FLAT ACCELERATION SOURCE SPECTRUM IS AN ORDINARY PROPERTY OF STOCHASTIC SELF-SIMILAR EARTHQUAKE FAULT WITH PROPAGATING SLIP PULSE

A.A. Gusev

Institute of Volcanology and Seismology, Russian Ac. Sci. and
Kamchatka Branch, Geophysical Service, Russian Ac. Sci.
Petropavlovsk-Kamchatsky, Russia

e-mail: gusev@emsd.ru

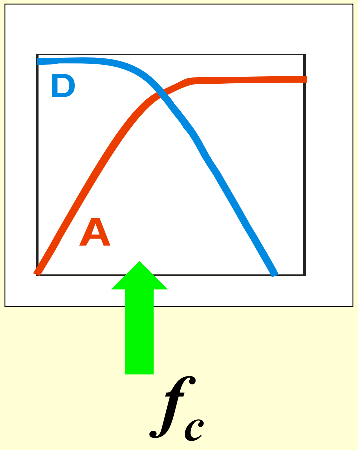
The "Double Stochastic Fault Model" (DSFM) is proposed in order to explain 3 common properties of earthquakes:

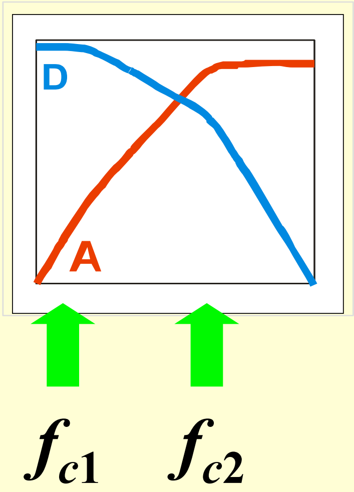
- **(1)** ω^{-2} **["omega-square"]** shapes of (*displacement*) source spectra [Aki 1967; Brune 1970]

  [or, equivalently, **flat** ***acceleration*** source spectra [Brune 1970; Hanks&McGuire 1981]]

- **(2) two-corner** ($\omega^{0} - \omega^{-1} - \omega^{-2}$) source spectra (typical for larger magnitudes) [Brune 1970;Gusev 1983]

- **(3) frequency-dependent directivity** [e.g. Somerville 1999]



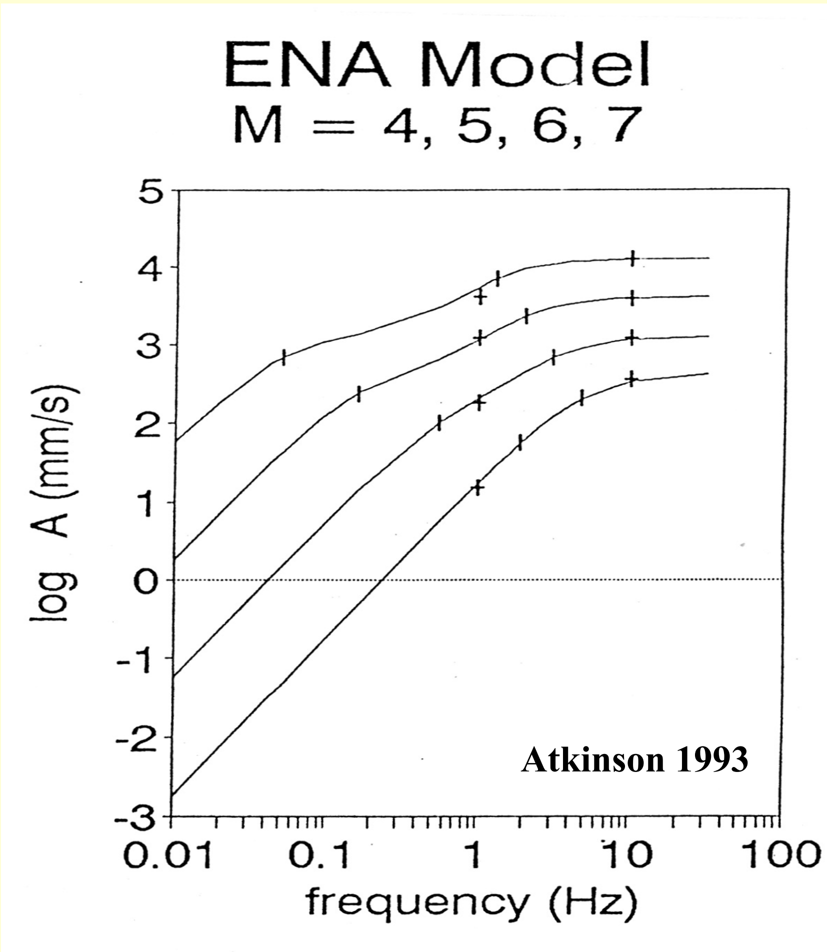


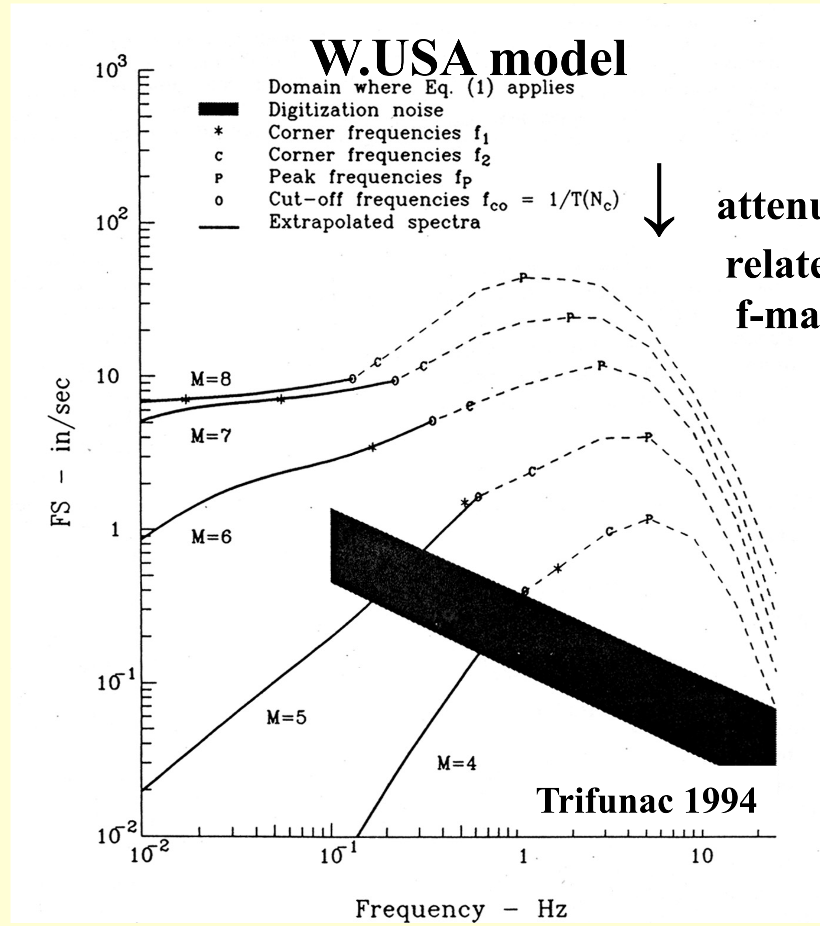
these three properties are well-known,

all three lack consistent theoretical explanation

Empirical scaling laws for Fourier acceleration spectra, with flat HF part
they approximate source acceleration spectral shapes shapes

note clear two-corner spectral shapes
with gap between corners increasing with magnitude

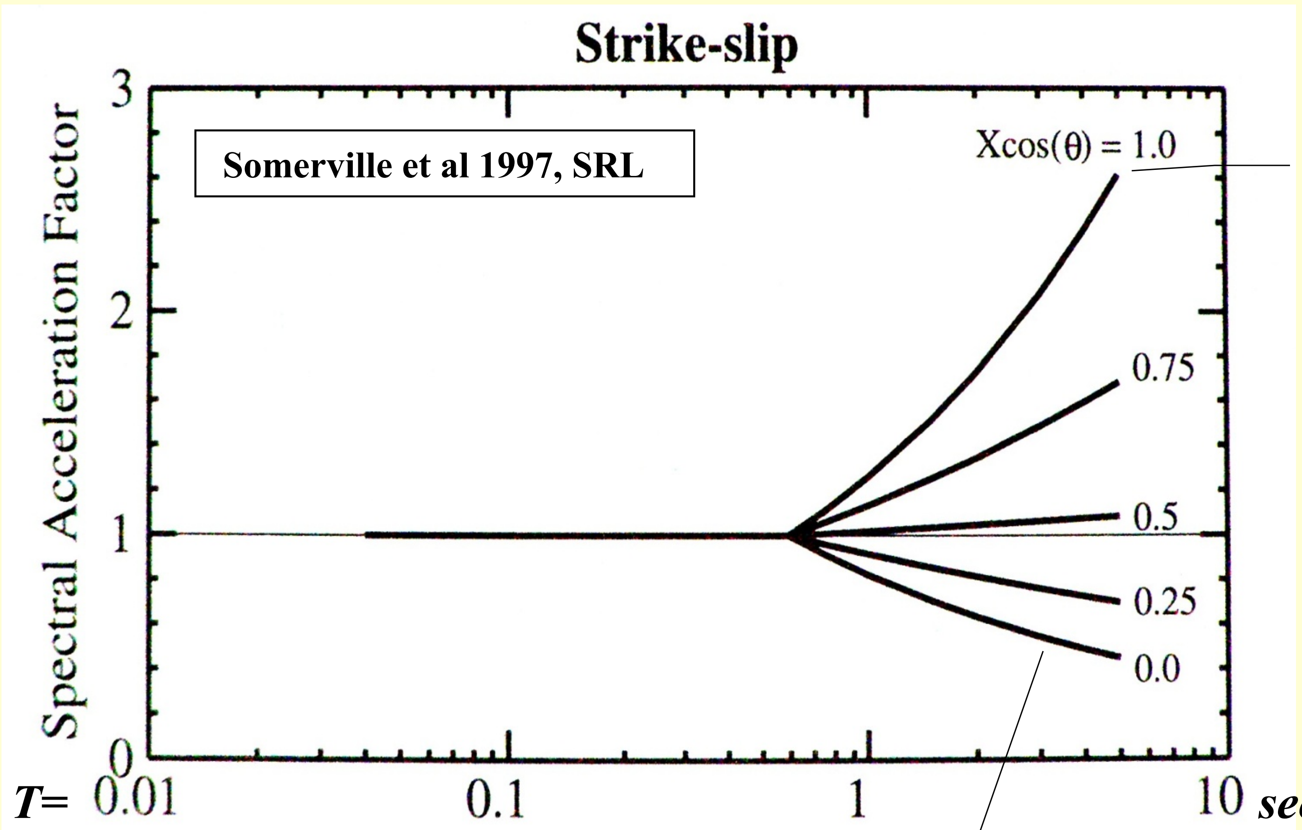




attenuation-related f-max

Period/frequency dependence of the average directivity factor

of response-spectrum acceleration RSA (RSA ≈ peak of narrow-band-filtered acceleration) for a set of angles between forward direction of propagation and the ray to receiver



***f*= 100 10 0.1 Hz**

forward

backward

Xcos(theta):

X measures degree of unilaterality

(0 for bilateral)

theta is anglular deviation from forward direction (0 to 180)

Key components of DSFM

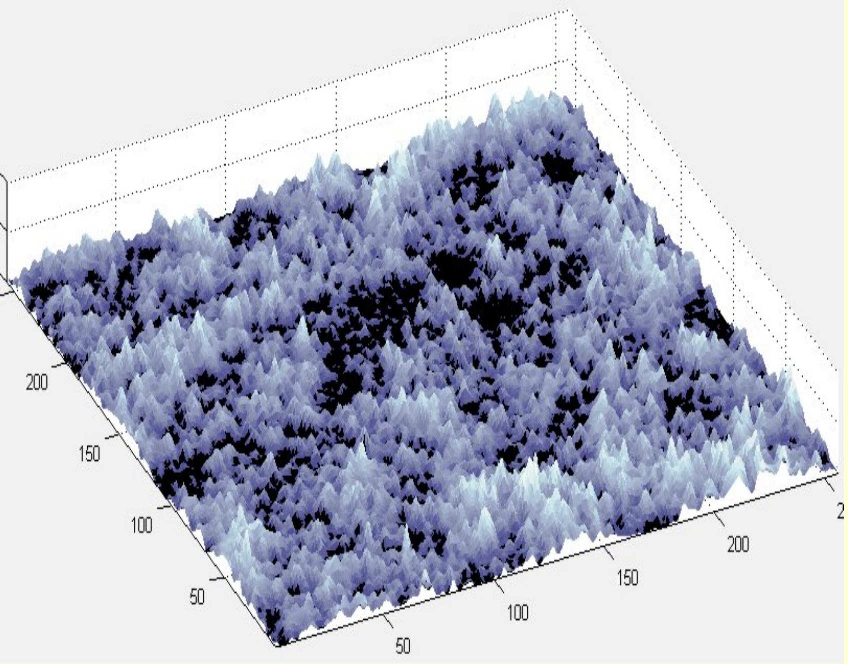
- 1. Final (local) stress drop field **$\Delta\sigma(x,y)$** on a fault :
  random, fractal [Andrews 1980]

- 2. Rupture-front structure **at high *k*, *locally*:**
  random, fractal, disjointed, tortuous [Gusev 2012];

- 3. Rupture-front propagation mode **at low *k*, [smoothed picture]:**
  systematic, following the concept of running **slip pulse**
  [Heaton 1990; Haskell 1964,1966]

- 4. Formation of seismic waves:
  according to the **fault asperity failure model**
  [Das&Kostrov 1983,1986; Boatwright 1988]

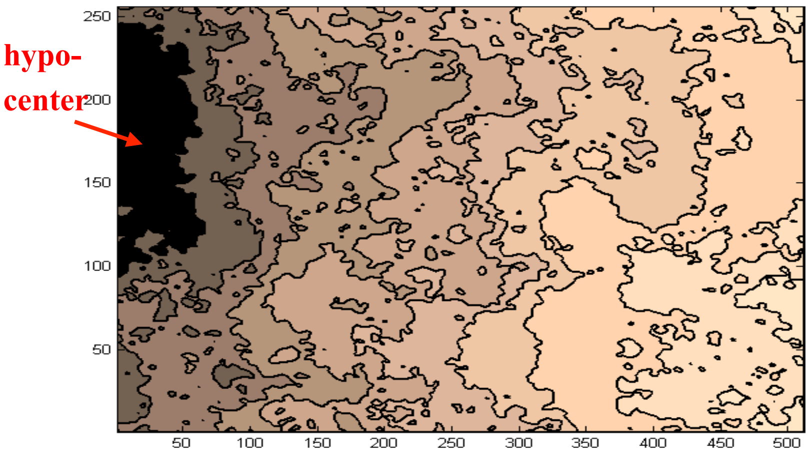
The stochastic/random component #1 of the DSFM (*time-independent*) : local stress drop $\Delta\sigma(x, y)$ field [*Andrews* 1980]



$\Delta\sigma(x, y)$ is a random (assumedly isotropic) 2D field, defined through its power spectrum $P(k)$, or through mean amplitude spectrum $\boldsymbol{S(k)} \propto P^{0.5}(k)$ [Andrews 1980] (k=|**k**|)

- $\boldsymbol{S(k)}$ is power law ($\boldsymbol{S(k)} \propto k^{-\beta}$) [self-affine, or broad-sense self-similar or fractal behavior] [Andrews 1980]; and in particular:
- $\boldsymbol{\beta=1}$ and $\boldsymbol{S(k)} \propto k^{-1}$ [narrow-sense self-similar behavior] [Andrews 1980]
- $\Delta\sigma(x, y)$ is rigidly tied to final dislocation/slip field $\boldsymbol{D(x, y)}$ [with spectrum $\propto k^{-\beta-1}$]

The stochastic component #2 of the DSFM (defines space-time evolution): local rupture-front structure at high *k* [*Gusev* 2012]



example isolines of $t_{fr}(x, y)$
shading: the later, the lighter

- defines **time history of rupture** through **rupture front arrival time**

  $t_{fr}(x, y)$

- $t_{fr}(x, y)$ has "lacy" general appearance, with:
  (a) tortuous or wiggling isolines
  (b) fragmented, with "islands" and "lakes"

- $t_{fr}(x, y)$ can be represented as superposition of

(1) smooth [***low-k***] global/"macroscopic" rupture propagation, with well-defined rupture velocity (*traditional element*); and

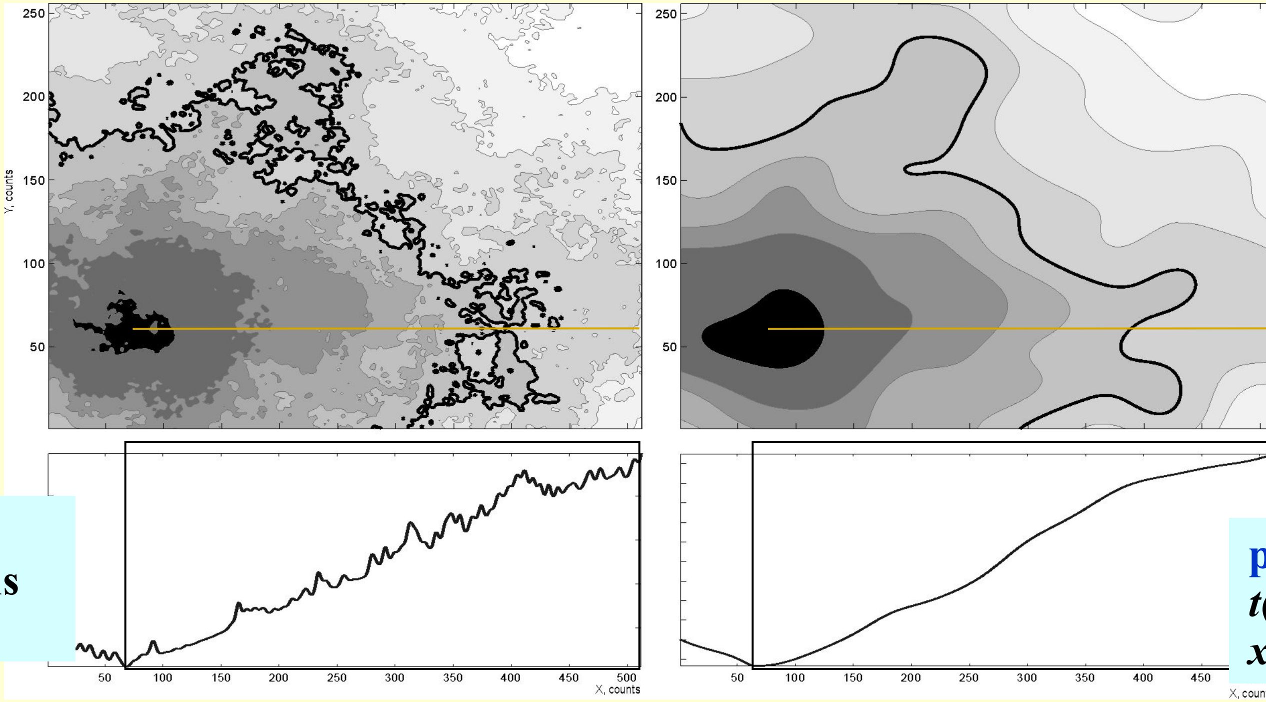
(2) random ***high-k*** local/"microscopic" rupture propagation at random directions (*novel element; **creates incoherence***)

local rupture-front structure at high k (cont. 1)

$t_{fr}(x, y)$: original, $t_{fr}(x, y)$: smoothed,

full k-spectrum isolines fragmented

low-k part only isolines continuous



profile: yellow line:
t(x) **non-monotonous**
x(t) **multiple-valued**

profile: yellow line:
t(x) **monotonous**
x(t) **single-valued**

Creating $t_{fr}\ (x,y)$

$$t_{fr}\ (x, y) = R\ (x, y) + S\ (x, y)$$

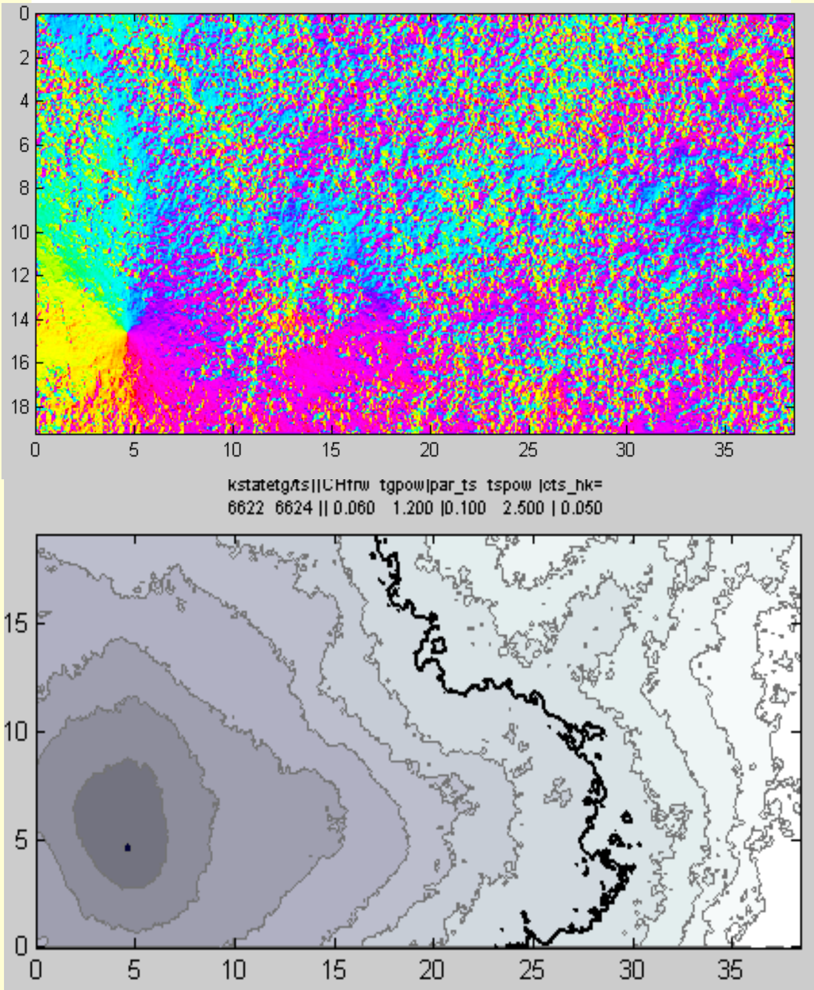
$R\ (x, y)$: **random** self-similar, spectrum $\propto 1/k^{\delta}$; δ=1-1.5;

$S\ (x, y)$: **smooth**; deterministic term with constant velocity + perturbation

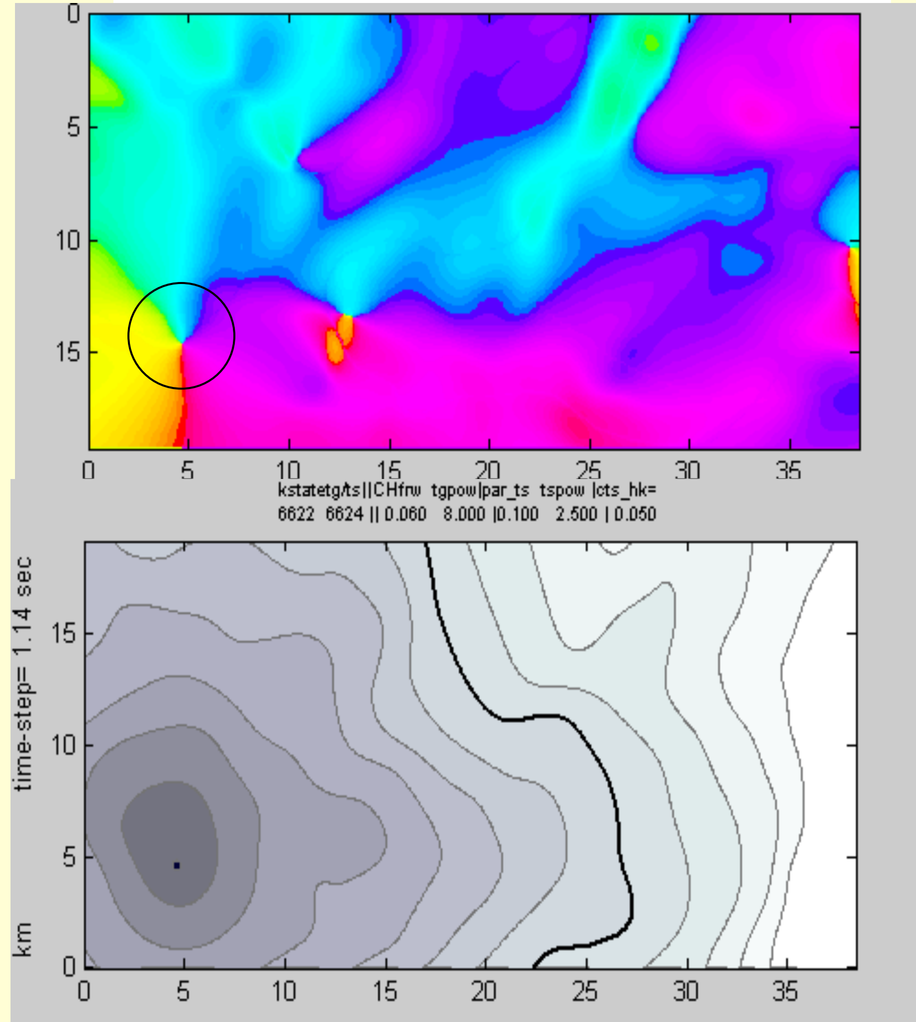
The cause of incoherence is manifested in local rupture front orientation

Color code: local direction of rupture-front normal

incoherent case (used in simulation)

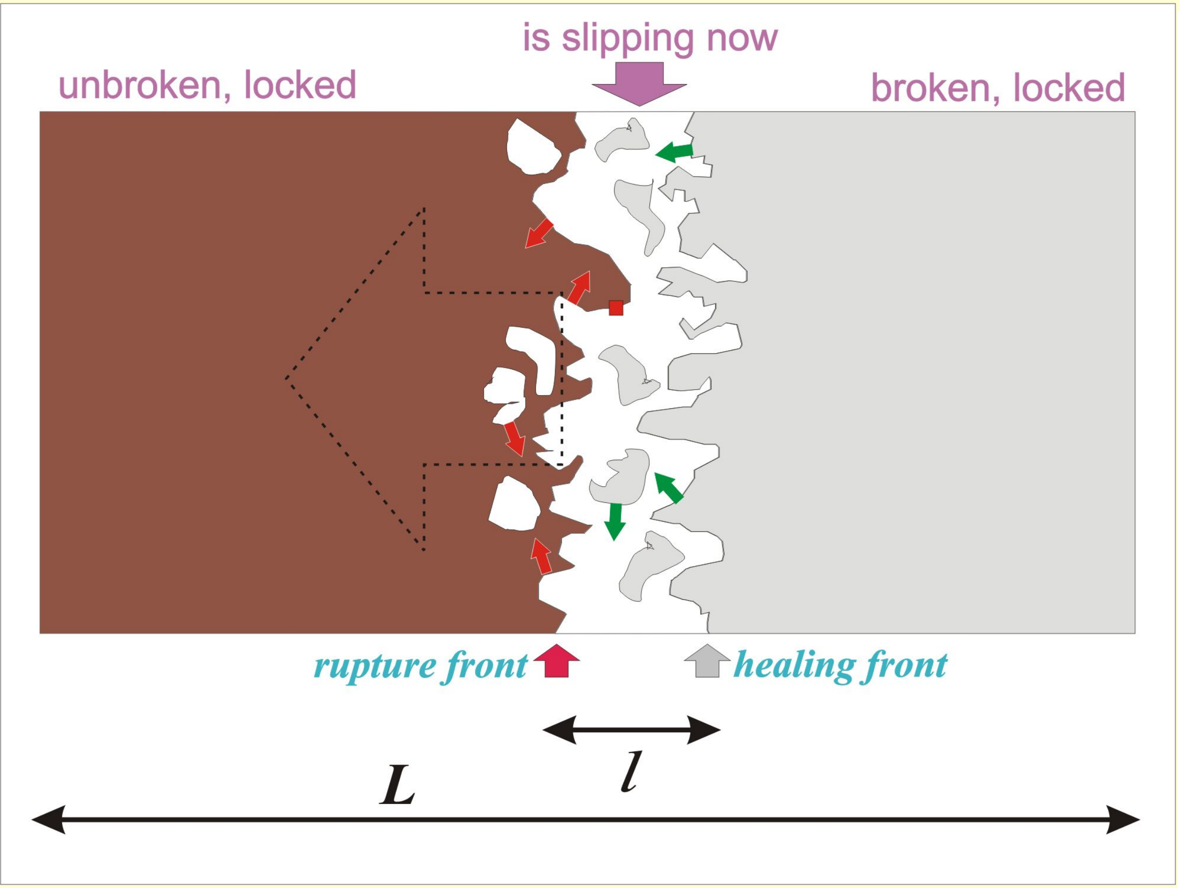


coherent case (not used, example only)



Component #3 of the DSFM (non-stochastic):

"slip-pulse" rupture propagation, with **healing front** **[Heaton 1990]**



• detailing time history of rupture through **healing front** evolution

• introducing critical parameter **"relative pulse width"**

$$C_H = {}^{l}\!/_{L}$$

where: ***L*** is **fault length**;
l is **slip pulse width**

in other terms

$$C_H = \frac{T_{rise}}{L / v_{rup}}$$

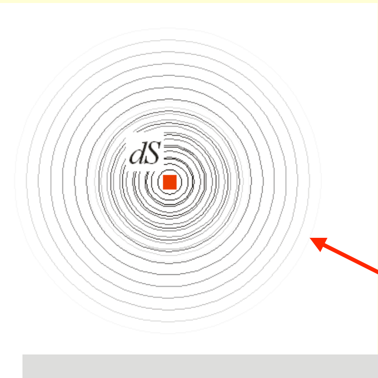
where:
T_{rise} is **rise time**;
v_{rup} is **(mean) rupture velocity**;

typical values of C_H :
around 0.1 [*Heaton* 1990]

Component #4 of the DSFM: Using **fault asperity failure theory** [Das&Kostrov 1983,1986; **Boatwright1988**] to describe body wave generation

Consider failing fault spot *dS* at position *(x, y)* on a fault **Σ** surrounded by a region of negligible cohesion: infinite (*Case* 1) or finite, size $2R_r$ (*Case* 2). Rupture front arrives to *dS* at time t_{fr}.

For far-field *SH* body wave, consider velocity time history on along-normal ray: $\dot{u}^{SH}(\xi, t + R/c_S)$

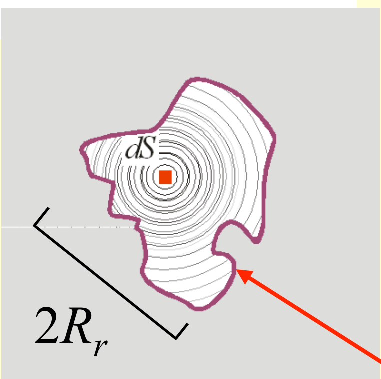


***Case* 1**: infinite fault

$$d\dot{u}^{SH,\infty}(\xi, t + R/c_S) = A\Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

$$\dot{u}^{SH,\infty}(\xi, t + R/c_S) = A\int_\Sigma \Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

Fault-guided waves (P, inhomogeneous S and R) go to infinity



***Case* 2**: finite fault

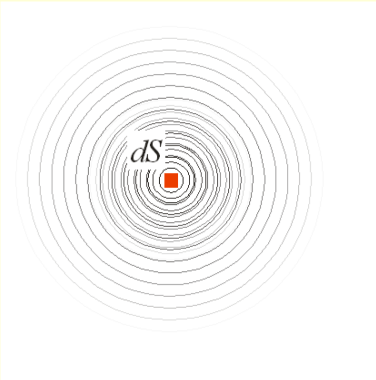
$$d\dot{u}^{SH}(\xi, t + R/c_S) = G(t) * A\Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

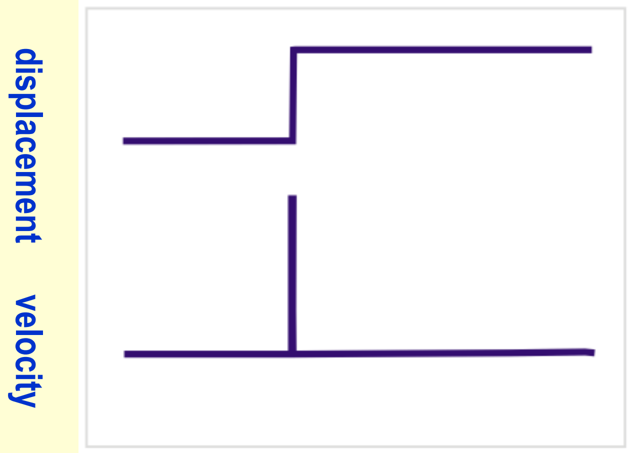
$$\dot{u}^{SH}(\xi, t + R/c_S) = G(t) * A\int_\Sigma \Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

with specific *G*(*t*)=*G*(*t,x,y*), **of zero integral** and of duration on the order $2R_r/c_R$

Fault-guided waves (P,S and R) diffract/transform to regular body waves at the boundary and die off

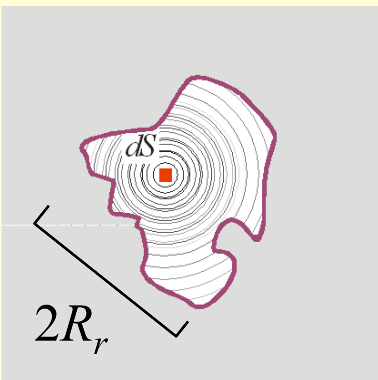
Fault asperity failure theory, continued: far field body waves

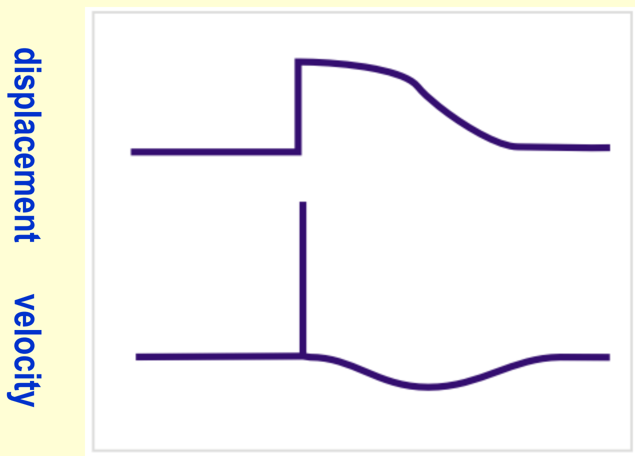




Case 1: infinite fault with asperity ***dS***

infinite seismic moment





Case 1: finite fault with asperity ***dS***

seismic moment is on the order

$dM\mathrm{o} = 2R_r\, dF$

where dF is **seismic force**

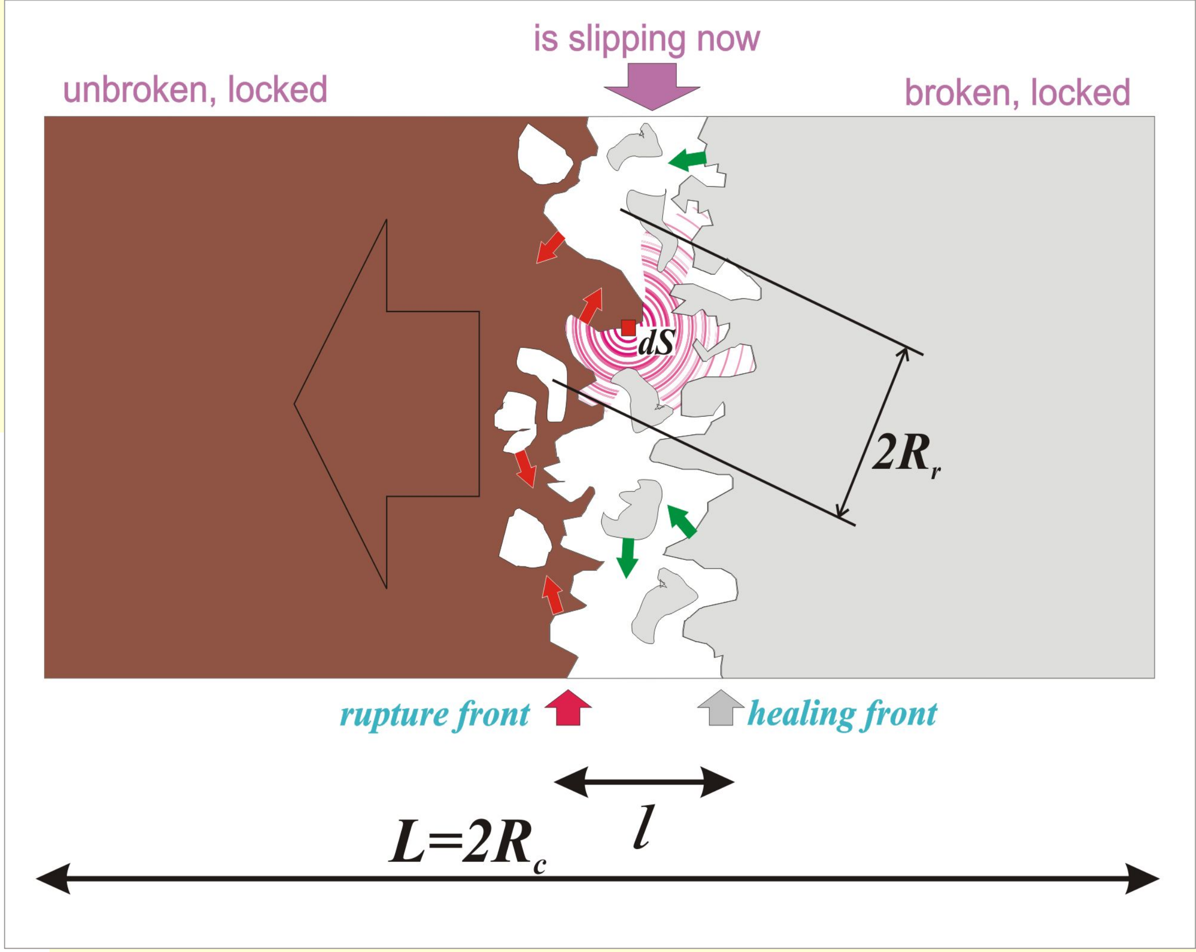
$dF = \Delta\sigma(x, y)\, dS$

Note that abruptness of pulse front causes formation of accurately ω^{-1} factor to source spectrum

Key assumption is made here to justify the application of Das-Kostrov theory:

a low-cohesion spot (of size $2R_r$)
can be associated
with a piece of slip-pulse strip (of width l)
and corresponding sizes are close one to another:

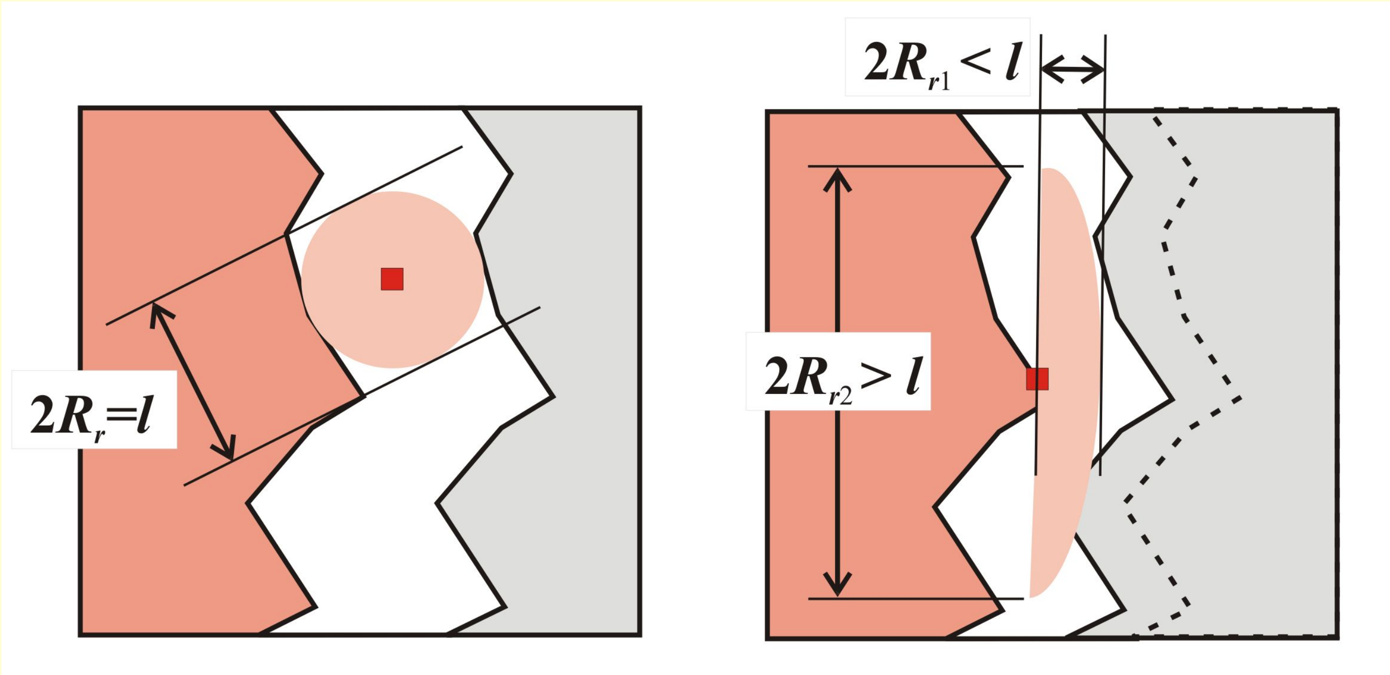
$$2R_r \approx l$$



For each rupture front element *dS*, there is an individual, corresponding low cohesion/slipping patch

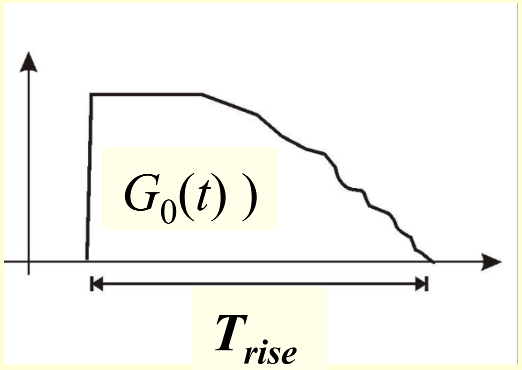
Two possible ajustments may be needed; both ignored in further simplified simulation

- **Along-front size of slipping spot above *l*:**
  as there is more free space for alog-fault waves to propagate
- **Across-width size of slipping spot below *l*:**
  as the healing front does not stand and approaches at comparable velocity



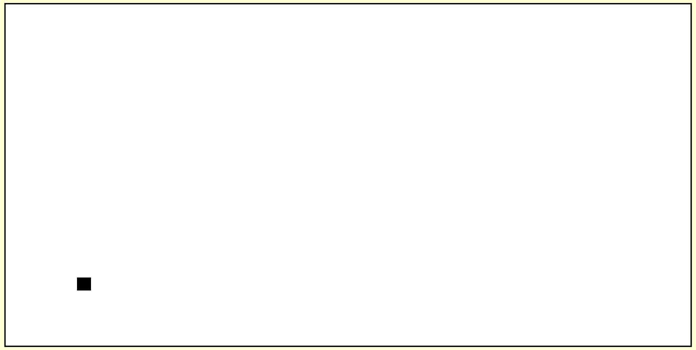
More simplifications adopted in simulation:

- Stress *drops instantly* at the arrival of rupture front (slip-weakening distance / cohesion length: very small)
- Only *SH* waves are considered
- T_{rise} or l vary only weakly over fault area
- Function $G(t,x,y)$ is identical for all fault spots ($G(t,x,y) = G_0(t)$)

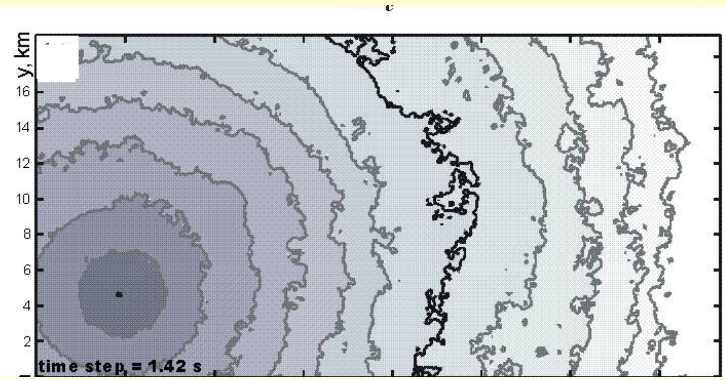


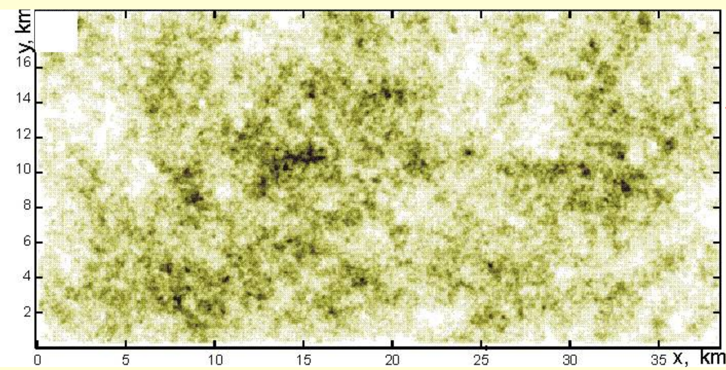
Simulation stages

the accepted parameter values in parentheses



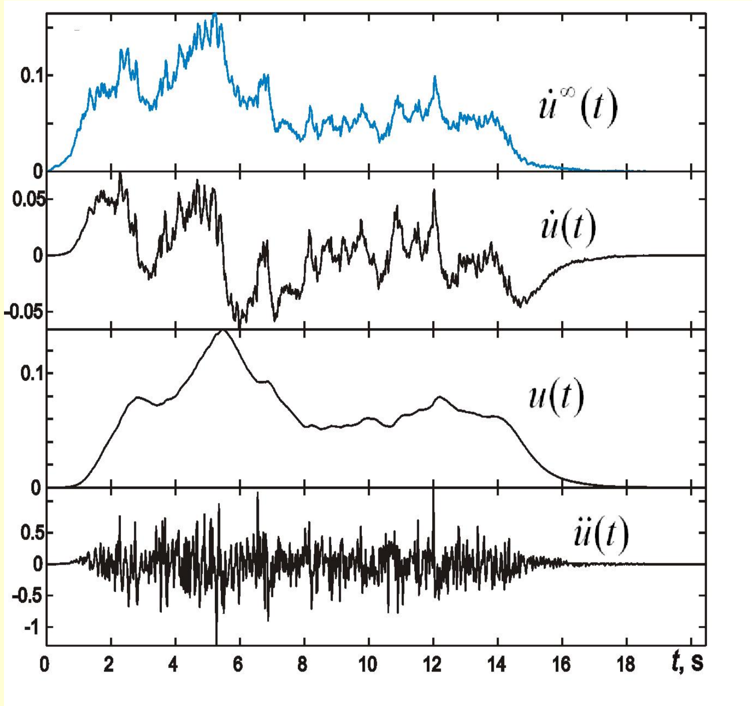
- (**a**) select source rectangle (38 ×19 km), nucleation point etc
- (**b**) set control parameters: β (1.0), C_H (0.06), $CV_{\Delta\sigma}$ (0.8), δ (1.4);
- (**c**) generate sample random fields $t_{fr}(x, y)$ and $\Delta\sigma(x, y)$;

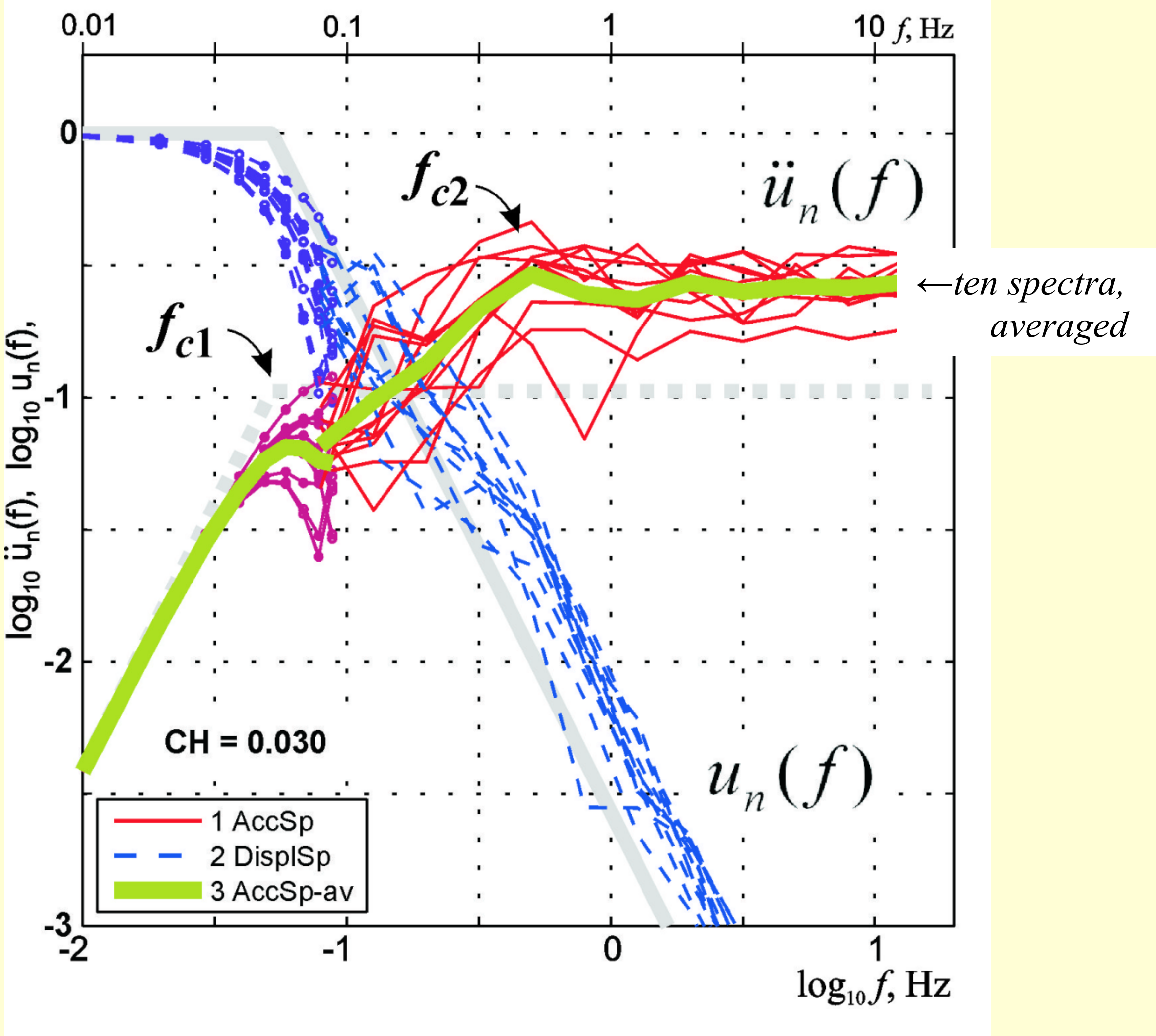




- (**d**) calculate time functions at a receiver for the cases of infinite and finite fault
- (**e**) determine normalized displacement spectrum and associated acceleration spectrum

Simulation: example signals and spectra



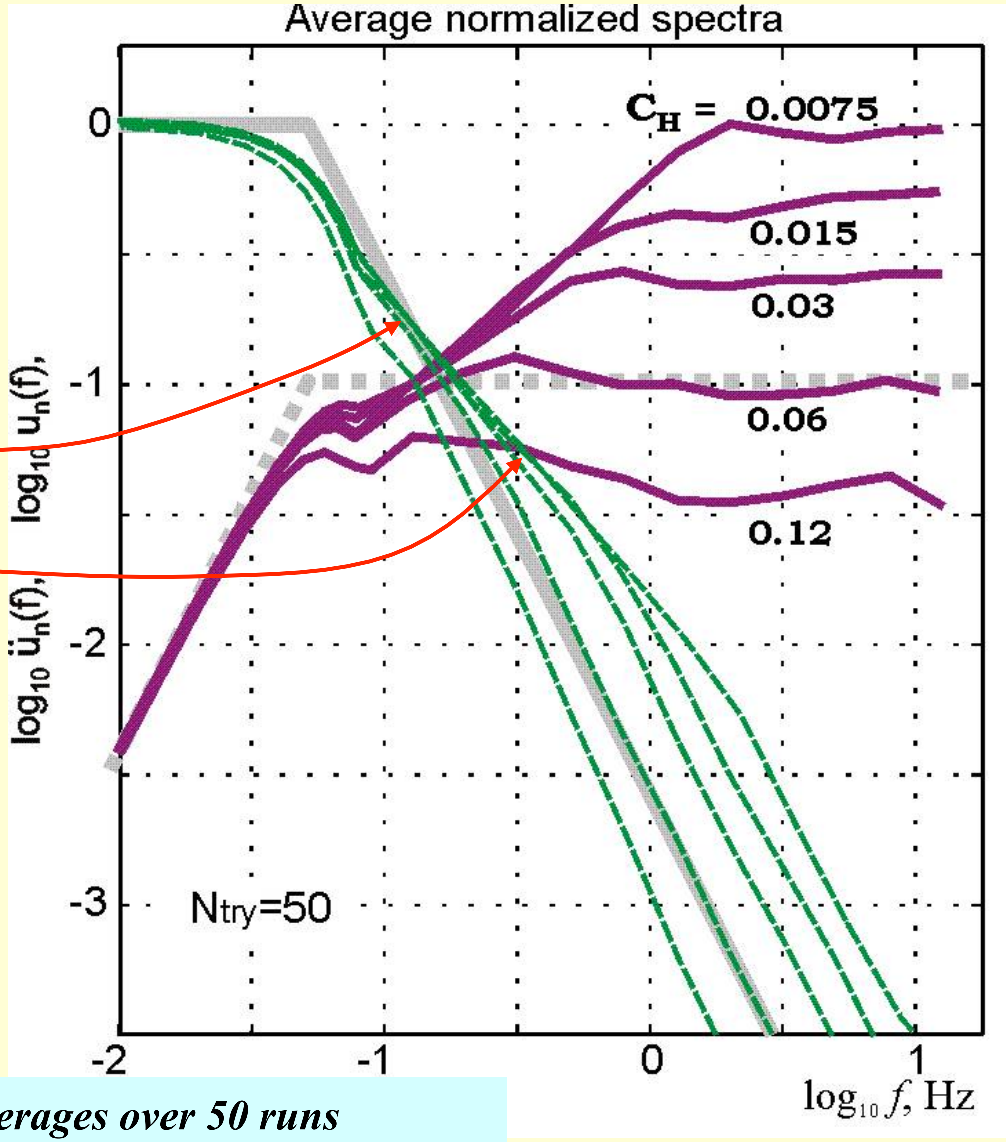


- Receiver position is assumed to be positioned at the along-normal ray

Simulation: how spectral shapes depend on $C_H = l/L$

Receiver position is assumed at along-normal ray

“ω^{-1}” intermediate slope



shown: averages over 50 runs

clear 2-corner shapes at $C_H \leq 0.05$

apprx. classical “omega-square” at $C_H \approx 0.06\text{-}0.10$

Example scaling:
of f_{c2} and
of HF spectral level A_{HF}:

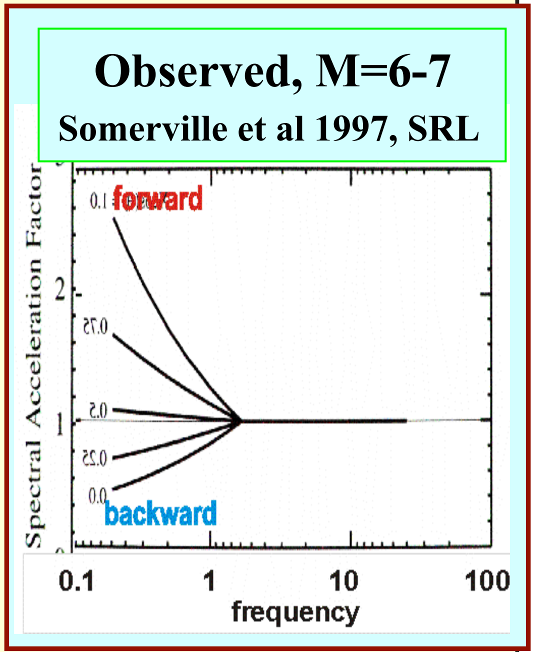
$$f_{c2}/f_{c1} \approx 0.2/C_H$$

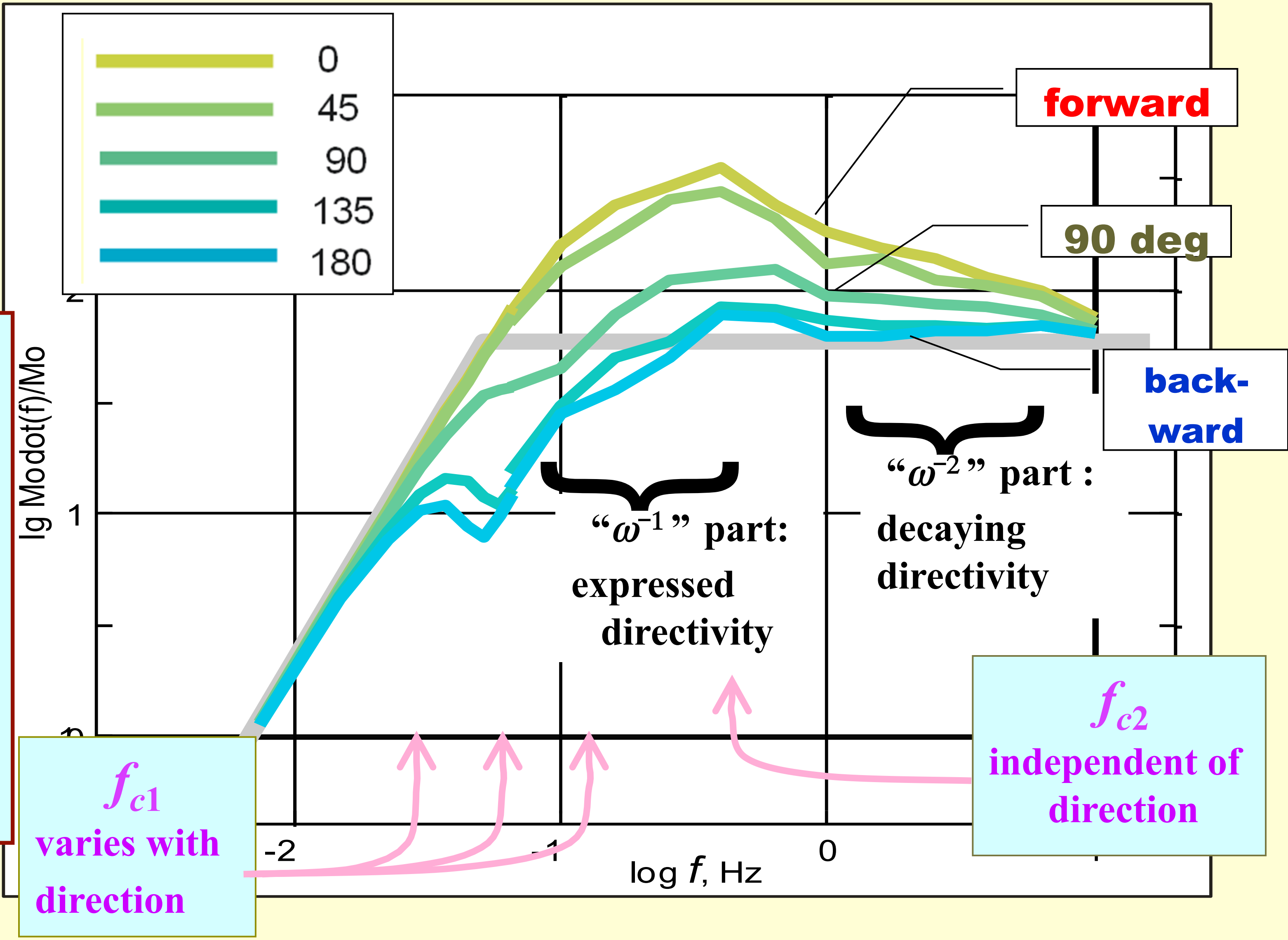
$$A_{HF}/A_{1HF} \approx 0.063/C_H$$

Simulation: spectral shapes depending on receiver position w.r.t. mean rupture propagation direction; note frequency-dependent directivity

$v_{rup}/c_S = 0.85$

$M \approx 6.7\text{-}6.9$





Conclusions

- **1. The proposed approach permits to reproduce, through numerical modeling, the following observed features of radiated earthquake waves:**
  - **ω^{-2} HF spectral slope;**
  - **2-corner spectral shapes, and**
  - **frequency-dependent directivity (high at LF, low at HF)**

- **2. To achieve this result, "double stochastic fault model" is proposed, that incorporates two self-similar/fractal structures, one in spatial domain, and another in space-time domain**

- **3. The presented model is kinematic and numerical. It is however versatile and can be adapted for practical strong motion simulation**