Propagation of 1-Point and 2-Point Statistics from Dynamic Source through Kinematic to Ground Motions

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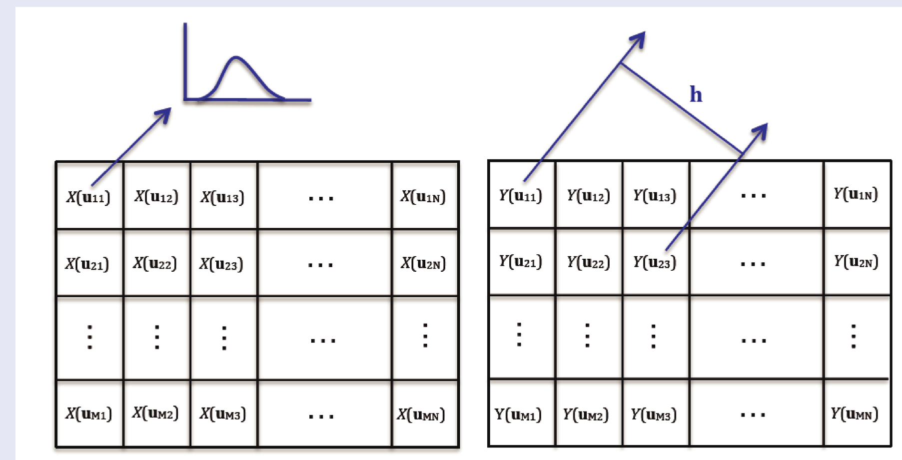
SUMMARY

Both rupture and wave propagation affect strong ground motions on the surface. The complexity of finite earthquake source process play a significant role in determining near-source ground motion characteristics, especially for large events. Spontaneous dynamic rupture modeling has been efficiently adopted for physics-based source and ground motion modeling for the last several decades, but input dynamic parameters and friction laws are still not well constrained in general. We demonstrate that at least two statistical measures, i.e., 1-point and 2-point statistics, are needed to quantify the heterogeneity of spatial data and that 1-point statistics, such as mean, standard deviation, and shape of probability density function (PDF) of dynamic input parameters, is a separate quantity that we need to consider in addition to 2-point statistics in earthquake source modeling. We show that 1-point statistics of input dynamic parameters such as stress drop can significantly affect resulting kinematic source and near-source ground motions. The standard deviations of input stress drop affect both 1-point and 2-point statistics of kinematic source parameters derived from spontaneous rupture modeling significantly and even systematically. They also strongly control near-source ground motions, especially in the rupture directivity region. Quantifying the characteristics of both earthquake source and ground motions in the same format of 1-point and 2-point statistics may help us to construct a consistent framework for studying the effect of finite source process on near-source ground motions.

Source Modeling with Spatial Random Field Models



Continuous Random Field Model

$\{X(\mathbf{u}), \forall \mathbf{u} \in A\}$

Discrete Random Field Model

$\{X(\mathbf{u}_{ij}), i=1,\dots,M, j=1,\dots,N\}$

1-point statistics in source modeling

$$E\left(X(\mathbf{u}_{ij})\right) = E\left(X(\mathbf{u}_{pq})\right)$$

$$Var\left(X(\mathbf{u}_{ij})\right) = Var\left(X(\mathbf{u}_{pq})\right)$$

$$f_{X(\mathbf{u}_{ij})}(x) = f_{X(\mathbf{u}_{pq})}(x) \qquad i \text{ and } p = 1, 2, \dots, M, j \text{ and } q = 1, 2, \dots, N$$

1-point statistics at a given point (or subfault patch) is represented by a marginal probability density function (mPDF). Mean and standard deviation are two main representative parameters. Stationarity is often assumed to infer statistical features from data at a given point. This assumption may need to be reconsidered if significant variation is expected with location, especially with depth.

2-point statistics in source modeling

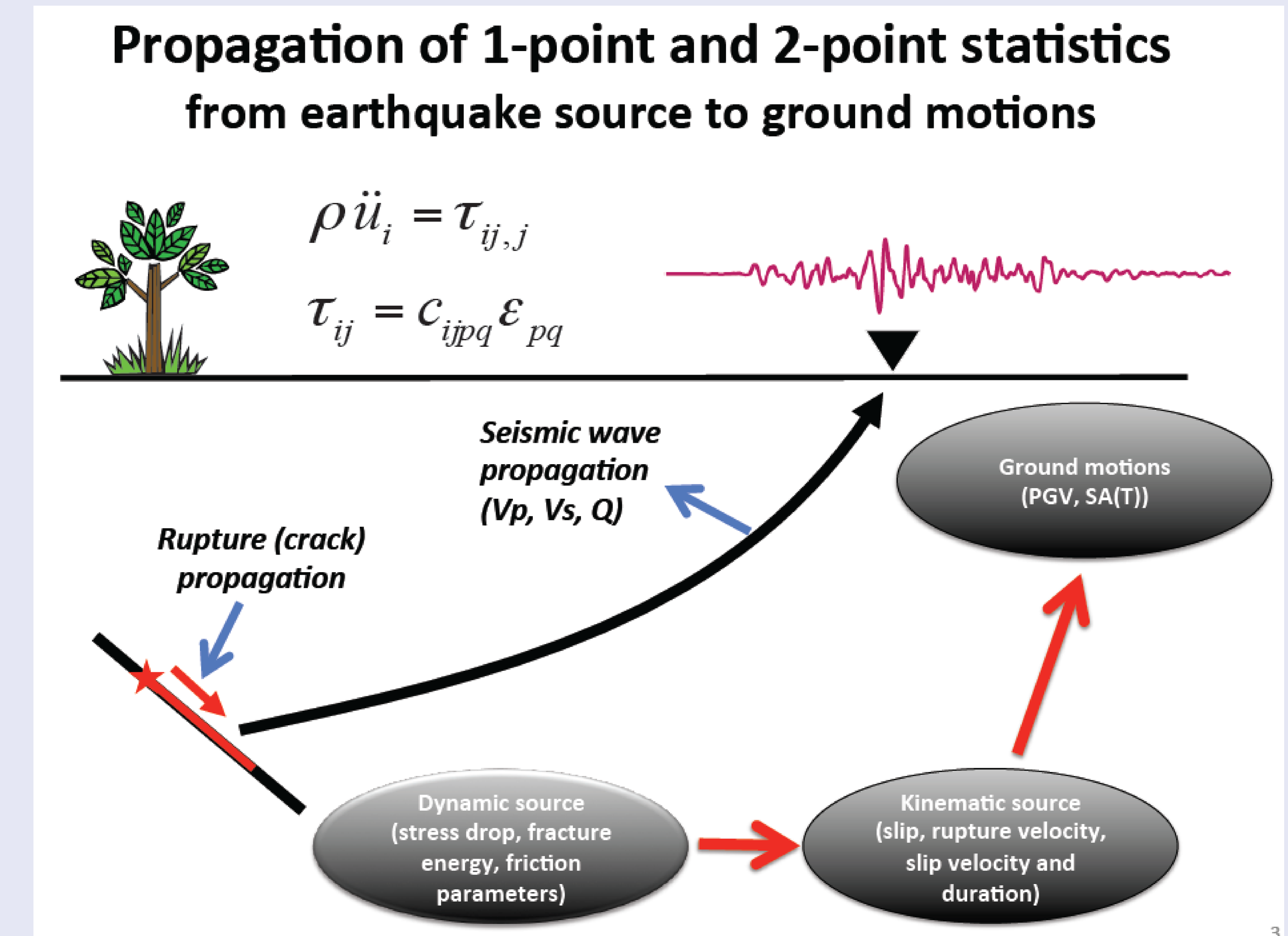
Auto-covariance

$$Cov\left(X(\mathbf{u}_{ij}), X(\mathbf{u}_{ij}+\mathbf{h})\right) = Cov\left(X(\mathbf{u}_{pq}), X(\mathbf{u}_{pq}+\mathbf{h})\right) = Cov_{XX}(\mathbf{h})$$

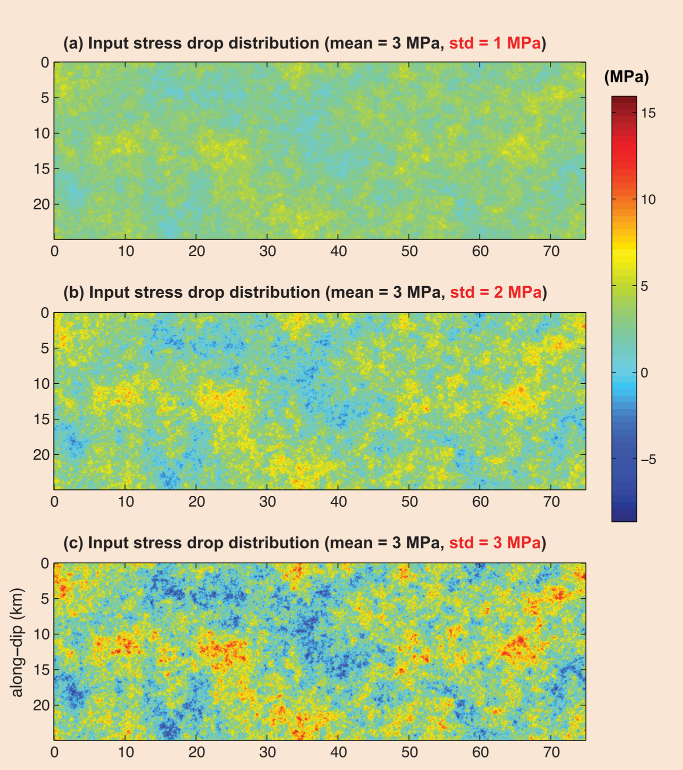
Cross-covariance

$$Cov\left(X(\mathbf{u}_{ij}), Y(\mathbf{u}_{ij}+\mathbf{h})\right) = Cov\left(X(\mathbf{u}_{pq}), Y(\mathbf{u}_{pq}+\mathbf{h})\right) = Cov_{XY}(\mathbf{h})$$

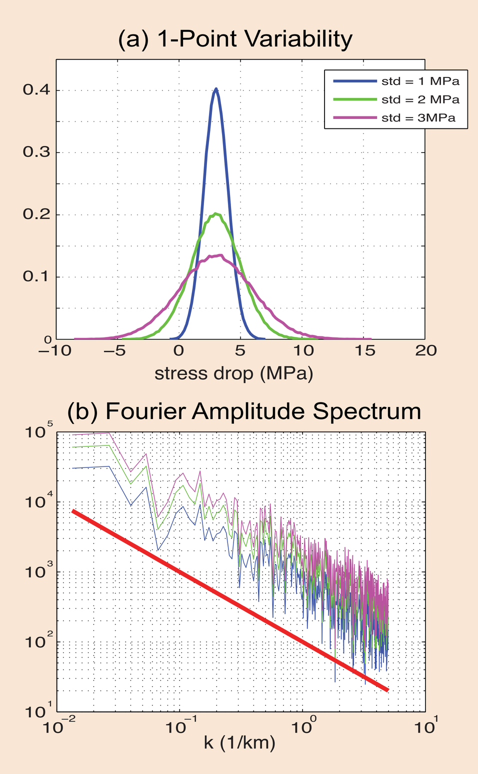
2-point statistics is composed of both auto and cross-correlation. Stationarity is assumed in the 2-point statistics as well. Correlations depend only on the sepration vector, **h**, not on the location vector, **u**. Non-stationarity in 2-point statistics is more difficult to handle compared to that in the 1-point statistics.



Changing Standard Deviation in Input Stress Drop

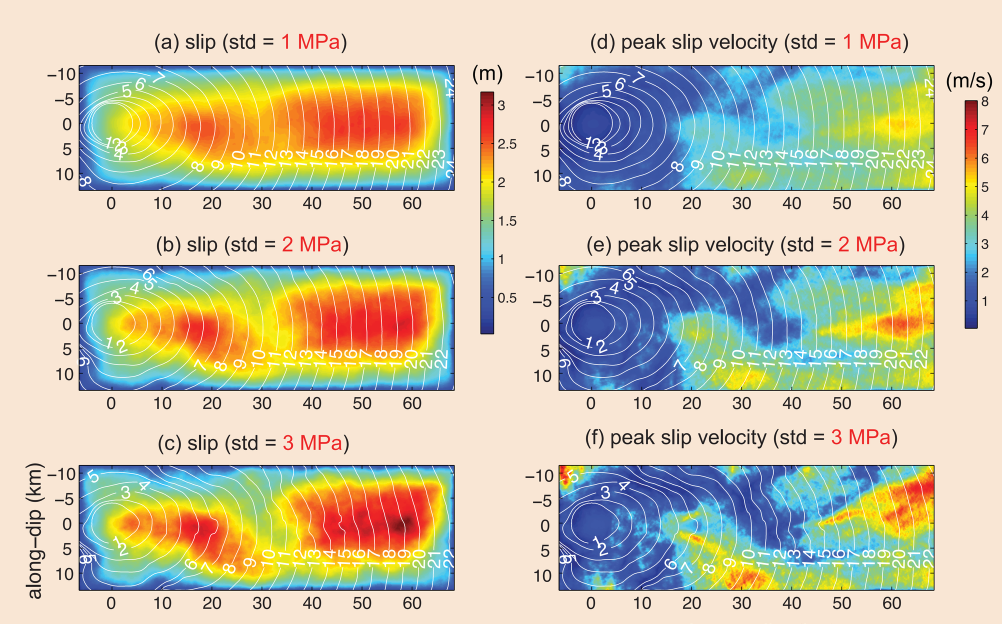


[Top] Input stress drop distributions with the same Gaussian 1-point statistics, but different standard deviations, i.e., (a) 1 MPa, (b) 2 MPa, and (c) 3 MPa. All three distributions have the same mean stress drop (= 3 MPa) and follow the same spectral decay rate (k^{-1}).

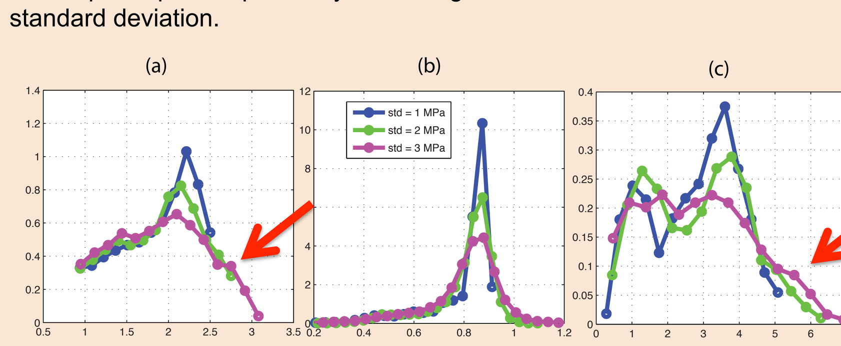


[Top] (a) 1-point probability density function (PDF) and (b) Fourier amplitude spectrum of three input stress drop distributions in the left panel. Note that they all have the same spectral decay rate (k^{-1}) although the standard deviation varies from 1 MPa to 3 MPa. The red solid line in (b) shows a reference spectral decay rate (k^{-1}).

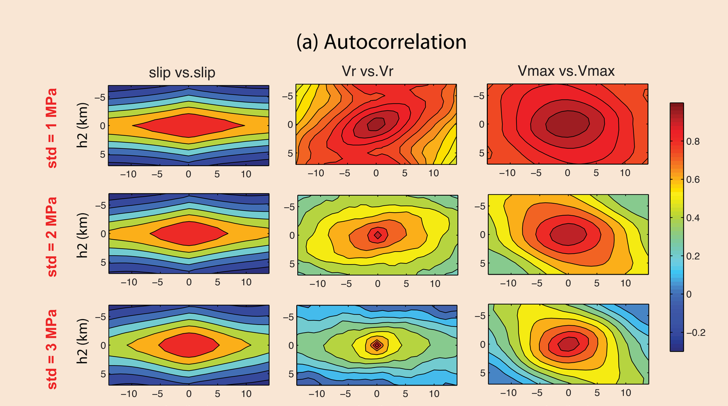
Kinematic Motions from Different Standard Deviaitons



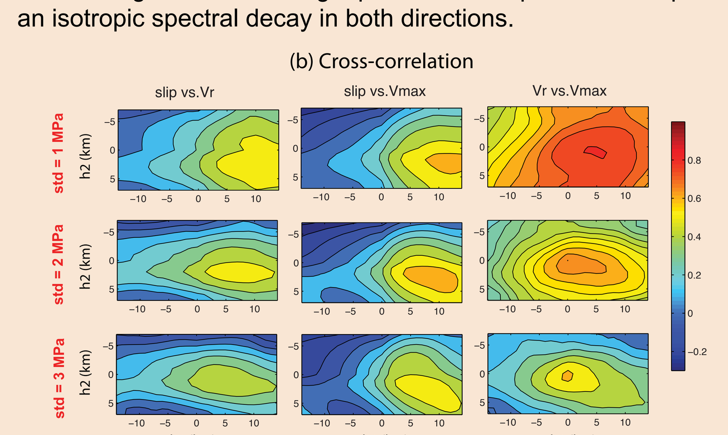
[Top] Kinematic rupture motions obtained from spontaneous dynamic rupture modeling with the slip weakening friction law with different standard deviations of input stress drop shown in the top right box. Final slip is plotted on the left with rupture time while peak slip velocity is plotted on the right with the same rupture time. We can easily observe that rupture front is more distorted and both slip and peak slip velocity have rougher distributions as we increase standard deviation.



[Top] Empirical 1-point PDFs for three kinematic source parameters produced by dynamic rupture modeling, (a) final slip, (b) rupture velocity, and (c) peak slip velocity. As the standard deviation increases, the upper tails of three parameters also increase as indicated by red arrows.

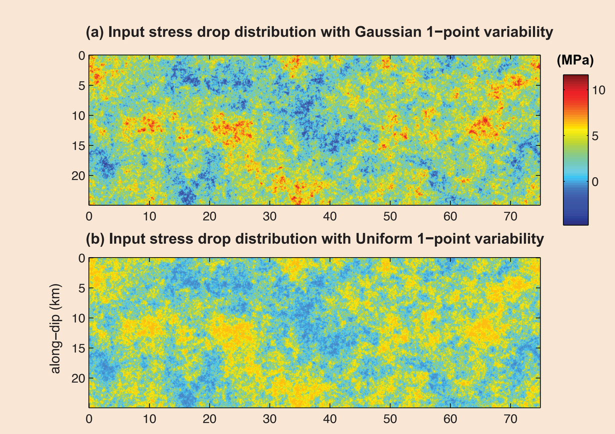


[Top] Autocorrelation of three kinematic source parameters shown in the left panel. The autocorrelation decays faster with increasing standard deviation in general. This pattern is relatively insignificant in earthquake slip, but it is clearly observed in temporal source parameters. Also note the anisotropic decay of the autocorrelation in the along-strike and along-dip directions. Input stress drop has an isotropic spectral decay in both directions.

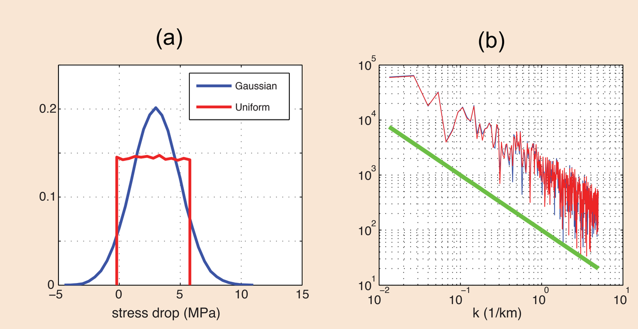


[Top] Cross-correlation between three kinematic source parameter. Significant correlations are observed in all three pairs of cross-correlogram. In addition, correlation maximum points are shifted in the forward rupture direction up to 12 km, which are consistent with findings by Song et al. (2009) and Song & Somerville (2010). Both cross-correlation maximum and response distance estimates decrease in general as the standard deviation of input stress drop increase. It is important to understand what controls cross-correlation structure between kinematic source parameters. The standard deviation of input stress drop may be one of the key elements that affect cross-correlation between kinematic source parameters.

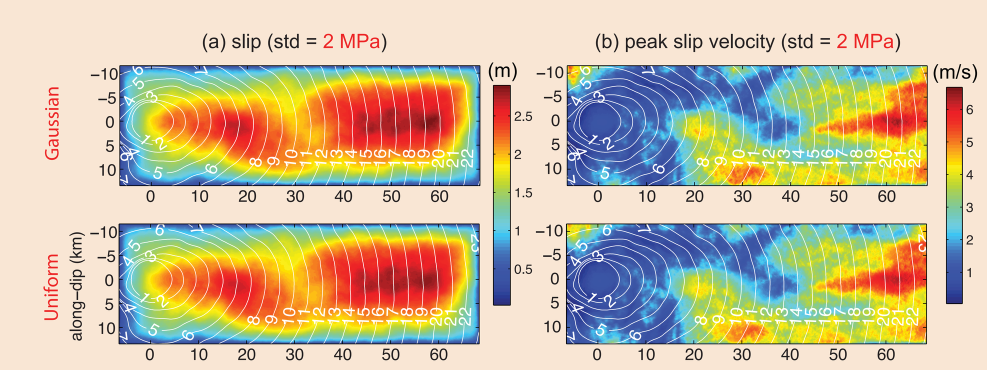
Gaussian vs. Uniform in Input Stress Drop



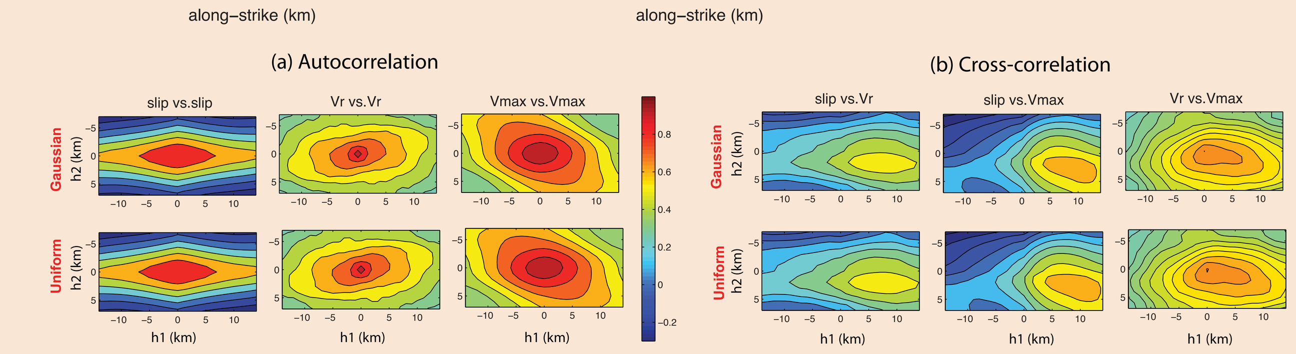
[Top] Input stress drop distributions with the Gaussian (a) and Uniform (b) 1-point statistics. Both distributions have the same mean stress drop (= 3 MPa), same standard deviation (= 2 MPa), and spectral decay rate (k^{-1}).



[Top] 1-point statistics (a) and Fourier amplitude spectrum (b) of two input stress drop distributions in the left panel. Note that both have the same spectral decay rate as shown in (b) although the shapes of their 1-point PDFs are different. The green solid line in (b) shows a reference spectral decay rate (k^{-1}).

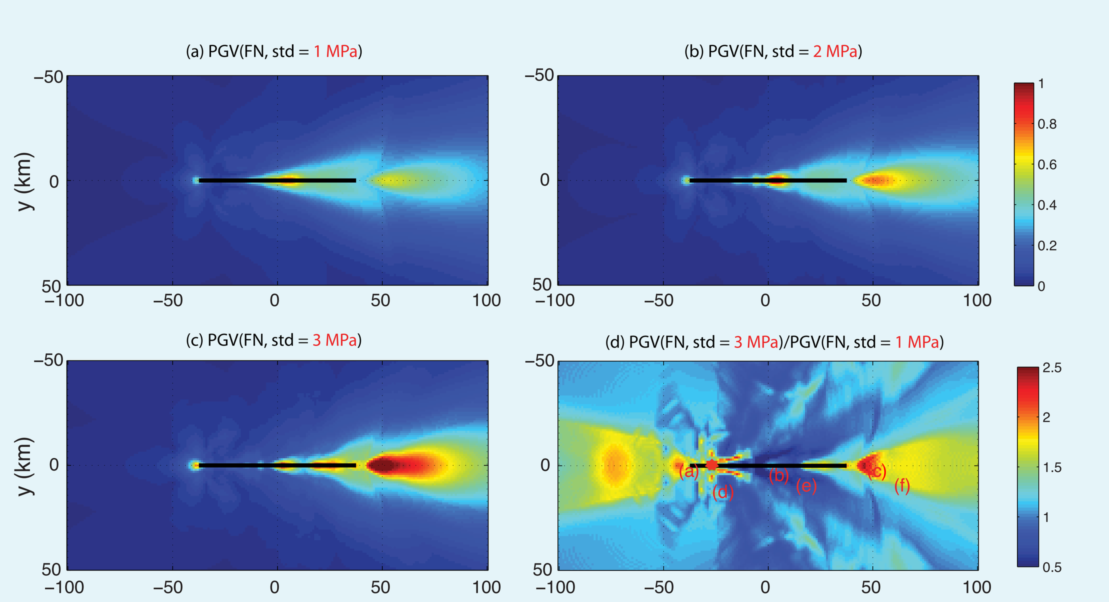


[Left] Kinematic rupture motions obtained from dynamic rupture modeling with different shapes of 1-point statistics, i.e., Gaussian and Uniform. Although the input stress drop distributions have different shapes of 1-point statistics, they produce almost identical kinematic motions.

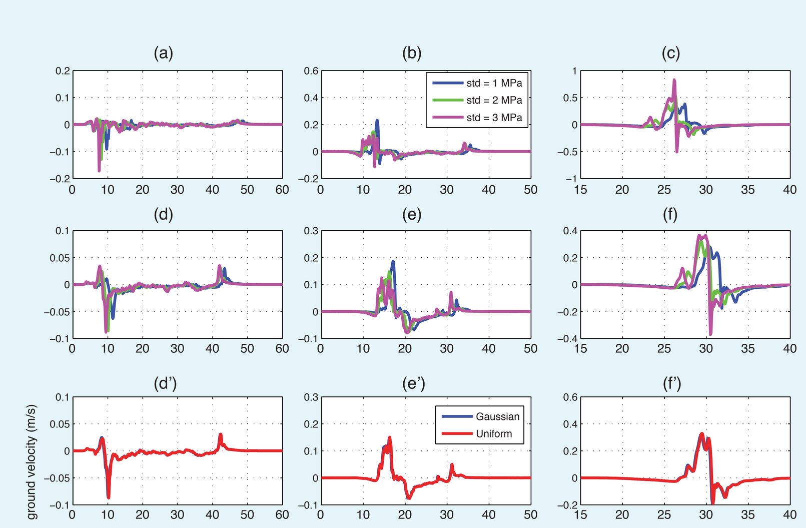


[Top] Both auto- and cross-correlation of kinematic motions from Gaussian and Uniform 1-point statistics. Both correlation structures are almost identical. Standard deviation of input dynamic parameters may affect resulting kinematic motions than detailed shape of PDFs.

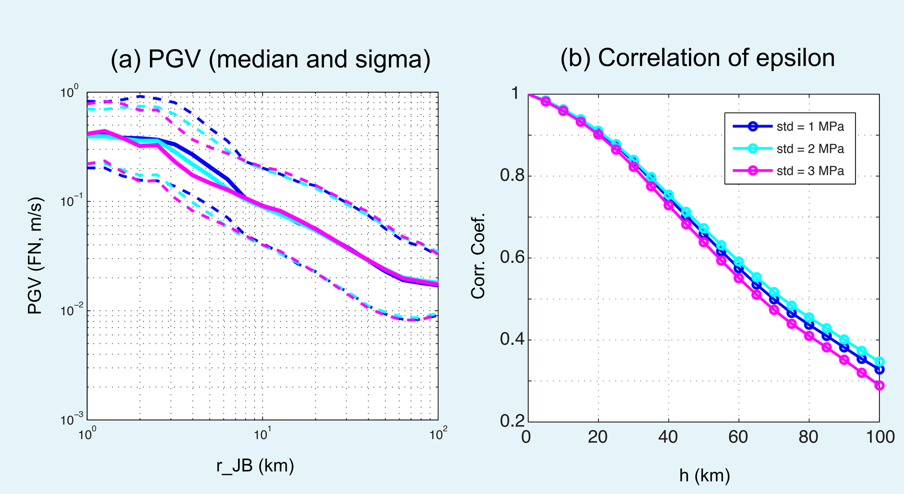
1-Point and 2-Point Statistics in Near-source Ground Motions



[Top] Distribution of peak ground velocity (PGV, fault normal component) with three different standard deviation values of input stress drop, (a) 1 MPa, (b) 2 MPa, and (c) 3 MPa. The bottom right (d) panel shows the ratio of PGV(std = 3MPa) to PGV (std = 1 MPa). PGV values increase significantly with increasing standard deviation, especially in the forward and backward rupture directivity regions. The solid black line and red star indicates the surface projection of the faulting plane and nucleation point, respectively. Red labels ((a) – (f)) denote the location of recorded waveforms shown in the bottom panel.



[Top] Waveforms (Fault normal) recorded at 6 different locations denoted in the top panel. Note that PGV increases by a factor of 2 at a certain location and waveform duration is reduced (i.e., more high frequencies) with increasing standard deviation, as shown in panel (c). Bottom three panels (d', e', f') show waveforms with std = 2 MPa, but for both Gaussian and Uniform distributions. As expected in kinematic motions, they show almost identical ground motions.



[Top] (a) 1-point statistics of peak ground velocity as a funciton of source-to-site distance, (b) 2-point statistics of peak ground velocity, i.e., spatial correlation of epsilon. Neither of 1-point and 2-point statistics are perturbed signifiantly by the perturbation of standard deviation in input stress drop.