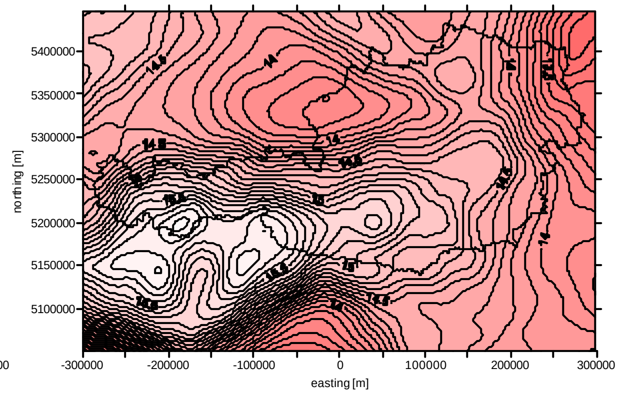
Inserting (2) into (3) results to:

$$BA(P) = \int_0^N \frac{\partial g}{\partial z} dz + \underbrace{g_S(P_E) + \int_0^N \frac{\partial g_S}{\partial z} dz + \int_N^{N+H} \frac{\partial g_S}{\partial z} dz}_{g_S(P)} + dg_G(P) = g_S(P) + \underbrace{\int_0^N \frac{\partial g}{\partial z} dz}_{\text{gradient effect}} + \underbrace{dg_G(P)}_{\text{mass effect}} \tag{4}$$

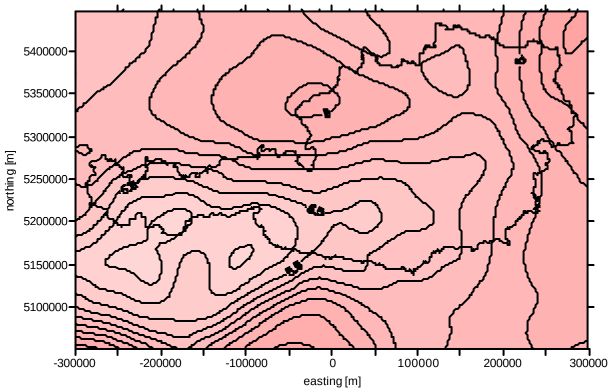
$g_S(P)$ is the gravity effect of all sources below the ellipsoid on topography. The Bouguer anomaly differs from $g_S(P)$ by the last two terms in (4) that can be estimated easily, if we know the geoid undulation and the density distribution. Rough estimates of the effect in the Eastern Alps show, that ignoring both expressions introduces both offset and height dependant error components.



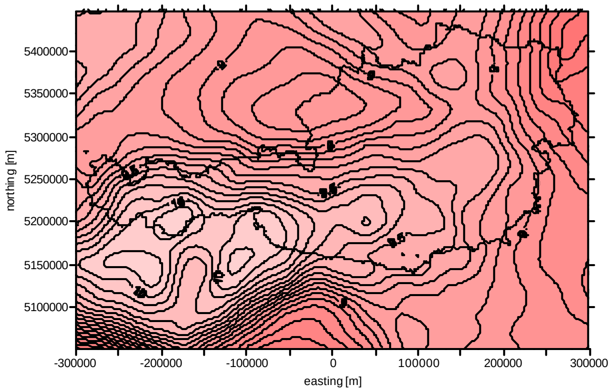
Geoid $[m]$



Gradient effect $[mGal = 10^{-5}\ ms^{-2}]$



Topographic mass effect $[mGal]$



Total effect $[mGal]$

2. *Bouguer anomaly determination based on ellipsoidal heights:*

$$BA(P) = g(P) - g(P_E) - \int_0^N \frac{\partial g}{\partial z} dz - \int_N^{N+H} \frac{\partial g}{\partial z} dz - \delta g_T(P) - \delta g_G(P) \tag{5}$$

Inserting (2) into (5) results to:

$$BA(P) = g_S(P_E) + \int_0^N \frac{\partial g_S}{\partial z} dz + \int_N^{N+H} \frac{\partial g_S}{\partial z} dz = g_S(P) \tag{6}$$

Now the Bouguer anomaly is identical with the gravity effect of all sources below the ellipsoid on topography. Therefore ellipsoidal height systems should be preferred.

The synthetic concept (eqs (4) and (6)) easily permits to explain which gravity effects are left in the Bouguer anomaly and tells us how to proceed when interpreting the anomaly in the model space. This even holds if we do not know exactly the density distributions assumed for the mass corrections. We have to consider that there are a lot of simplifications used for the practical calculation of the Bouguer gravity, which is closely related to its error budget (limitation of the mass correction area, density distribution within topographic masses, normal gravity at point *P*).

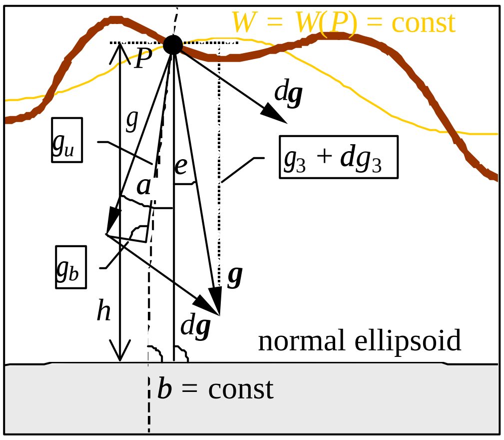
The geophysicist has to know these errors in order to interpret the anomalies correctly. In forward modeling he has to consider all density inhomogeneities with respect to the density distribution of the normal ellipsoid. Of course, from a theoretical point of view, this might be a problem. The normal ellipsoid tells us nothing about the internal density structure as it is fully defined by only four Stokes' constants. However, Moritz (1990) has shown that structures like those of the preliminary earth model have a gravity distribution close to the normal gravity. This permits to use the normal gravity as a reference. Today there is no need to apply simplified formulae to calculate the normal gravity at the observation site. Either closed formulae including atmospheric correction should be used instead or more precise Taylor series representations like that of Wenzel (1985).

Ignoring gravity vector misalignments

In principle we should formulate equation (2) as a vector instead of a scalar equation. However, the scalar approximation is inevitable to be left with harmonic quantities. The norm of a potential gradient vector does not meet Laplace differential equation while its projection to any direction does. To be more precisely we would need to know the direction of the observed gravity vector and not only its norm as resulting from gravity measurements. Only in that case we can calculate the projections of all contributing gravitational vectors to the direction of the observed gravity vector. We should keep in mind that even in geodesy the gravity vector misalignment is ignored to end up with the fundamental equation of physical geodesy. The error can easily be estimated. Ignoring the angle α between the true direction of the normal gravity at *P* and the ellipsoid normal passing *P* we get

$$g = |\mathbf{g}| = \sqrt{dg_1^2 + dg_2^2 + (g_3 + dg_3)^2} = (g_3 + dg_3)\underbrace{\sqrt{\frac{dg_1^2 + dg_2^2}{(g_3 + dg_3)^2} + 1}}_{\mathrm{tg}^2 e \ll 1} = g\underbrace{\cos a}_{\cong 1} + dg_3 \quad \Rightarrow \quad dg_3 \cong g - g$$

dg_3 fulfills Laplace equation in contrast to $|d\mathbf{g}|$, that means, $BA(P)$ is harmonic.



ε [″]	error [nms^{-2}]	
10	11	Flat terrain
30	104	Mountainous area
60	415	Maximum estimate

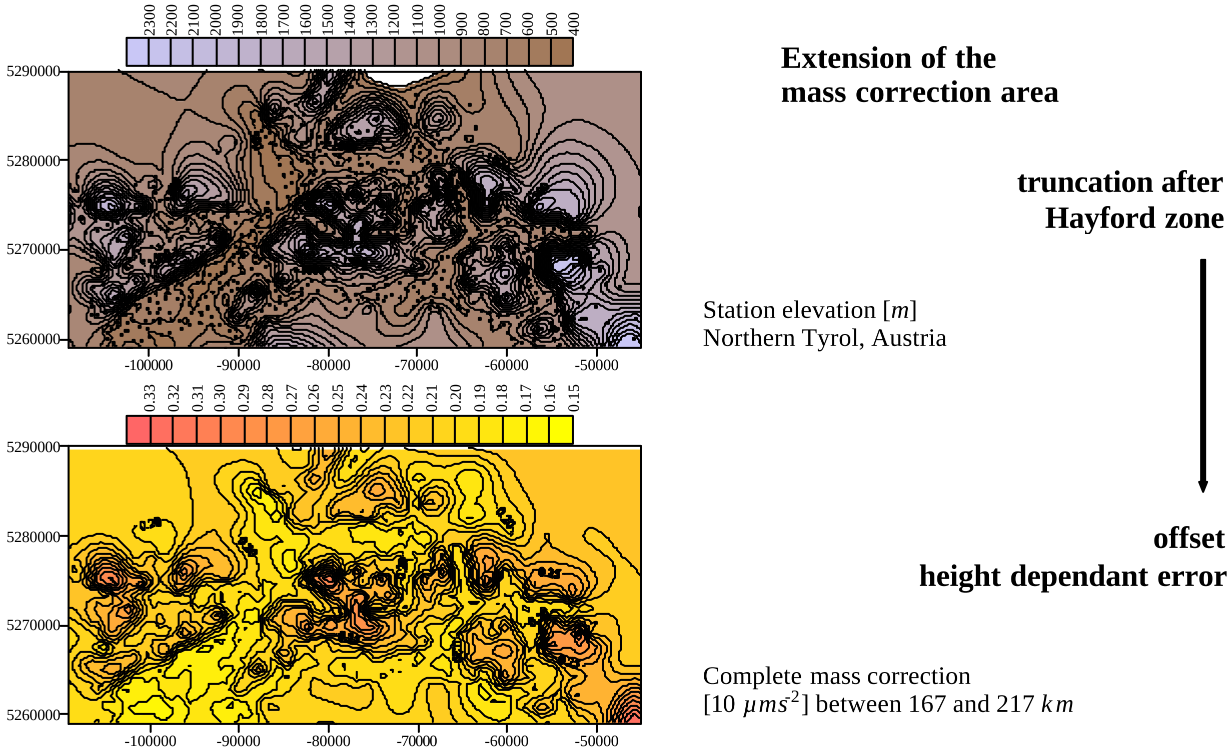
Mass correction errors

Several errors sources are limiting the accuracy of mass corrections:

- geometrical approximation of topography
- limited correction area
- density distribution

They show up both as random and systematical components that often correlate with the elevation of the gravity stations. Therefore those errors are able to disturb severely the gravity anomaly determined in high mountainous areas. The first problem can be easily solved today, as both precise terrain models and computing facilities are available. This permits to approximate the topography by arbitrary shaped bodies represented by polyhedra instead of flat topped prisms (e.g. Meurers et al. 2001). The gravity effect of polyhedral bodies can be calculated exactly (e.g. Götze and Lahmeyer 1988). The second problem concerns the limitation of the correction area under consideration. In principle the correction has to be performed over the whole earth surface. In practice only masses up to a distance of about 167 km (Hayford zone O_2) is considered. This introduces a bias and, to some extent, small height dependant errors. The first aspect is important for crustal balancing investigations based on isostatic anomalies.

The most severe error is introduced by applying average crustal density for the mass correction instead of the real density distribution which is a priori unknown. If information about the surface density can be utilized, the calculation of the Bouguer anomaly based on 2D density models gives an essential improvement. In fact, especially in areas with tectonic regime like an orogene with its complex nappe structure, even this approach is still incorrect as density does not only vary laterally. However, this method permits to estimate the effects and to detect apparent anomalies that are closely correlated with the topography in many cases (e.g. Meurers 1992).



Station elevation [m]
Northern Tyrol, Austria

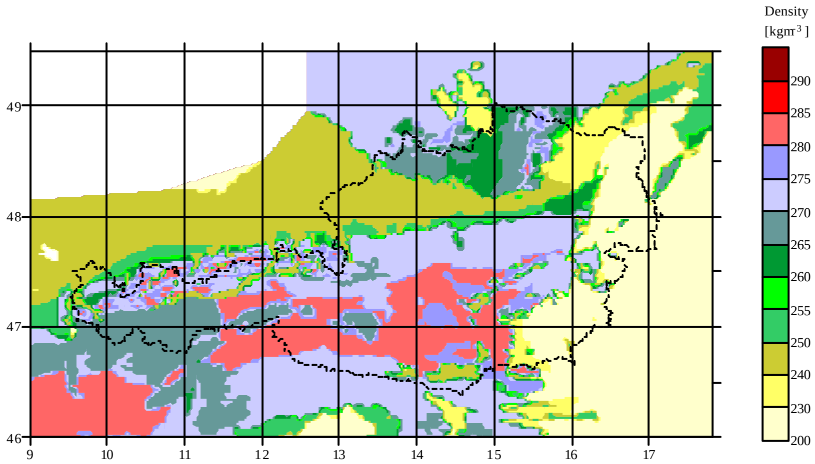
Complete mass correction
[10 $\mu m s^{-2}$] between 167 and 217 km

Field continuation between arbitrary surfaces

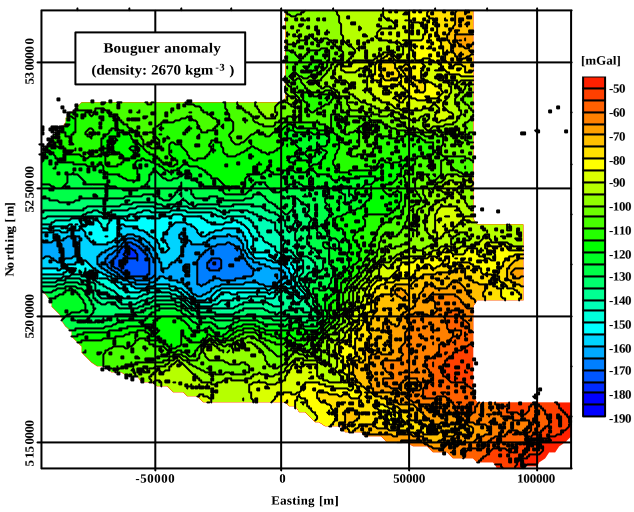
Many interpretation steps like interpolation or field transformation require the potential field to be known on a flat surface. As Bouguer anomalies are still related to the true observation site, that means to an irregular surface, we need accurate methods to perform the field continuation upward from the topography to a reference level. Downward continuation is not allowed as due to the incompleteness of mass corrections sources still remain below the topopgraphy and therefore the anomaly is not harmonic there. Several approaches have been published in the past, among others the procedures by Xia et al. (1993) and Ivan (1994), that are based on the equivalent source concept. Xia et al. compare the observed field with that continued to the observation sites by DFT and adjust iteratively a flat surface density distribution situated below the deepest topography. Ivan uses a dipole distribution spread over the irregular topography, which is represented by a polyhedral surface, and solves a Fredholm integral equation of 2nd kind, whereby the surface integral for calculating the potential of the equivalent source is evaluated exactly. While in Ivan's the space between equivalent source and topography has not necessarily to be harmonic, this requirement has to be fulfilled in Xia et al.'s algorithm. However, the method of Xia et al. can be applied even on Bouguer anomalies, as long as the high frequency content is sufficiently suppressed (Meurers and Pail 1998).

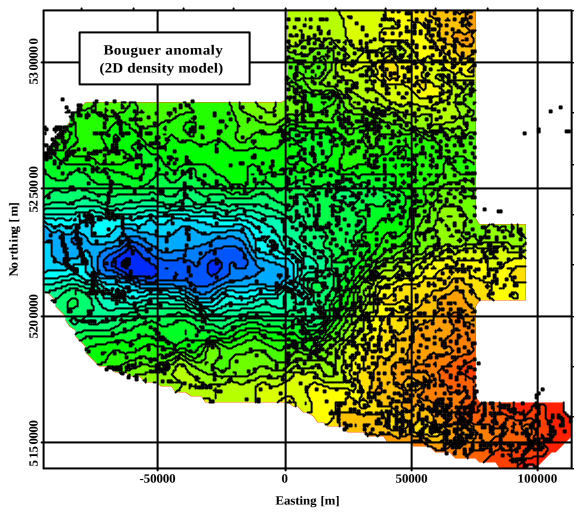
Conclusion

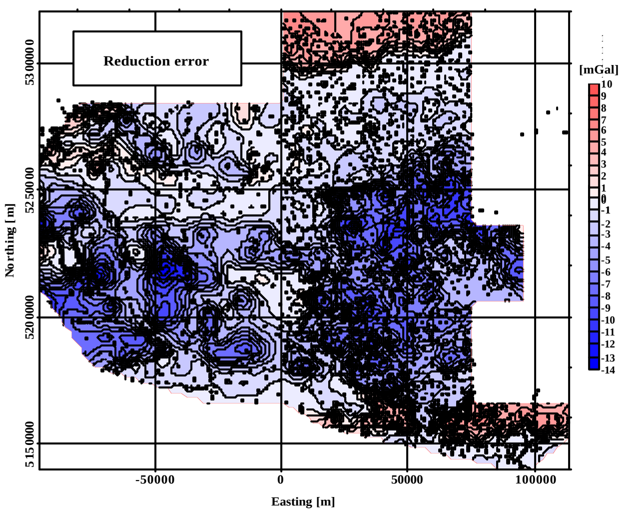
Modern processing and interpretation methods are required to obtain high accurate gravity field determination in rugged terrain:



2D density distribution of Austria (area of investigation indicated by square)







Comparison of Bouguer gravity calculated by using different density distributions
upper left: constant denstity
upper right: laterally varying density
left: error due to wrong density assumption

Normal gravity correction

- Ellipsoidal heights (station) are proposed to be used.

Height correction

- Closed expressions for the normal gravity as a function of the ellipsoidal height or Taylor series expansions have to be applied in order to consider the non-linearity of the vertical gradient.
- Atmospheric correction has to be included.
- Gravity field continuation between irregular surfaces (equivalent source methods) removes the effect of the anomalous vertical gradient.

Mass correction

- High precise terrain modeling is required especially in close vicinity of gravity stations.
- High precise station coordinates are required.
- Gravity effect of lakes and glaciers in the vicinity of gravity stations has to be corrected because of the extreme density contrast between rocks and water or ice.
- Corrections have to be calculated spherically up to a radius of at least 167 *km*.
- DTMs have to be based on ellipsoidal heights.
- 2D density models help to detect and to reduce mass correction errors.

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