TEL AVIV UNIVERSITY אוניברסיטת תל-אביב
RAYMOND AND BEVERLY SACKLER FACULTY OF EXACT SCIENCES
DEPARTMENT OF GEOSCIENCES
הפקולטה למדעים מדויקים ע"ש ריימונד ובברלי סאקלר
החוג למדעי כדור הארץ

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P-wave Attenuation with Implications for Earthquake Early Warning

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Overview

- P-wave attenuation equations
  - Used for real-time magnitude estimations
  - Empirically derived (up to now)

- Deriving P-wave theoretical attenuation laws
- Relying on well established models
  - Brune omega-squared model
  - Parseval's theorem
- Verification against Japan and California data
- Implications for Earthquake Early Warning (EEW)
- Prospective

Theory

Source Spectra

$\omega = 2\pi f$: Frequency
$\Omega(\omega)$: Displacement spectral amplitude
$\dot{\Omega}(\omega)$: Displacement spectral amplitude
Ω_0: Low Frequency plateau
$\omega_0 = 2\pi f_0$: Corner frequency

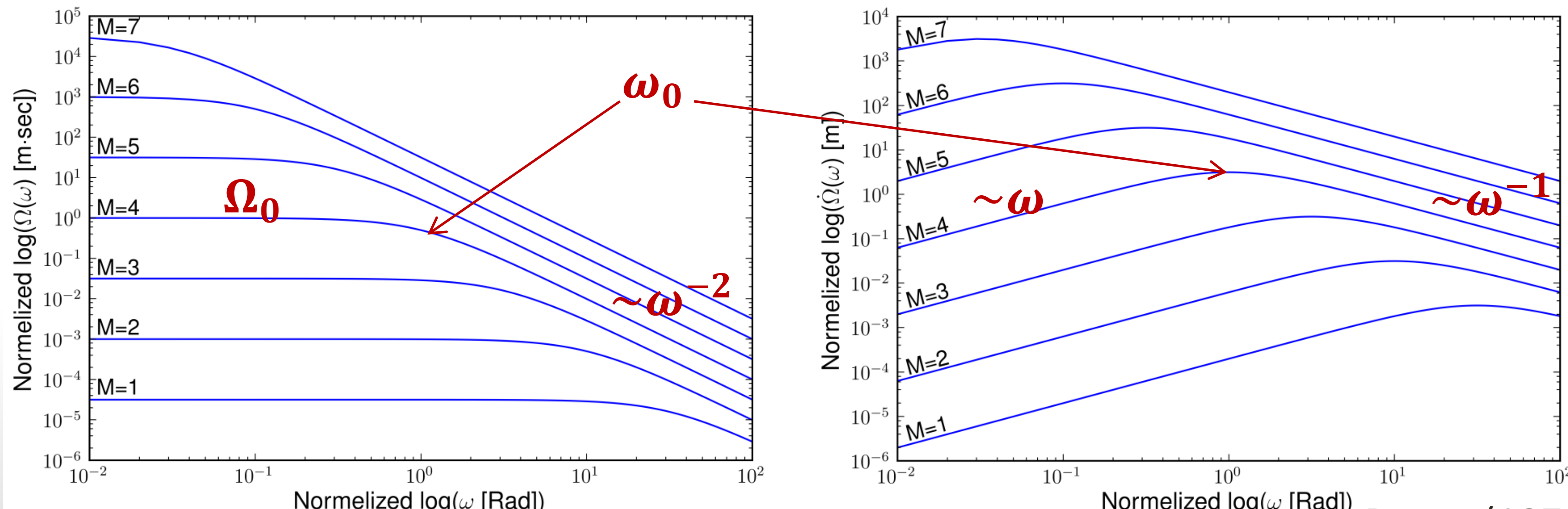
- Brune omega-squared model

- <u>Displacement</u>

$$\Omega(\omega) = \frac{\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$

- <u>Velocity</u>

$$\dot{\Omega}(\omega) = \frac{\omega\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$



Brune (1970)

Theory

Source Spectra

- Displacement

$$\Omega(\omega) = \frac{\Omega_0}{1+\left(\frac{\omega}{\omega_o}\right)^2}$$

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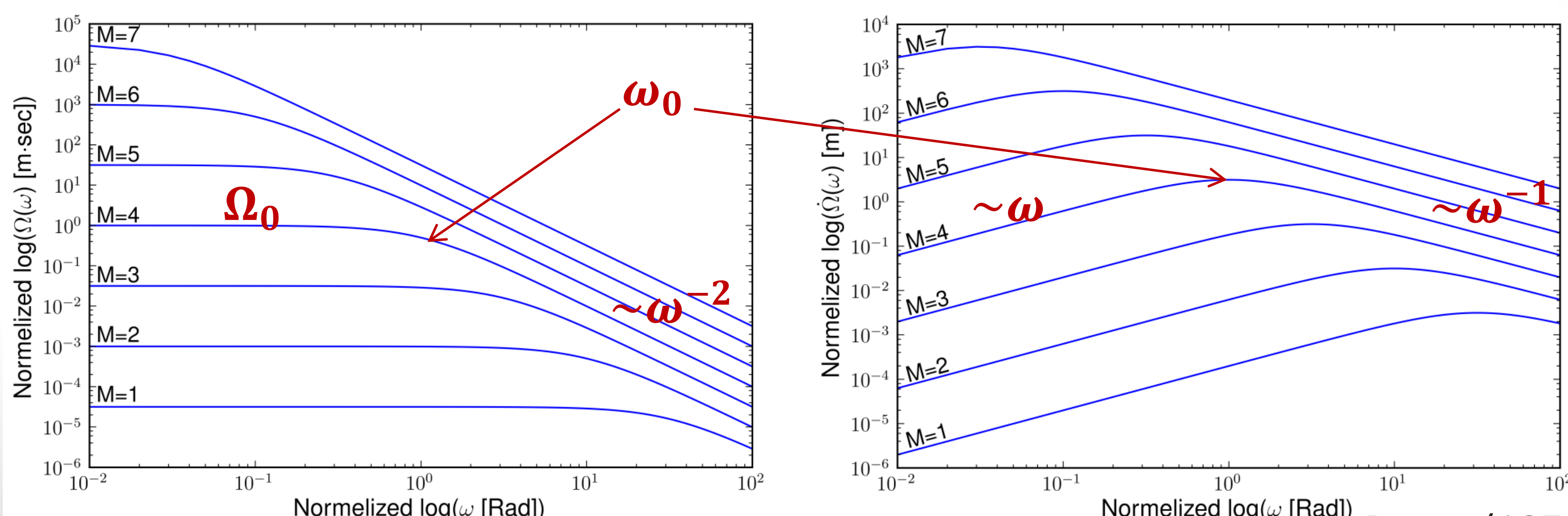


Brune (1970)

Theory

Source Spectra

M_0: Seismic moment
$U_{\varphi\theta}$: Radiation pattern
F_s: Free surface correction
ρ: Density
C_P: P-wave velocity
R: Hypocentral distance

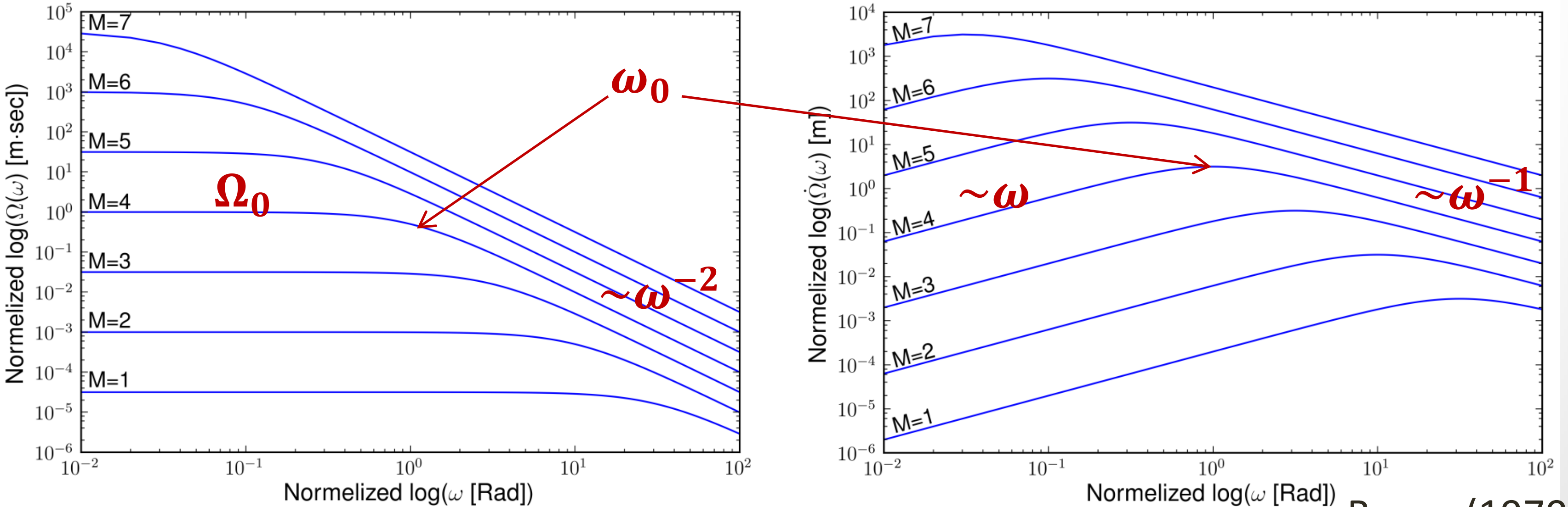
- Displacement

$$\Omega(\omega) = \frac{\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$

- Velocity

$$\dot{\Omega}(\omega) = \frac{\omega\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$

$$\Omega_0 = \frac{M_0 U_{\varphi\theta} F_s}{4\pi\rho C_P^3 R}$$



Brune (1970)

Theory

Source Spectra

M_0: Seismic moment
$U_{\varphi\theta}$: Radiation pattern
F_s: Free surface correction
ρ: Density
C_P: P-wave velocity
R: Hypocentral distance
k: Constant
C_S: S-wave velocity
r: Rupture radius

- Displacement

$$\Omega(\omega) = \frac{\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$

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$$\dot{\Omega}(\omega) = \frac{\omega\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$

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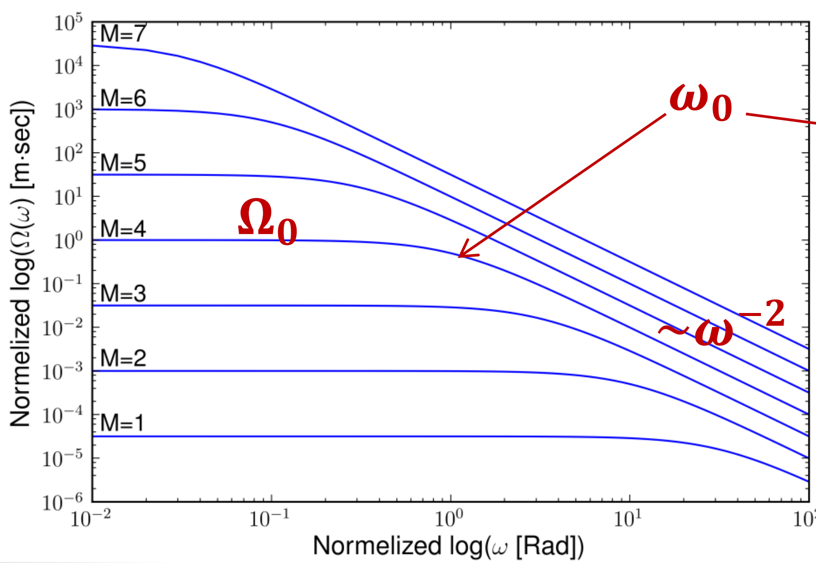

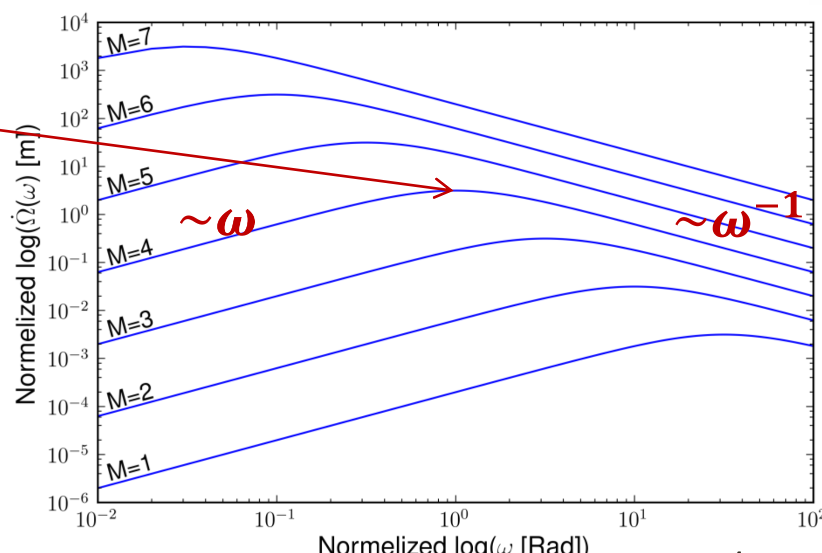

Brune (1970)

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ρ: Density
C_P: P-wave velocity
R: Hypocentral distance
k: Constant
C_S: S-wave velocity
r: Rupture radius
$\Delta\tau$: Stress drop

- Displacement

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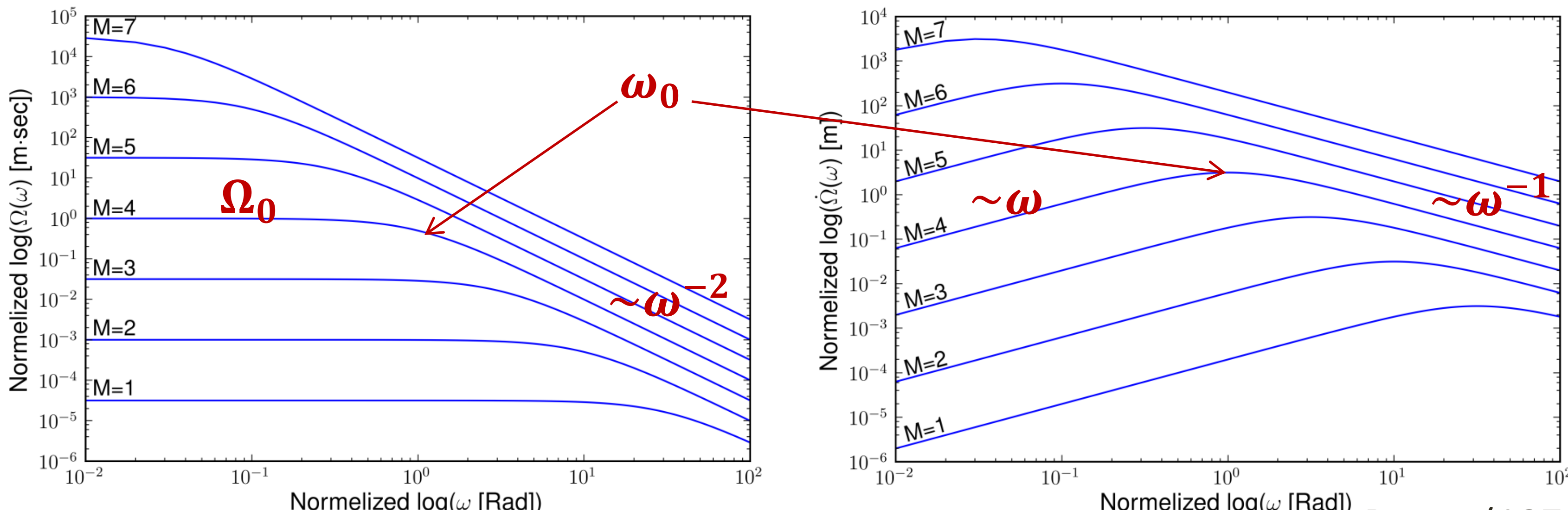
$$\Omega_0 = \frac{M_0 U_{\varphi\theta} F_s}{4\pi\rho C_P^3 R} \qquad f_0 = \frac{kC_S}{r}$$

$$M_0 = \frac{16}{7}\Delta\tau r^3$$ Eshelby (1957)



Brune (1970)

$y(t)$: Time series
$Y(f)$: Fourier transform of $y(t)$

Theory

Frequency Domain Root-Mean-Square

- Time domain rms:

$$y_{rms} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} |y(t)|^2 dt}$$

- Parseval's theorem:

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-\infty}^{\infty} |Y(f)|^2 df$$

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$$y_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} |\mathrm{Y}(f)|^2 df}{T_{S-P}}}$$

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- Time Window:

$$T_{S-P} = R\left(\frac{1}{C_S} - \frac{1}{C_P}\right) \stackrel{\text{def}}{=} R\eta$$

Theory

Attenuation Law Derivation

$$\Omega(\omega) = \frac{\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2} \qquad \dot{\Omega}(\omega) = \frac{\omega\Omega_0}{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$

$$y_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} |\mathrm{Y}(f)|^2 df}{T_{S-P}}}$$

$$f_0 = \frac{kC_S}{r}$$

$$\Omega_0 = \frac{M_0 U_{\varphi\theta} F_S}{4\pi\rho C_P^3 R}$$

$$M_0 = \frac{16}{7}\Delta\tau r^3$$

$$T_{S-P} = R\eta$$

Theory

Attenuation Law Derivation

$$d_{rms} = \Omega_0 \sqrt{\frac{\pi}{2} \frac{f_0}{T_{S-P}}}$$

$$v_{rms} = 2\pi\Omega_0 \sqrt{\frac{\pi}{2} \frac{f_0^3}{T_{S-P}}}$$

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$$y_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} |\Upsilon(f)|^2 df}{T_{S-P}}}$$

$$f_0 = \frac{kC_S}{r}$$

$$\Omega_0 = \frac{M_0 U_{\varphi\theta} F_S}{4\pi\rho C_P^3 R}$$

$$M_0 = \frac{16}{7} \Delta\tau r^3$$

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$$d_{rms} = \Omega_0 \sqrt{\frac{\pi}{2} \frac{f_0}{T_{S-P}}}$$

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$$d_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/6} M_0^{5/6} \frac{1}{R^{3/2}}$$

$$v_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/2} M_0^{1/2} 2\pi k C_S \frac{1}{R^{3/2}}$$

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$$\Omega_0 = \frac{M_0 U_{\varphi\theta} F_s}{4\pi\rho C_P^3 R}$$

$$M_0 = \frac{16}{7}\Delta\tau r^3$$

$$\epsilon = \frac{U_{\varphi\theta} F_s}{4\pi\rho C_P^3} \sqrt{\frac{\pi k C_S}{2\eta}}$$

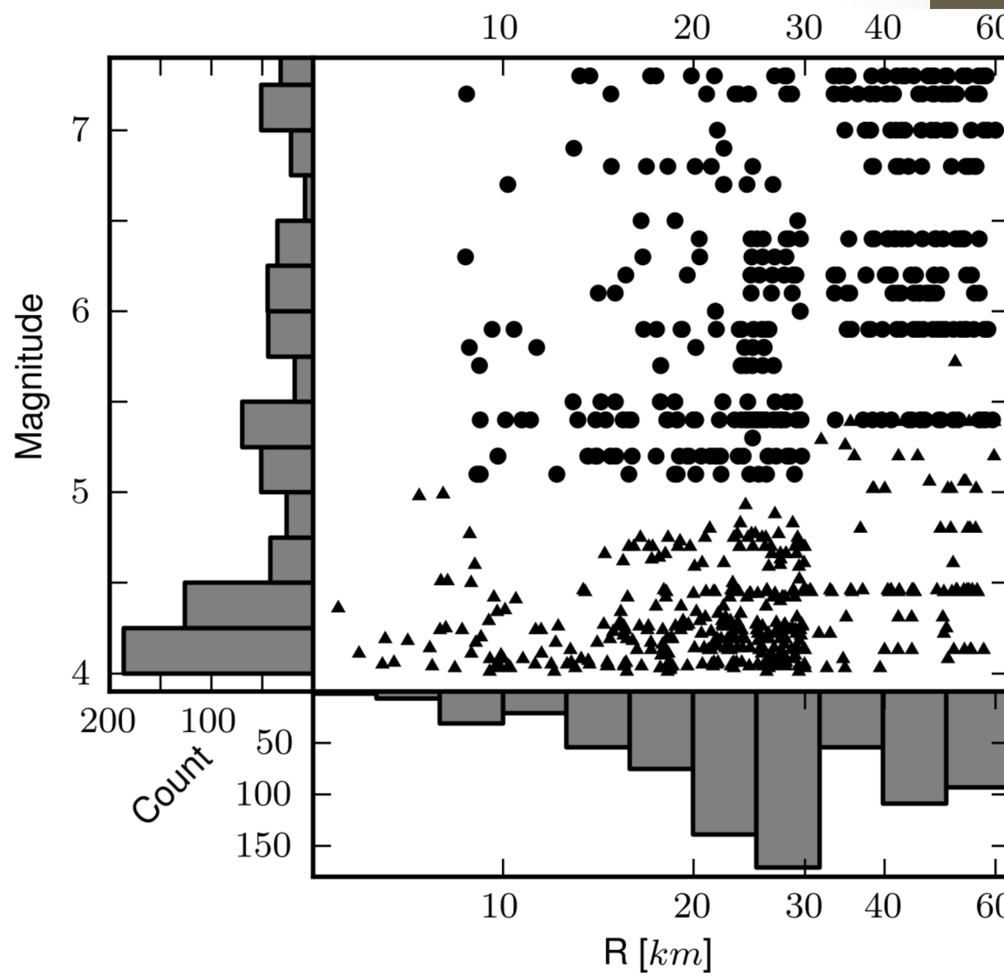
$$T_{S-P} = R\eta$$

Validation

Data

- Japan
  - K-net + KiK surface
  - 353 accelerograms
  - 42 earthquakes
  - 5.1<M<7.3

- California
  - CISN
  - 403 velocity seismograms
  - 120 earthquakes
  - 4<M<5.7



Validation

Parameters

$$d_{rms} = \epsilon \left(\frac{16}{7} \Delta\tau \right)^{1/6} M_0^{5/6} \frac{1}{R^{3/2}}$$

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$$\epsilon = \frac{U_{\varphi\theta} F_s}{4\pi\rho C_P^3}\sqrt{\frac{\pi k C_S}{2\eta}}$$

- $U_{\varphi\theta} = 0.52$
- $F_s = 2$
- $\rho = 2600\ kg/m^3$
- $C_S = 3200\ m/s$
- $C_P = 5333\ m/s$
- $k = 0.32$
- $\eta = 1/8\ \ s/km$
- $\epsilon = 753 \cdot 10^{-15}\ 1/Pa$

AZ, AE, AN: Velocity or Displacement samples

Validation

Parameters

$$d_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/6} M_0^{5/6} \frac{1}{R^{3/2}}$$

$$v_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/2} M_0^{1/2} 2\pi k C_S \frac{1}{R^{3/2}}$$

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- $k = 0.32$
- $\eta = 1/8\ \ s/km$
- $\epsilon = 753 \cdot 10^{-15}\ 1/Pa$
- M_o, R – Catalog
- $rms = \sqrt{\sum_{i=1}^{h} \left[AZ_i^2 + AE_i^2 + AN_i^2\right]/h}$

Validation

Stress Drop Determination

$$d_{rms} = \epsilon\left(\frac{16}{7}\Delta\tau\right)^{1/6} M_0^{5/6}\frac{1}{R^{3/2}}$$

$$v_{rms} = \epsilon\left(\frac{16}{7}\Delta\tau\right)^{1/2} M_0^{1/2} 2\pi k C_S \frac{1}{R^{3/2}}$$

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Stress Drop Determination

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$$\Delta\tau = \frac{7}{16} M_0 \left(\frac{1}{kC_S} \cdot \frac{1}{2\pi} \frac{v_{rms}}{d_{rms}}\right)^3$$

Validation

Stress Drop Determination

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- Recalling Eshelby:

$$M_0 = \frac{16}{7}\Delta\tau r^3$$

$1/r$

Validation

Stress Drop Determination

$$d_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/6} M_0^{5/6} \frac{1}{R^{3/2}} \qquad v_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/2} M_0^{1/2} 2\pi k C_S \frac{1}{R^{3/2}}$$

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- Recalling Eshelby:

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$1/r$

- And brune:

$$f_0 = \frac{kC_S}{r}$$

f_0

$$f_0 = \frac{1}{2\pi}\frac{v_{rms}}{d_{rms}}$$

Andrews (1986) and Snoke (1987)

T_r: Rupture duration

Validation

Stress Drop Determination

- Condition:

$$T_{S-P} > T_r$$

$$T_r = \frac{2r}{0.9C_S}$$

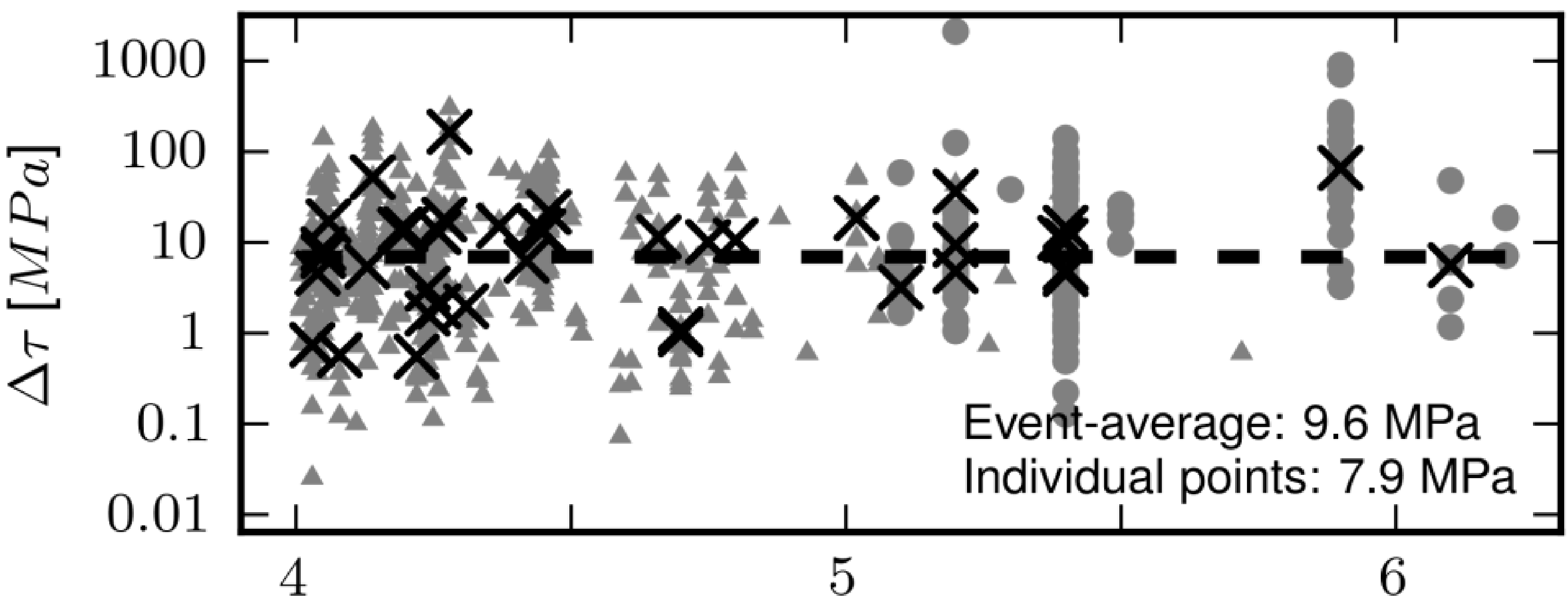
- 67% of data satisfies the condition

Validation

Stress Drop Determination

$$\Delta\tau = \frac{7}{16} M_0 \left(\frac{1}{kC_S} \cdot \frac{1}{2\pi} \frac{v_{rms}}{d_{rms}} \right)^3$$

Median: $\Delta\tau = 7.9 MPa$



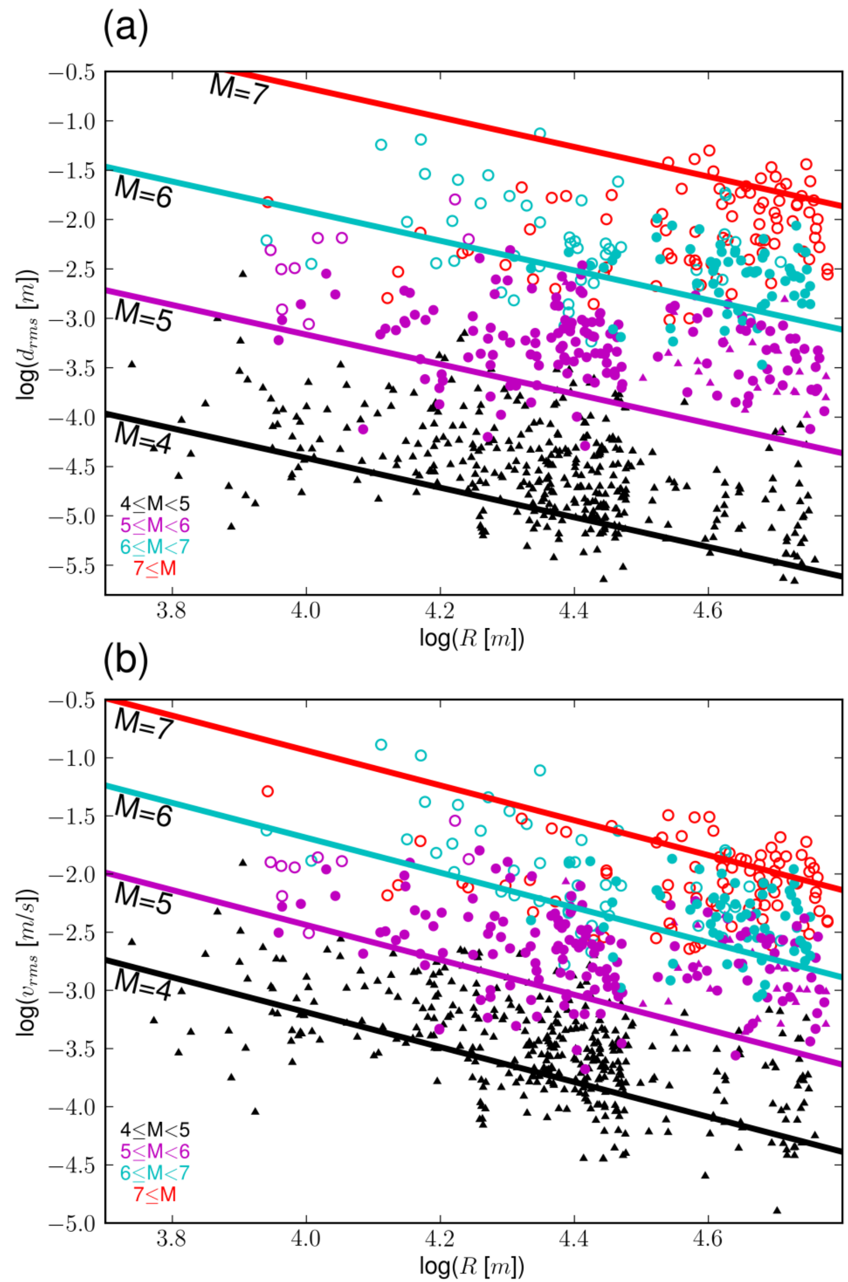
Magnitude

Validation

Observed Attenuation

$$d_{rms} = \epsilon\left(\frac{16}{7}\Delta\tau\right)^{1/6} M_0^{5/6}\frac{1}{R^{3/2}}$$

$$v_{rms} = \epsilon\left(\frac{16}{7}\Delta\tau\right)^{1/2} M_0^{1/2}2\pi k C_S\frac{1}{R^{3/2}}$$



Validation

Observed Attenuation

- Displacement

$$d_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/6} M_0^{5/6} \frac{1}{R^{3/2}}$$

- Velocity

$$v_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/2} M_0^{1/2} 2\pi k C_S \frac{1}{R^{3/2}}$$

Validation

Observed Attenuation

- Displacement

$$d_{rms} = \epsilon \left(\frac{16}{7} \Delta\tau \right)^{1/6} M_0^{5/6} \frac{1}{R^{3/2}}$$

- Velocity

$$v_{rms} = \epsilon \left(\frac{16}{7} \Delta\tau \right)^{1/2} M_0^{1/2} 2\pi k C_S \frac{1}{R^{3/2}}$$

$$M_0 = \frac{16}{7} \Delta\tau r^3$$

Validation

Observed Attenuation

- Displacement
- Velocity

$$d_{rms} = \epsilon \left(\frac{16}{7}\Delta\tau\right)^{1/6} M_0^{5/6} \frac{1}{R^{3/2}}$$

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$$M_0 = \frac{16}{7}\Delta\tau r^3$$

$$d_{rms} = \epsilon \frac{16}{7}\Delta\tau \left(\frac{r^{5/3}}{R}\right)^{3/2}$$

$$v_{rms} = \epsilon \frac{16}{7}\Delta\tau 2\pi k C_S \left(\frac{r}{R}\right)^{3/2}$$

$$\epsilon = \frac{U_{\varphi\theta} F_S}{4\pi\rho C_P^3} \sqrt{\frac{\pi k C_S}{2\eta}}$$

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Observed Attenuation

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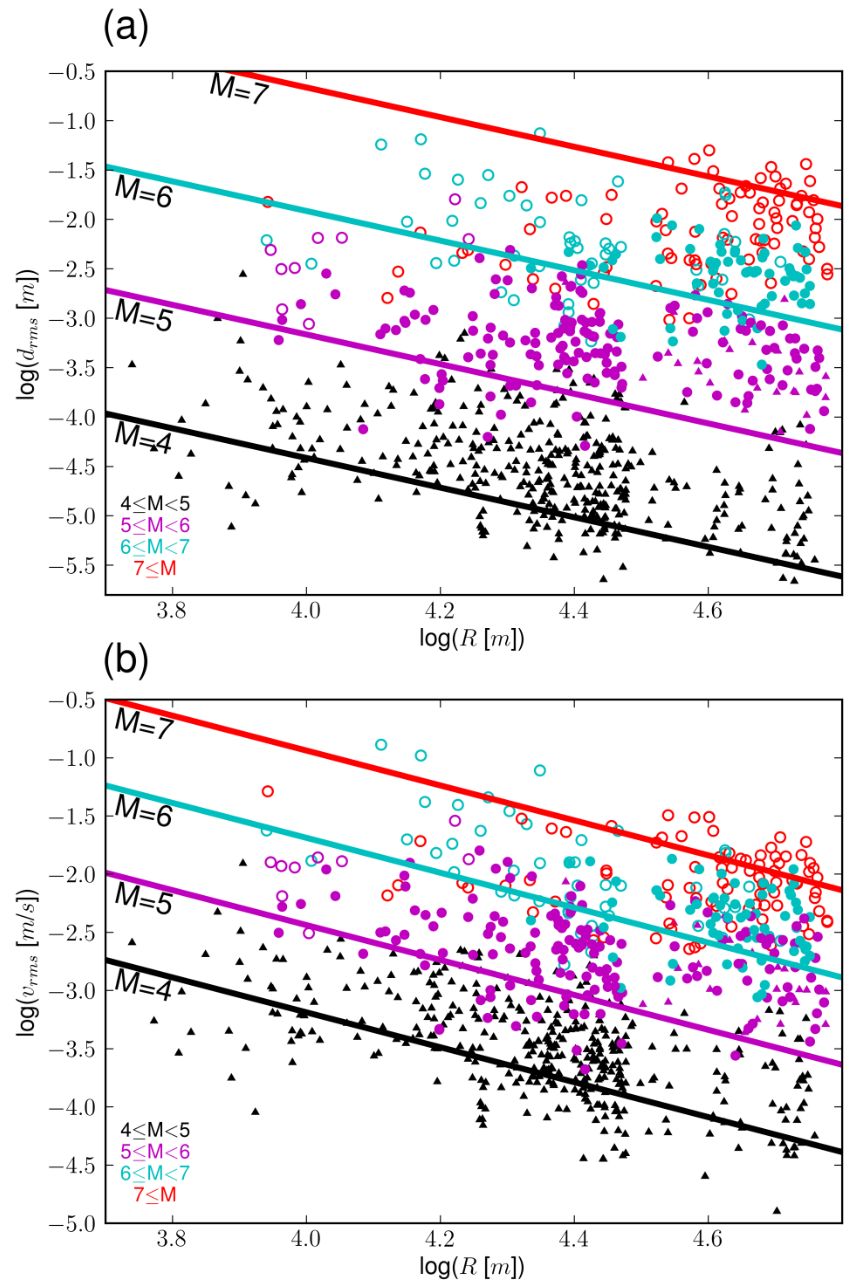

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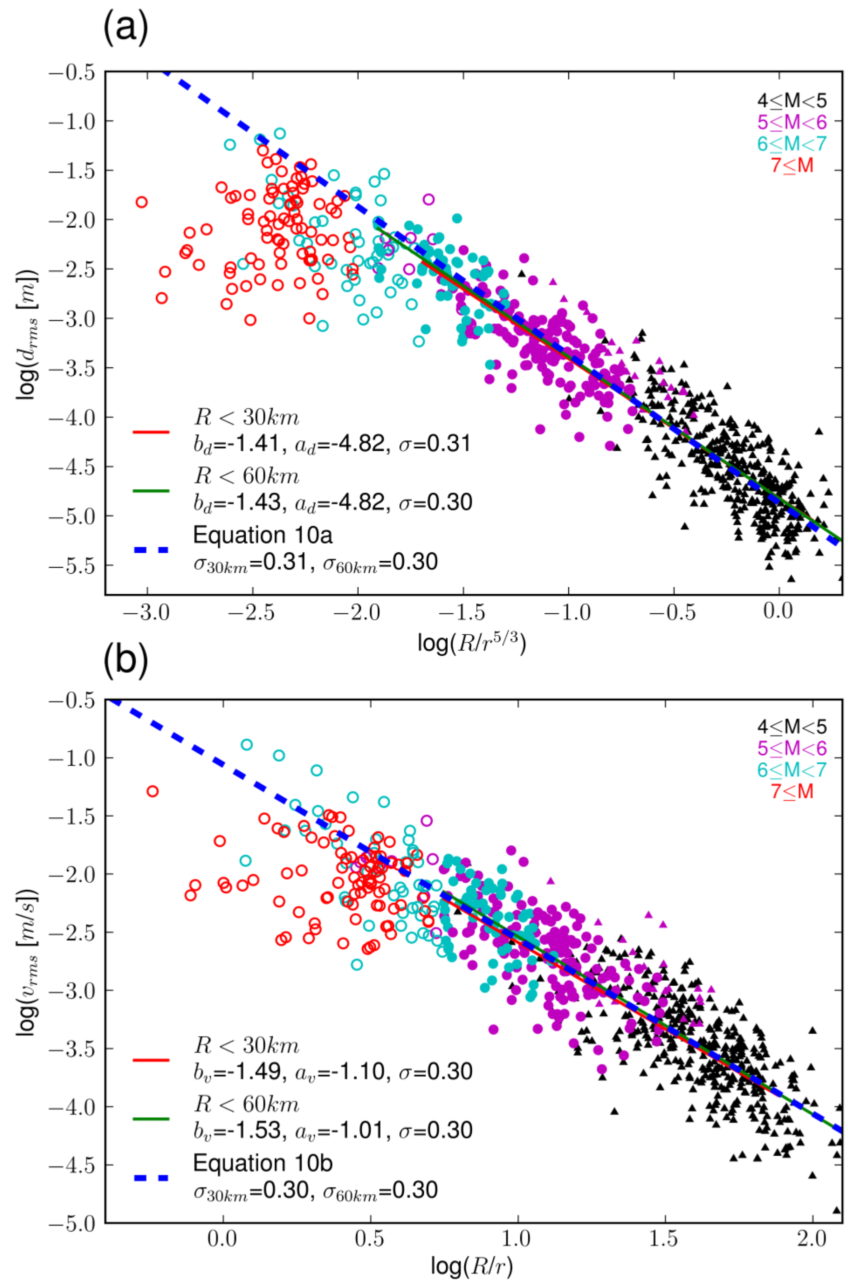


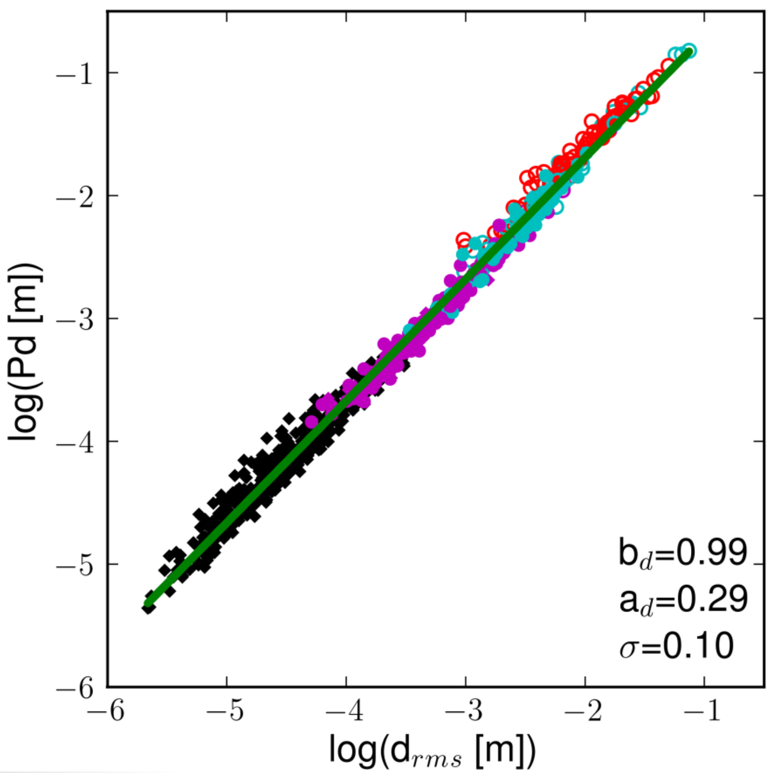
Validation

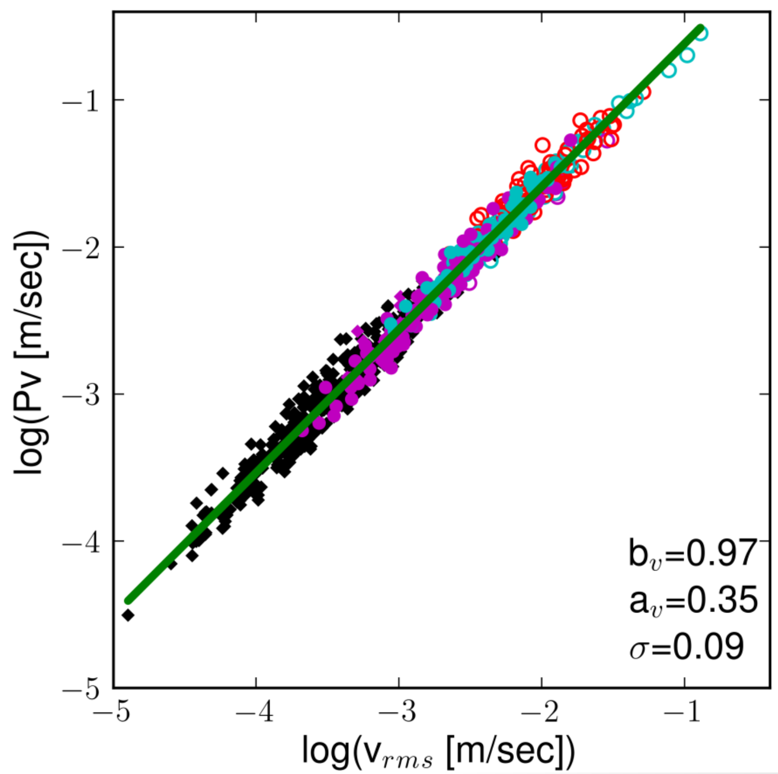
Implications for EEW: *rms* to Peak Amplitude

- Displacement
- $Pd = 2.0(\pm 0.5)d_{rms}$

- Velocity
- $Pv = 2.3(\pm 0.5)v_{rms}$

$$Pd\ or\ Pv = \max\left[\sqrt{AZ_i^2 + AE_i^2 + AN_i^2}\right]$$





u: Displacement time series
v: Velocity time series

Validation

Implications for EEW: τ_c

- Characteristic period
- Real-time magnitude proxy
  - $\tau_c - Pd$ EEWS

$$\tau_c \stackrel{\text{def}}{=} 2\pi \sqrt{\frac{\int_{t_0}^{t_0+\Delta t} u^2 dt}{\int_{t_0}^{t_0+\Delta t} v^2 dt}} = 2\pi \frac{d_{rms}}{v_{rms}}$$

Wu and Kanamori (2005)

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v: Velocity time series

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$$f_0 = \frac{1}{2\pi} \frac{v_{rms}}{d_{rms}}$$

Wu and Kanamori (2005)

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$$\tau_c = \frac{1}{f_0}$$

$$f_0 = \frac{1}{2\pi} \frac{v_{rms}}{d_{rms}}$$

$$\tau_c = \frac{1}{kC_s} \sqrt[3]{\frac{7}{16} \frac{M_0}{\Delta\tau}}$$

$$\Delta\tau = \frac{7}{16} M_0 \left(\frac{1}{kC_s} \cdot \frac{1}{2\pi} \frac{v_{rms}}{d_{rms}} \right)^3$$

Wu and Kanamori (2005)

Discussion

Magnitude Estimations

- Attenuation laws show:

$$d_{rms} \propto \Delta\tau^{1/6} {M_0}^{5/6} R^{-3/2}$$

$$v_{rms} \propto \Delta\tau^{1/2} {M_0}^{1/2} R^{-3/2}$$

Discussion

Magnitude Estimations

$$M_0 \propto {d_{rms}}^{1.2} \Delta\tau^{-0.2} R^{1.8}$$

$$M_0 \propto {v_{rms}}^{2} \Delta\tau^{-1} R^{3}$$

Discussion

Magnitude Estimations

$$M_0 \propto {d_{rms}}^{1.2} \Delta\tau^{-0.2} R^{1.8}$$

$$M_0 \propto {v_{rms}}^{2} \Delta\tau^{-1} R^{3}$$

$$\tau_c \longleftrightarrow M_0 \propto {v_{rms}}^{3} {d_{rms}}^{-3} \Delta\tau$$

Discussion

Magnitude Estimations

$$M_0 \propto {d_{rms}}^{1.2} \Delta\tau^{-0.2} R^{1.8}$$

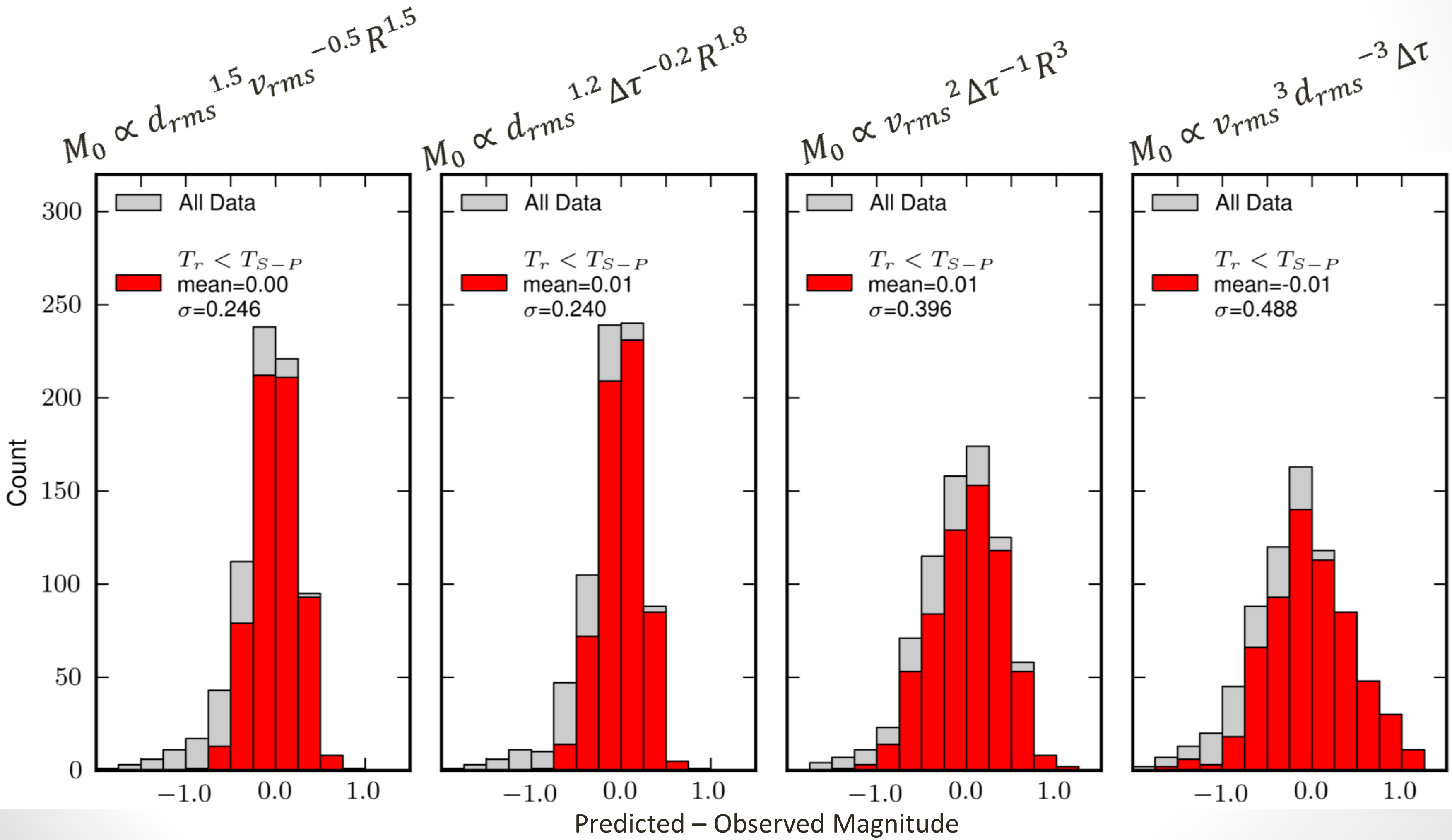
$$M_0 \propto {v_{rms}}^{2} \Delta\tau^{-1} R^{3}$$

$$\tau_c \longleftrightarrow \quad M_0 \propto {v_{rms}}^{3} {d_{rms}}^{-3} \Delta\tau$$

$$M_0 \propto {d_{rms}}^{1.5} {v_{rms}}^{-0.5} R^{1.5}$$

Discussion

Magnitude Estimations



Conclusion

- New **theoretical** attenuation laws
  - Provide source parameter weightings
  - Better stress drop and seismic moment understanding
- New stress drop estimation schemes
- Verified for Japan and California
- Earthquake Early Warning (EEW) implications
  - Little calibration needed
  - rms – peak amplitude relation
  - ${d_{rms}}^{1.5}/{v_{rms}}^{0.5}$ magnitude estimation
  - $\tau_c = 1/f_0$
- prospective
  - Stress drop estimation
  - Ground motion prediction equation