The folding-averaging algorithm applied to precisely determine periodical signals from time series with application to the detection of free oscillations using super-conducting gravimeter observation data

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The folding-averaging algorithm is applied to precisely determining periodical signals which may be present in a time series. The basic principle is to rebuild for every test period a new short time series by cutting the the original times series to shorter ones of which the length is equal to the test period, and then stacking the short time series by averaging them. In this stacking process of averaging, the amplitude of the possible signal with period equal to the test period remains the same, but signals of different periods are averaged out and the random error is reduced. Amplitude of the possible signal with a period equal to the test period can then be estimated according to the short averaged time series. By searching for the maximum of the amplitude by varying the test period, the periods of signals which may be present in the time series can be very precisely determined. With the precise values of periods, the phases of the signals can also be determined precisely. In this work the method is applied to study Earth's free oscillation using super-conducting gravimeter observation data after an Earthquake.

In spectrum analysis, the Fourier transform is extensively used nowadays. Having an inverse transform formula, the Fourier transform is an exact presentation of the time series using waves of periods P, $P/2$, $P/3$, $\cdots$, P/i, $\cdots$, where P is the time span of the time series. But periods of real physical signals are related to the real physical causes, and is independent of the time span of observation. So the periods of signals may be 1.2, 1.5 or any number times P/i. The folding-averaging algorithm can be used to remedy this weakness of Fourier transform for determining periods of real physical phenomenon.

The folding-averaging algorithm is in fact an old method. It is was used by Darwin to analyze tides (*Melchior* 1978), and was widely used before computer age (*Bartels* 1935). Recently this method is used to detect non-harmonic periodical signals in biology (linear stacking method of *Hoenen el al.* (2001)).

We denote the N data in the original time series sampled at time $t = 0, \Delta t, 2\Delta t, 3\Delta t, \cdots, (N-1)\Delta t$ as $V_0, V_1, V_2, V_3, \cdots, V_{N-1}$, and we search for a signal with a period T, i.e. there are $N_s = (T/\Delta t)$ data in every cycle. The shor time series is constructed as follows:

$$\begin{array}{lllll} R_0: & V_0, & V_1, & \cdots, & V_{N_s-1} \\ R_1: & V_{(T/\Delta t)}, & V_{(T/\Delta t)+1}, & \cdots, & V_{(T/\Delta t)+N_s-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{M-1}: & V_{(M-1)(T/\Delta t)}, & V_{(M-1)(T/\Delta t)+1}, & \cdots, & V_{(M-1)(T/\Delta t)+N_s-1} \\ \hline \bar{R}: & \bar{R}_0, & \bar{R}_1, & \cdots, & \bar{R}_{N_s-1} \end{array} \tag{1}$$

$$\bar{R}_j = \frac{1}{M} \sum_{i=0}^{M-1} V_{i(T/\Delta t)+j} \,. \tag{2}$$

The amplitude and phase can then be estimated according to

$$a\cos\phi = \frac{2}{T}\Delta t \sum_{i=0}^{N_s-1} \bar{R}_i \sin[(2\pi/T)(i\Delta t)] \,, \qquad a\sin\phi = \frac{2}{T}\Delta t \sum_{i=0}^{N_s-1} \bar{R}_i \cos[(2\pi/T)(i\Delta t)] \,, \tag{3}$$

whence

$$a = \sqrt{(a\sin\phi)^2 + (a\cos\phi)^2} \,, \qquad \phi = \mathrm{atan2}(a\sin\phi, a\cos\phi) \,, \tag{4}$$

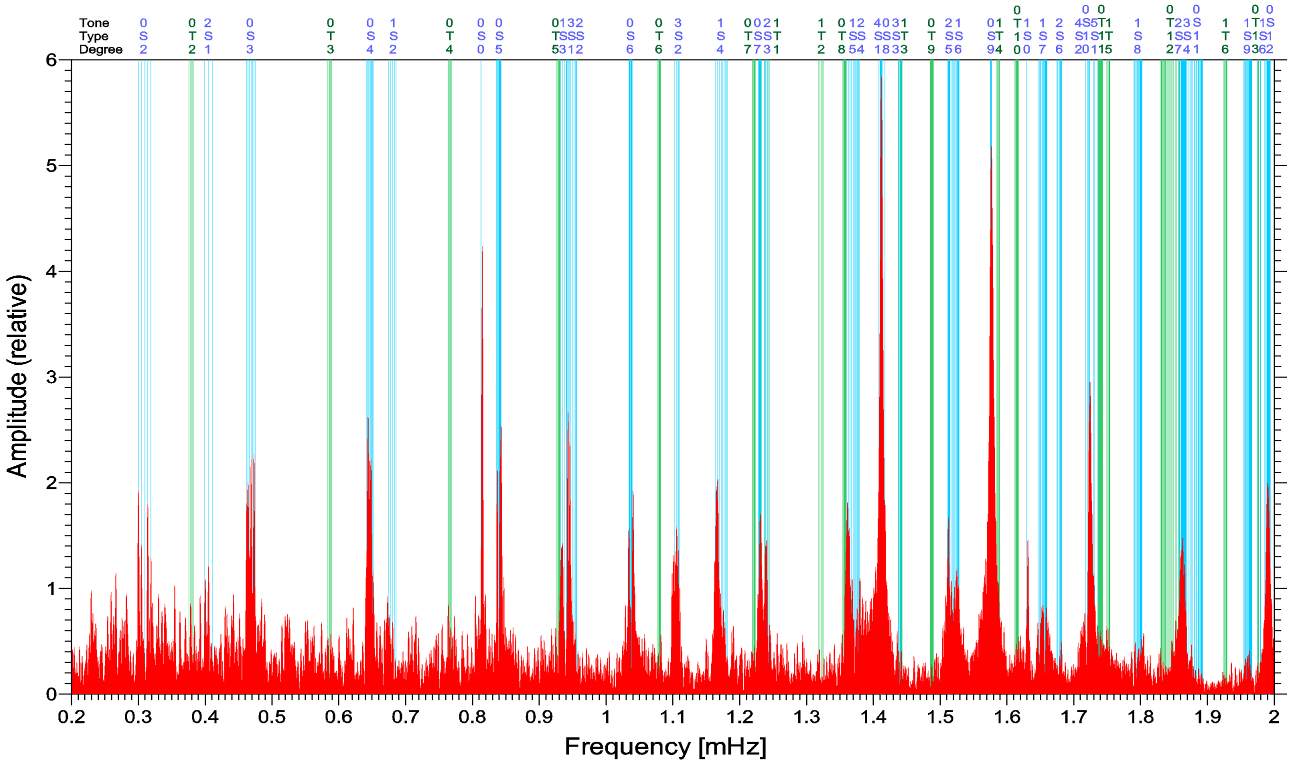

Figure 1. Free oscillation spectrum

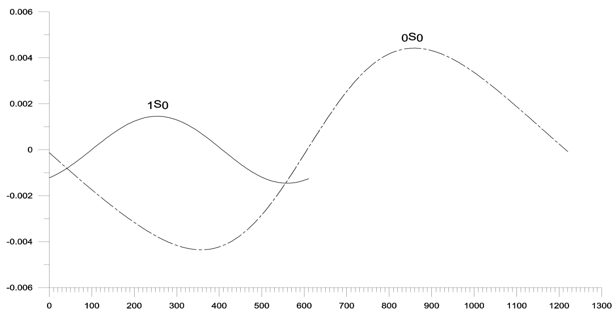

Figure 2. Short time series built for ${}_0S_0$ and ${}_1S_0$

where the function $\mathrm{atan2}(x, y)$ is provided in practically all programming languages.

We applied this method to build the amplitude spectrum of long period free oscillation using the super conducting gravimeter data of 152 hours after an Earthquake. See Figure 1. The short time series built for ${}_0S_0$ and ${}_1S_0$ are shown in Figure 2.

By stacking 9 records, *Riedesel el al.* (1980) estimated the period of ${}_0S_0$ to be 1227.50 s using 2000 hours of record, and the period of ${}_1S_0$ to be 612.93 s using 300 hours of record. Our estimates are respectively 1227.40 s and 612.97 s using 152 hours of record.

REFERENCES

Bartels, J., Random fluctuations, persistence and quasipersistence geophysical and cosmical periodicities, *Terr. Magn. Atmos. Electricity, 40*, 1-60, 1935.

Dahlen, F. A. and Sailor, R. V., Rotational and elliptical splitting of the free oscillation of the Earth, *Geophys. J. R. Astr. Soc., 58*, 609-623, 1979.

Hoenen, S., Schimmel, M. and Marques, M. D., Rescuing rhythms from noise: A new metrhod of analysis, *Biological Rhythm Research, 32*, 271-284, 2001.

Melchior, P., *Tides of the planet Earth*, Pergamon Press, 1978.

Riedesel, M. A., Agnew, D. C., Berger, C. and Gilbert, F., Stacking for the frequencies and Q's of ${}_0S_0$ and ${}_1S_0$, *Geophys. J. R. Astr. Soc., 62*, 457-471, 1980.