How much can the core possibly contribute to the secular variation of LOD ? [1]

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1. The length of day (LOD) in the geological past

The observational material used for obtaining palaeo-LOD data consists essentially of fossil corals, brachiopods, and bivalves for the Phanerozoic, stromatolites and tidal deposits (tidal rhythmites, also called tidalites) for the Proterozoic. Fig. 1 shows some samples discussed by Scrutton (1978), Lambeck (1980), Evans (1975; 1972), and Williams (1989; 2000). Varga *et al.* (1998) published a compilation of values of the length of day (LOD) throughout geological times (see Fig. 2), both for the Proterozoic (Ptz; here conventionally defined as the era extending from 2.50×10^9 years ago to 0.64×10^9 years ago) and the Phanerozoic (Pz; from 0.64×10^9 years ago to the present). Notice, however, that according to Piper (1982), the geologically determined transition from the Ptz to the Pz eras actually took place a bit later, some 0.57×10^9 years ago.

Linear regressions performed on the Pz and Ptz LOD-values yield (Varga *et al.*, 1998)

$$\mathtt{LOD} = 24.00 - 4.98\,\tau \quad \text{for} \quad 0.00 < \tau < 0.64\,, \tag{1}$$

respectively

$$\mathtt{LOD} = 21.435 - 0.974\,\tau \quad \text{for} \quad 0.64 < \tau < 2.50\,. \tag{2}$$

The parameter τ represents time before the present, expressed in aeons [1 aeon=10^9 years=1 Ga]. We notice that the ratio of the slopes of these straight lines, which represents the ratio of the average despinning rates in the Pz and the Ptz, is $\frac{4.98}{0.974} = 5.1$, indicating that the length of the day increased on the average slightly more than 5 times faster in the Pz than in the Ptz.

At first sight, this result seems paradoxical if we remember that, concomitantly with a lengthening of the day on geological time scales brought about mainly by the lunar tidal torque N_* associated with the lunar tide M_2 (Varga and Denis, 1991; Varga *et al.*, 1991), there is a transfer of the angular momentum associated with the Earth's spin to the angular momentum associated with the Moon's orbit. Consequently, the Moon must have been recessing from the Earth ever since the Earth-Moon system came into existence, and the average Earth–Moon distance c_* must have been increasing ever since. Now, N_* is inversely proportional to the sixth power of c_* (Jeffreys, 1970):

$$N_* \propto \chi_*^2 \sin 2\varepsilon\,, \quad \text{with} \quad \chi_* = \frac{3GM_*}{2c_*^3}\,, \tag{3}$$

where ε is the tidal lag angle caused by tidal dissipation, M_* is the mass of the Moon, and G is the gravitational constant. Hence, owing to the smaller Earth–Moon distance in the past, tidal friction should have been enhanced, and the tidal despinning rate should have been greater in the Ptz than in the Pz, contrarily to the information yielded by palaeontological and sedimentological clocks. Notice that this argument holds only if we assume that ε did not change significantly in the

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past. In fact, because coastlines change with time, it seems impossible for the latter hypothesis to be valid (*e.g.* Varga *et al.*, 1998).

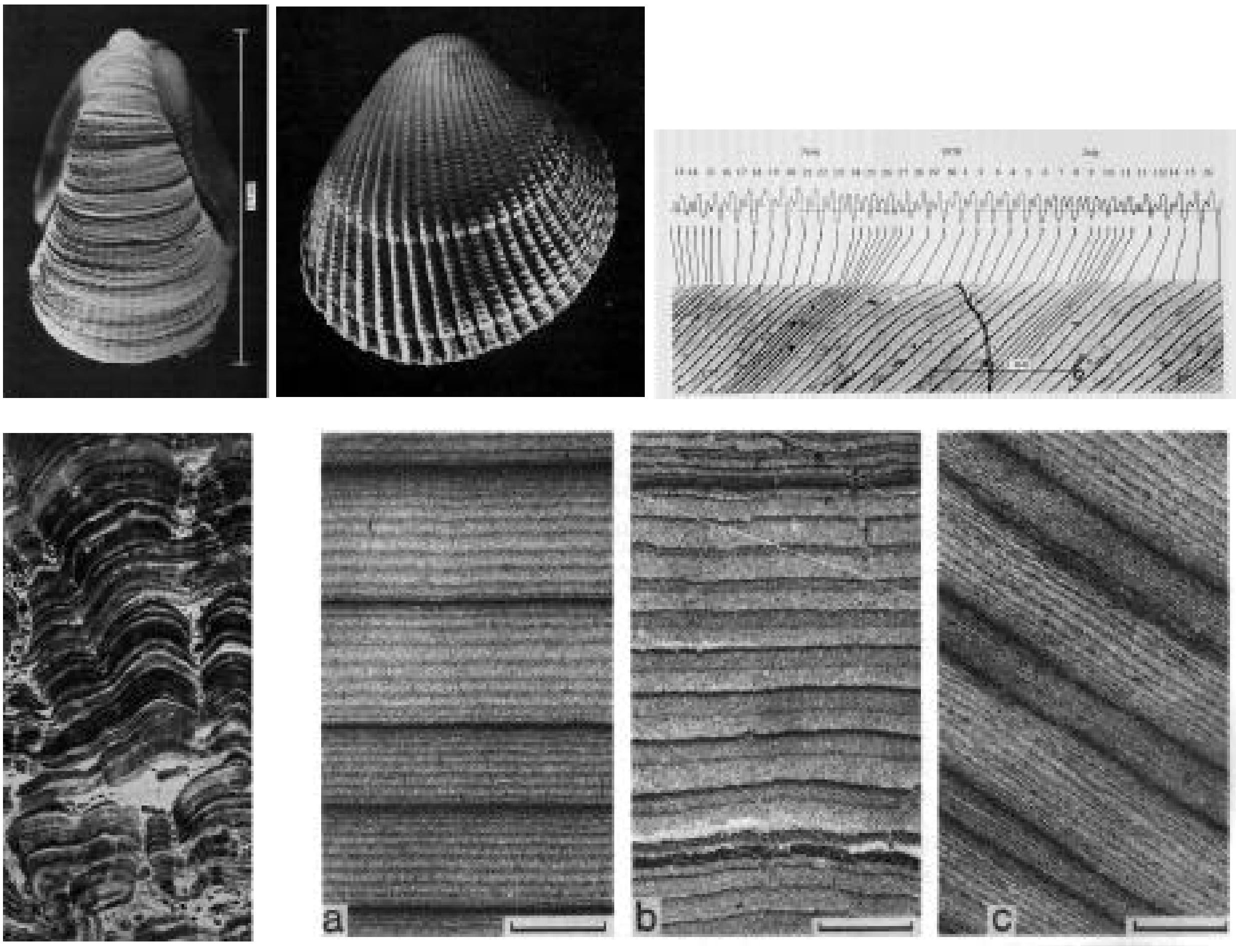

Figure 1.– Some fossil clock samples used in the determination of palaeo-`LOD` data. The banded structures are attributed to complex astronomical cycles: alternation of day and night, tidal modulation, seasonal modulation, annual modulation, *etc.* *Upper row, from left to right:* [1] Middle Devonian coral epitheca from Michigan, U.S.A., illustrating 13 well-developed bands, each with an average of 30.8 ridges (Scrutton, 1978). [2] Bivalve *Clinocardium nuttalli* showing the external growth ridges (Evans, 1975). [3] The rightmost picture in the upper row shows an experiment carried out on a modern shell similar to the fossil one in [2], during the summer of 1970. In this experiment, the internal growth lines in *Clinocardium nuttalli* are compared with tidal predictions for the same period. The horizontal line drawn through the tide curve marks the intertidal position at which the specimen was found. Lines form when the tide drops below this position (Evans, 1972). An analysis of the ridges corresponding to different growth increments yields a value of `LOD` of about 24.5 hours. This experiment shows that we should expect for each determination of `LOD` based on a single specimen an absolute error of some tens of minutes. For the published fossil data, it is most often extremely difficult to assess the actual error bounds, both for the `LOD` determination and for the age determination. *Lower row, from left to right:* [4] The banded structure of a Precambrian stromatolite from Montana (Runcorn, 1966). [5-a,b,c] Three pictures showing alternations of lighter and darker colorations in late Proterozoic tidal deposit from South Australia. The different astronomical periods reflected by these colorations are most easily found by performing a power spectral analysis (Williams, 1989; 2000).

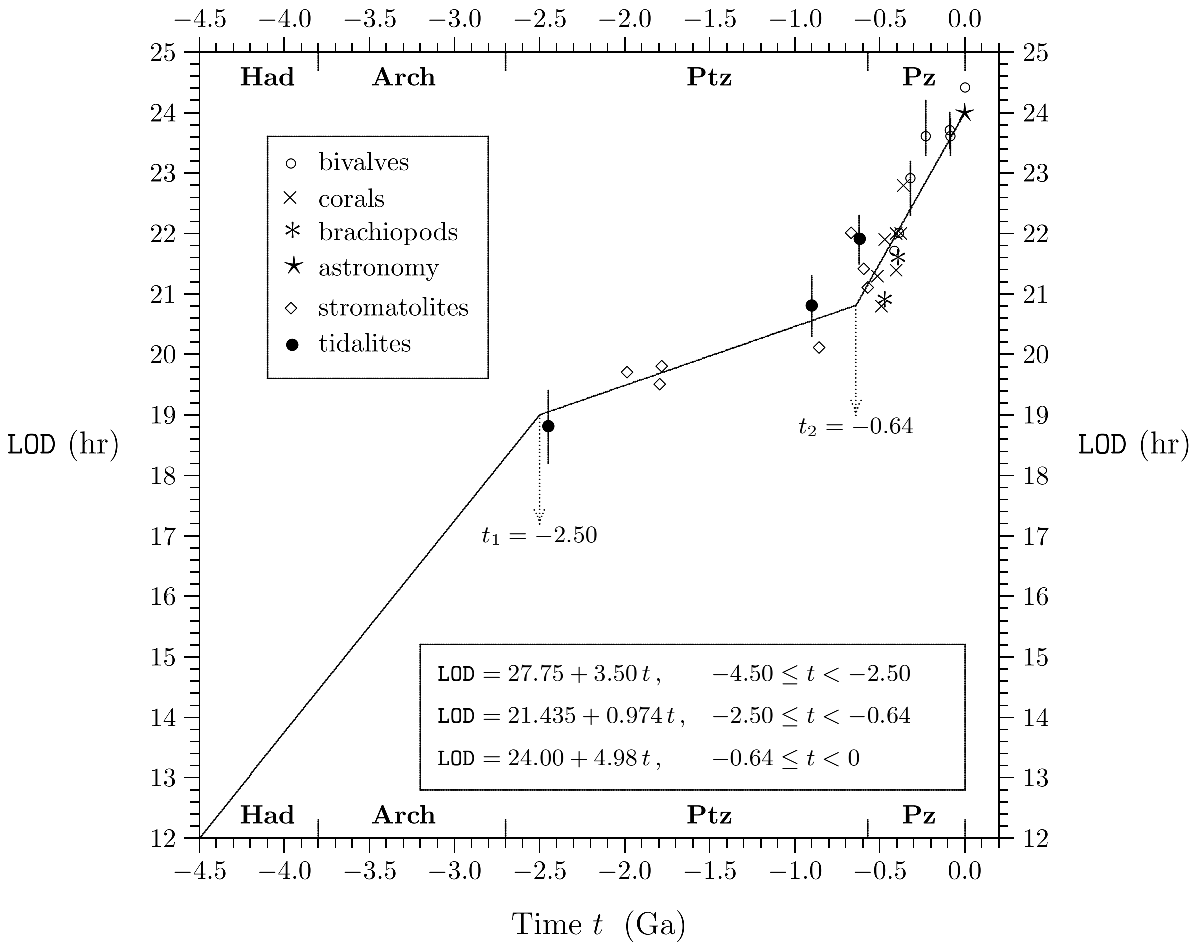

Figure 2.– Palaeovalues of LOD determined from fossils and tidal deposits, and inferred variation of LOD during the Phanerozoic (Pz: 0.57×10^9 years ago until the present), the Proterozoic (Ptz: 2.7×10^9 until 0.57×10^9 years ago), the Archaean (Arch: 3.8×10^9 until 2.7×10^9 years ago), and Pre-Archaean or Hadaean (Had: 4.7×10^9 until 3.8×10^9 years ago, *cfr.* Emiliani, 1992, p. 406). A star (★) denotes the modern astronomical value (24 hr), the symbols ∘, ×, ∗ refer respectively to bivalve, coral and brachiopod data, a diamond (⋄) denotes information from stromatolites, and a bullet (•) information from tidalites. Some error bars are given. The errors involved are generally difficult to extract from the relevant literature, but in any case they are rather large both for time and LOD. In particular, for LOD, they are typically ±0.5 hr (Varga *et al., 1998).* The straight line for the Archaean and Pre-Archaean, for which no determinations of LOD are available, is speculative and will briefly be discussed below (*cfr.* section 3).

2. Rates of lunar recession and evolution of the Earth–Moon distance

There is compelling observational evidence, in particular from the geological record and from age determinations of lunar rocks, that the Earth and the Moon already existed as a double-planet system in the Archaean, 3.5×10^9 years ago, and most probably also in the early Hadaean, 4.5×10^9 years ago. The present-day recession rate of the Moon, determined by lunar laser ranging (Christodoulidis *et al.*, 1988; Newhall *et al.*, 1990; Dickey *et al.*, 1994) is 3.82 ± 0.07 m/cy [1 cy=1 century=10^2 years]. Expressed in units of the equivolumetric average Earth radius $R = 6.371 \times 10^6$ m, the present-day value of the average Earth–Moon distance is $\gamma_o = 60.27$. Walker and Zahnle (1986; see also Williams, 2000) provided expressions that allow to estimate the mean

Earth–Moon distance $\gamma(\tau)$ in the past from the rate of lunar recession $\dot{\gamma}(\tau)$ at different times τ in the past, considering only the lunar semi-diurnal tide M_2 and neglecting the inclination and eccentricity of the lunar orbit.

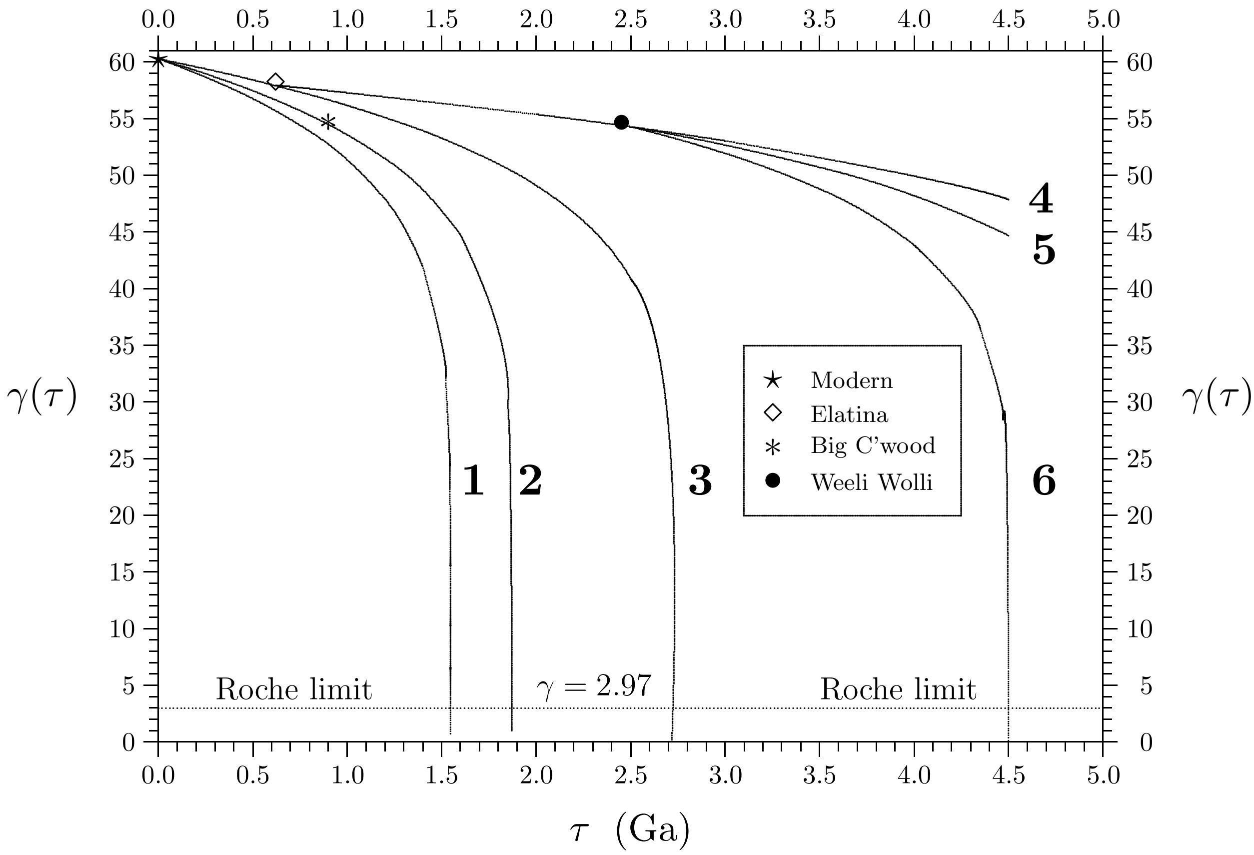

Figure 3.– Changes in the mean Earth–Moon distance γ, expressed in Earth radii, as a function of time τ in the past, expressed in aeons, for diverse average rates of tidal dissipation. The curves are drawn using Eqn.(4′) and the parameter values listed in Table 1. See text for a discussion of the different curves.

Generalizing the formula provided by Walker & Zahnle (1986) for any interval of time $[\tau_i\,,\,\tau_{i+1}]$ referenced by the subscript 'i', we write for the latter interval

$$\gamma(\tau) \approx \gamma_i \left[1 - \frac{13\,\langle\dot{\gamma}_i\rangle\,(\tau-\tau_i)}{2\,\gamma_i}\right]^{2/13}, \tag{4}$$

where γ_i is the mean radius (expressed in Earth radii) of the lunar orbit at time τ_i, and $\langle\dot{\gamma}_i\rangle$ is the mean rate of lunar recession (expressed in Earth radii per aeon) during the interval $[\tau_i\,,\,\tau_{i+1}]$. Let $\alpha_i = 6.5\,\langle\dot{\gamma}_i\rangle\,\gamma_i^{-1}$, then Eqn.(4) becomes

$$\gamma(\tau) \approx \gamma_i\,[\,1 + \alpha_i\,\tau_i - \alpha_i\,\tau\,]^{2/13} \tag{4′}$$

or, inversely,

$$\tau \approx \frac{1}{\alpha_i}\left\{1 + \alpha_i\,\tau_i - \left[\frac{\gamma(\tau)}{\gamma_i}\right]^{13/2}\right\} \tag{4″}$$

Associated with each interval $[\tau_i\,,\,\tau_{i+1}]$, there exist three critical instants which may lie within or beyond the interval, namely:

(1) The instant T_{lim} when the Moon's centre would merge, on the average, with the Earth's centre, *i.e.* $\gamma(T_{lim}) = 0$. Eqn.(4″) shows that $T_{lim} \approx (\alpha_i)^{-1}\,(1 + \alpha_i\,\tau_i)$.

(2) The instant T_{col} when the Moon's surface would hit, in a grazing collision, the surface of the Earth. Thus, T_{col} is defined in a first order approximation by the fact that $\gamma(T_{col}) = (R{+}R_*)/R \approx 1.273$. Here, $R = 6371$ km denotes, as before, the mean radius of the Earth, and $R_* = 1738$ km is the radius of the Moon. Eqn.(4″) yields $T_{col} \approx (\alpha_i)^{-1}\,[1 + \alpha_i\,\tau_i - (1.273/\gamma_i)^{13/2}] < T_{lim}$.

(3) The instant T_{Roche}, or "Roche time", when the Moon's orbit reaches the Roche radius R_{Roche}, which we define here as the smallest Earth–Moon distance below which tidal forces would disrupt the Moon. According to Stacey (1992, p. 132), $R_{Roche} = 2.97\,R$. For $\gamma(\tau) < 2.97$, Earth and Moon cease to exist as distinct bodies. Therefore, the associated critical "Roche time" τ_{Roche} is such that $\gamma(\tau_{Roche}) = 2.97$, *i.e.* $T_{Roche} \approx (\alpha_i)^{-1}\,[1 + \alpha_i\,\tau_i - (2.97/\gamma_i)^{13/2}] < T_{col} < T_{lim}$.

curve	#1	#2	#3	#4	#5	#6
τ_1	0.00	0.00	0.00	0.00	0.00	0.00
τ_2				0.62	0.62	0.62
τ_3					2.50	2.50
γ_1	60.27	60.27	60.27	60.27	60.27	60.27
γ_2				57.92	57.92	57.92
γ_3					54.28	54.28
$\langle\dot{\gamma}_1\rangle$	6.00	4.96	3.41	3.41	3.41	3.41
$\langle\dot{\gamma}_2\rangle$				1.63	1.63	1.63
$\langle\dot{\gamma}_3\rangle$					3.00	4.18
α_1	0.647	0.535	0.368	0.368	0.368	0.368
α_2				0.183	0.183	0.183
α_3					0.359	0.500
$T_{lim} \leq$	1.545	1.869	2.717	6.082	5.286	4.500

Table 1.– Parameters describing the five curves given in Fig. 3. Curve #1 corresponds to the present-day dissipation rate of tidal energy projected into the Cryptozoic, curve #2 is based on a smaller dissipation rate consistent with the Big Cottonwood, Utah, tidalite datum corresponding to 900 Ma ago (Sonett *et al.*, 1996a,b). Curve #3 is consistent with the modern and Elatina data (see Williams, 2000) but, similarly to the curves #1 and #2, would bring the Moon critically close to the Earth in too recent a past, a fact which is incompatible with geological observations. Curve #4 assumes two different recession rates, namely the same as in curve #3 until 620 Ma ago, and a much smaller one (consistent with the Weeli Wolli datum of 2450 Ma ago, see Williams, 2000). According to curve #4, the Moon would never have come closer to the Earth than roughly 47.5 Earth radii (see, *e.g.*, Denis, 1993, p. 110). There are reasons to believe, however, that in the Archaean and Pre-Archaean, tidal energy dissipation might have been larger than in the Proterozoic. Curve #5, based on three different lunar recession speeds, is an attempt to take this idea into account. It yields a critical time of about 5.28 Ga, which is longer than the age of the Earth, about 4.57 Ga and, thus, is not in conflict with the geological record. Curve #6 is based on the hypothesis that the Moon is a result of the collision of the Earth with a very large asteroid towards the end of the accretion process which formed the Earth and planets, about 4.5 Ga ago. We obtained this curve by putting $\tau_{lim} = 4.5$.

Although these three critical times are conceptually distinct, they are practically the same in most cases. This is because the numerator on the r.h.s. of Eqn.(4′′) is $1 + \alpha_i$ if $\gamma = 0.0$, and differs in general from $1 + \alpha_i$ by a negligible amount if $\gamma = 1.273$ or $\gamma = 2.97$. For instance, if we consider curve#1 of Fig. 3, these negligible amounts are $(1.273/60.27)^{13/2} = 1.29 \times 10^{-11}$ for the collision time, respectively $(2.97/60.27)^{13/2} = 3.18 \times 10^{-9}$ for the Roche time. By making the calculations with a sufficient number of digits using the `Mathematica` software, we find, *e.g.*, that T_{Roche} and T_{lim} differ from each other by slightly less than 5 years, for a total duration of more than 1.5×10^9 years.

Assuming that the present rate of tidal energy dissipation, corresponding to a lunar recession rate of 3.82 cm/year, which seems to be more or less typical of most of the Phanerozoic, was also the rate of tidal energy dissipation throughout the *whole* geological past, we obtain curve#1 of Fig. 3, described by the formula $\gamma(\tau) = 60.27\,(1 - 0.647\,\tau)^{2/13}$. The critical time T_{lim} for this curve#1 is $\frac{1}{0.647} \approx 1.5456\,\mathrm{Ga}$. The parameters (see Table 1) for this one-interval curve are: $\tau_1 = 0$, $\gamma_1 = 60.27$, $\langle\dot{\gamma}_1\rangle = 6.00\,\mathrm{Ga}^{-1}$, $\alpha_1 = 0.647\,\mathrm{Ga}^{-1}$. If our assumptions were correct, in particular if the tidal dissipation would have been approximately constant during the Phanerozoic *and* the Cryptozoic at its present-day value, the Earth-Moon system could not have existed for much more than 1.5×19^9 years. This, however, would mean that about 1.5 Ga in the past, a dramatic Gerstenkorn event should have happened, of which no trace can be found in the geological record. Therefore, at least during the Ptz, the lunar recession rate must have been significantly smaller than 3.82 m/cy, *i.e.* 6.00 Earth radii per aeon.

Curves #2 and #3, recalculated after Williams (2000), are also based on the assumption that the lunar recession rate was constant throughout geological time, but was smaller than for curve #1. Curve #2 uses a recession rate of 4.96 Earth radii per aeon, consistent with the Big Cottonwood (Utah, U.S.A.) datum discussed by Sonett *et al.* (1996). The corresponding limit time is about 1.8 Ga and, therefore, curve #2 is not acceptable for the Proterozoic. Curve #3, based on an average lunar recession rate of 3.41 Earth radii per aeon, is consistent with the values derived in a number of papers by Williams (see bibliography in Williams, 2000) for the Elatina (South Australia) formation. According to this author, the Elatina datum is particularly well determined and should be used as a reference datum for the Earth's rotational history. The limit time for curve #3, however, is close to 2.7 Ga and is not acceptable for the Archaean and before.

Curve #4, also recalculated after Williams (2000), considers that the lunar recession rate was constant during the Phanerozoic at the value of 3.41 Earth radii per aeon, and constant during the Cryptozoic at the value of 1.63 Earth radii per aeon. The latter value is consistent with the lunar recession speed which can be inferred from the Weeli Wolli tidalite datum from South Australia (age about 2.45 Ga) preferred by Williams (2000). As the limit time T_{lim} becomes in this case slightly over 6 Ga, the curve fulfills the geological constraints. Under the assumptions made for establishing curve #4, the Earth–Moon distance shortly after the Earth's formation (the 'initial' Earth–Moon distance) should have been about 47.5–47.9 Earth radii, lower with respect to the modern value by some 20% (see Denis, 1993, p. 110).

The existence of a supercontinent throughout most of the Proterozoic (Piper, 1982) must have enhanced significantly the diurnal tide O_1 and diminished the semi-diurnal tide M_2, thus giving rise to strongly reduced tidal friction and a strongly reduced lunar recession rate. This is not unlike what happened in the middle Mesozoic, from late Triassic (220 Ma ago) to early Jurassic (180 Ma ago), when the existence of the supercontinent *Pangaea* reduced strongly the slowing down of the Earth's rotation speed and, consequently, the secular increase rate of `LOD` (Denis and Varga, 1990, p. 151; Varga *et al.*, 1992).

There seems to be no observational evidence that the continental shields were grouped together earlier than 2.7 aeons ago, but there are arguments of a more theoretical nature (see § 3, below) that the tidal braking rate was larger in the Archaean and Pre-Archaean than in the Proterozoic and, thus, the lunar recession rate seems to have been larger. In curve #5 we adopt an average recession rate of 3.00 Earth radii per aeon for τ comprised between 2.5 and 4.5 Ga, and in this time interval we have $\gamma(\tau) = 54.28\,[\,1.898 - 0.359\,\tau\,]^{2/13}$. The corresponding critical time T_{lim} is close to 5.288 Ga, and the at the epoch of the formation of the Earth, 4.5 Ga ago, the Moon and Earth would have been orbiting around each other at an average distance of 44.7 Earth radii. Curve #5 is compatible with either of the two hypotheses, namely the Earth-Moon system being formed by accretion as a double-planet (co-formation model of Harris and Kaula, 1975) or the Moon being captured by the Earth a short time after its formation (Gerstenkorn, 1955; Gerstenkorn, 1969; Nakazawa *et al.*, 1983) [Long lists of references concerning the different lunar formation theories can be found, *e.g.*, in the books of Dermott (1978), Burns & Shapley (1986), and many others.].

We derived curve #6 in order to be consistent with the alternative type of theories, implying the ejection of the Moon out of the Earth's upper mantle, either according to the rather improbable scenario considered by Darwin (1879) and later revived, in a different form, by Ringwood (1960; 1970; 1979) and Wise (1963; 1969), or according to the 'great impactor theory' (Hartmann & Davis, 1975; Cameron & Ward, 1976; Ward & Cameron, 1978) favoured nowadays. In the latter scenario, a giant meteorite or asteroid collided with the Earth soon (say, between 4.5 and 4.2 Ga ago) after the Earth had been formed, and the kinetic energy transported by the meteorite is supposed to have been large enough to expell from the Earth's upper mantle the matter which gave birth to the Moon. Specifically, for curve #6 we take $\tau_3 = 2.5\,\mathrm{Ga}$, $\gamma_3 = \gamma(\tau_3) = 54.28$, and we compute $\langle\dot\gamma_3\rangle$ such that $\gamma(4.5) = 0.00$. We find $\langle\dot\gamma_3\rangle = 4.18$ Earth radii per aeon, and $\alpha_3 = \frac{13}{2}\,\langle\dot\gamma_3\rangle/\gamma_3 = 0.50$. Thus, the Earth-Moon distance from 4.5 to 2.5 aeons ago is described by the formula $\gamma(\tau) = 54.28\,[2.25 - 0.5\,\tau\,]^{2/13}$. If the impact hypothesis is correct, the proto-Moon must have resided for a short time within the Roche limit, but for a very short time only — at most a few years: 4.00 aeons ago the Earth-Moon distance was 43.85 Earth radii, 4.25 aeons ago it was 39.42 Earth radii, 4.45 aeons ago it was 30.77 Earth radii, 4.49 aeons ago it was 24.02 Earth radii, and 4.49999 aeons ago it was still 8.30 Earth radii.

3. On the 'initial' rotation speed of the Earth

There is no direct observational evidence available to infer the `LOD` for the Archaean, the epoch before the Proterozoic which spans the time from the Earth's birth as a planet (4.57 Ga ago) to the beginning of the Proterozoic (2.5 Ga ago). Nevertheless, there exists some indirect evidence that the spinning period of the early Earth had been very roughly about half its present-day value, namely 12 hours.

It is clear that the distribution of the angular momentum within the solar system and, in particular, the Earth's 'initial' spinning rate, is to a large extent a consequence of the accretion process by which the solar system was formed. The details of the accretion process are unknown, and so is the role played by magnetic fields, which might have been quite significant for the formation of the solar system and the transfer of angular momentum from the proto-sun to the planets (Alfvén, 1954; Dermott, 1978; Taylor, 1992). A possible clue concerning the Earth's initial angular velocity was dwelled upon by Alfvén (1964) and, later, by Alfvén and Arrhenius (1976). These authors point out that the observed spin periods of the asteroids for which the mass can be estimated from their magnitude, and of planets which did not undergo significant tidal despinning since their formation, vary only within a factor of 2 about a period of 8 hours, whereas the mass range comprises 12 orders of magnitude. Alfvén and Arrhenius (1976, p. 156) refer to this non-dependence of rotational periods on mass as the *law of spin isochronism.* On this basis, they infer (p. 226) for the primitive Earth a `LOD` of 6 hr, which tidal friction would have brought throughout geological times to the present-day value of 24 hr. Gerstenkorn (1969) provided a theoretical value for the initial `LOD` of about 4.8 hr on the assumption that the Moon was captured sometime in the early history of the Earth. Darwin's (1879) minimum value of `LOD`, assuming the Earth and Moon were originally a single body, is 4.25 hr.

Let us consider the ratio GM/R^3, where GM $(= 3.986005 \times 10^{14}\,\mathrm{m^3 s^{-2}})$ is the geocentric gravitational constant, and R $(= 6.371001 \times 10^6\,\mathrm{m})$ is the terrestrial mean radius at the present epoch. GM/R^3 has the dimensions of the square of a frequency. Therefore, we may write for the angular rotation speed of the Earth at any time τ

$$\Omega(\tau) = \sqrt{m(\tau)\,\frac{GM}{R^3}} \approx 1.24 \times 10^{-3}\sqrt{m(\tau)} \quad [\mathrm{rad/s}]. \tag{5}$$

As before, the variable τ denotes the time before the present epoch, expressed in aeons. Inasfar as a first-order hydrostatic theory holds, the function $m = m(\tau)$ represents the ratio of the centrifugal acceleration to the gravitational acceleration at the equator (Denis, 1989; Denis *et al.*, 1998). For the present epoch (τ=0), we have $m(0) \approx 1/289.873$ and $\Omega(0) \approx 7.29 \times 10^{-5}$ rad/s. The time τ=4.5 is close to the epoch of the Earth's formation. We shall denote by $\Omega(4.5)$ the initial spin rate, and by `LOD`(4.5) the corresponding initial length of day. Thus, $m(4.5) \geq 1$, corresponding to a spin rate $\Omega(4.5)$ greater or equal to 1.24×10^{-3} rad/s, *i.e.* a `LOD`(4.5) smaller or equal to $1^h 24^m$, would obviously have precluded the formation of the Earth. Plausible values for $m(4.5)$ range from 1% to 10% of the critical value m_{crit}=1, *i.e.* `LOD`(4.5) values may range from 4.5 to 14 hr. Tentatively, we shall suppose that `LOD`(4.5) was close to 12 hr, corresponding to m=0.014. The uncertainty attached to the latter value is probably 25 or 30%. For the purpose we have in mind here, the schematic `LOD`-curve given in Fig. 2 seems perfectly adequate. This curve represents the available information by three straight line segments, which correspond for the Phanerozoic and Proterozoic to linear regression lines for selected data sets. The discontinuities in the slope of `LOD`(t) at times $t_1 = -2.5$ Ga and $t_2 = -0.64$ Ga are of cause artefacts without a physical meaning.

4. Variation of the inertia moment for a simple model

Let us consider a simple mechanical system consisting of a central sphere of radius r_i and uniform density ρ_i, and of a surrounding spherical layer of inner radius r_i, outer radius r_o and uniform density ρ_o. The total mass M of this system is $M = M_i + M_o$, where $M_i = 4\pi \int_0^{r_i} \rho_i\, r^2 dr$ is the mass of the inner sphere, and $M_o = 4\pi \int_{r_i}^{r_o} \rho_o\, r^2 dr$ is the mass of the outer spherical layer. We assume that by some process, the inner radius r_i is changing with time, in such a way that the density in the inner sphere keeps the same value ρ_i and that the mass

$$M = \frac{4\pi}{3}[\,\rho_i\, r_i^3 + \rho_o\,(r_o^3 - r_i^3)] \tag{6}$$

of the total system remains constant as well. Assuming, moreover, that the outer radius r_o does not get changed significantly during this process, we find that only the density ρ_o of the outer layer changes with r_i, in such a way that ρ_o decreases when r_i increases, and vice-versa, according to the law

$$\rho_o(t) = \frac{M - \frac{4}{3}\pi\,\rho_i\, r_i^3(t)}{\frac{4}{3}\pi\,[r_o^3 - r_i^3(t)]} = \frac{\alpha - \rho_i r_i^3}{r_o^3 - r_i^3}\,, \qquad \alpha = \frac{3M}{4\pi} \tag{7}$$

where the variable t denotes time. The rate of change in time of the density in the outer layer is provided by

$$\frac{d\rho_o}{dt} = -\frac{3r_i^2\,[\,\rho_i\,(r_o^3 - r_i^3) - \alpha + \rho_i\,r_i^3]}{(r_o^3 - r_i^3)^2}\,\frac{dr_i}{dt}\,,. \tag{8}$$

Let $\Delta\rho(t)$ denote the changing density jump $\rho_i - \rho_o(t)$ at the changing inner boundary $r = r_i(t)$ at each instant t. Then Eqns.(6) and (7) can be recast into the form

$$M = \frac{4\pi}{3}[\,\Delta\rho(t)\,r_i(t)^3 + \rho_o(t)\,r_o^3]\,, \tag{6'}$$

$$\Delta\rho(t) = \rho_i - \frac{\alpha - \rho_i\,r_i^3(t)}{r_o^3 - r_i^3(t)}\,. \tag{7'}$$

The time derivative of $\Delta\rho(t)$ is, of course, the negative time derivative of ρ_o. After some straightforward algebraic operations, we obtain

$$\frac{d\Delta\rho}{dt} = -\frac{d\rho_o}{dt} = \frac{3r_i^2\,\Delta\rho}{r_o^3 - r_i^3}\,\frac{dr_i}{dt}\,. \tag{8'}$$

We wish to estimate quantitatively the rate of change in time of the moment of inertia I about an axis passing through the centre of the sphere, resulting from a prescribed rate of change in time of the inner radius r_i. The total inertia moment $I(t)$ at any instant of time t is the sum $I = I_i + I_o$ of the inner inertia moment $I_i(t) = \frac{8}{3}\pi\int_0^{r_i(t)} \rho_i\,r^4 dr$ and of the outer inertia moment $I_o(t) = \frac{8}{3}\pi\int_{r_i(t)}^{r_o} \rho_o(t)\,r^4 dr$. Unlike the total mass M, the total inertia moment I of the system does not remain constant during the assumed process. For our simple model, we have

$$I = \frac{8\pi}{15}[\,\rho_i\,r_i^5 + \rho_o\,(r_o^5 - r_i^5)] \tag{9}$$

or, alternatively,

$$I = \frac{8\pi}{15}[\,\Delta\rho(t)\,r_i(t)^5 + \rho_o(t)\,r_o^5]\,. \tag{9'}$$

Taking the time derivative of Eqn.(9′), using Eqn.(8′), and putting

$$\eta(t) = \frac{r_i(t)}{r_o}\,, \tag{10}$$

we find after some straightforward mathematical operations that

$$\frac{dI}{dt} = \frac{8\pi}{15}\,r_o^5\,\Delta\rho\left[5\eta^4 + 3\,\frac{\eta^7 - \eta^2}{1 - \eta^3}\right]\frac{d\eta}{dt}\,. \tag{11}$$

The latter formula is quite elegantly concise and relates the changes of I to the variable quantities η, $d\eta/dt$, and $\Delta\rho$ defining the variable inner boundary $r = r_i$. Knowing η or, equivalently, $r_i = \eta\,r_o$ as a function of time, we can use Eqn.(7′) to evaluate $\Delta\rho(t)$ and then Eqn.(11) to compute dI/dt. Nevertheless, even though we must perhaps accept the loss of some conciseness, it seems useful to provide a formula which relates dI/dt only to the inner radius r_i and its time rate of change, and not to $\Delta\rho$ or ρ_o as well. To achieve this goal, we may eliminate ρ_o right from the start in Eqn.(9) by using Eqn.(7) before taking the time derivative of Eqn.(9). After some fastidious but straightforward algebra, we find

$$\frac{dI}{dt} = \frac{8\pi}{15}\,r_o^5\,\rho_i\left[5\eta^4 - 3\eta^2\,\frac{1-\eta^5}{1-\eta^3} + \eta^2\,(\beta - \eta^3)\frac{2\eta^5 - 5\eta^2 + 3}{(1-\eta^3)^2}\right]\frac{d\eta}{dt}\,, \tag{12}$$

with

$$\beta = \frac{\alpha}{\rho_i\, r_o^3} = \frac{M}{\frac{4}{3}\pi \rho_i\, r_o^3}\,. \tag{13}$$

It is quite easy to check that substituting $\Delta\rho = \rho_i\,[\,1-(\beta-\eta^3)/(1-\eta^3)]$, *i.e.* Eqn.(7′) expressed in non-dimensional variables, into Eqn.(11), we obtain, indeed, Eqn.(12). The latter can be transformed to the following somewhat simpler form which we obtained by using the `Simplify`-command of the `Mathematica` software package:

$$\frac{dI}{dt} = \frac{8\pi}{15}\, r_o^5\, \rho_i\, \frac{(-1+\beta)\,\eta^2(3+6\eta+4\eta^2+2\eta^3)}{(1+\eta+\eta^2)^2}\, \frac{d\eta}{dt}\,. \tag{12′}$$

If $\rho_i > \rho_o$, $\beta < 1$, and Eqn.(12′) shows that under these circumstances the inertia moment decreases whenever the inner radius increases, and vice-versa.

Finally, we may wish to write Eqn.(12′) in an entirely non-dimensional form by using the non-dimensional inertia coefficient y instead of the inertia moment I:

$$y = \frac{I}{M r_o^2}\,. \tag{14}$$

Then, Eqn.(12′) can be written

$$\frac{dy}{dt} = \mu\, \frac{(-1+\beta)\,\eta^2(3+6\eta+4\eta^2+2\eta^3)}{(1+\eta+\eta^2)^2}\, \frac{d\eta}{dt}\,, \tag{12″}$$

where the constant μ represents two fifth of the ratio of the inner density ρ_i to the average density $\bar\rho = M/(\frac{4}{3}\pi r_o^3)$ of the model, *i.e.*

$$\mu = \frac{2}{5}\,\frac{\rho_i}{\bar\rho}\,. \tag{15}$$

5. Effect of crystallization of the inner core

Let us apply the foregoing formulae to the growth of the inner core, in order to get a quantitative idea about the secular rate of change of `LOD` this process may cause. Thermodynamic considerations dealing with the heat balance associated with the cooling and solidification of the inner core on one hand, thermal and compositional convection in the outer core on the other hand, led Buffett *et al.*, 1996) to the equation

$$f(\eta)\, d\eta = Q(t)\, dt\,. \tag{16}$$

The dependent variable η denotes the ratio of the (growing) inner core radius $r_i = r_i(t)$ with respect to the constant outer core radius $r_o = 3480\,\mathrm{km}$. The forcing term $Q(t)$ on the r.h.s. of Eqn.(16) represents the heat flux across the ICB associated with the cooling and solidification processes. The motion of the ICB upwards results from the fact that in the course of time, the temperature drops below the freezing point of the alloy at the base of the liquid outer core. Then, in this layer, iron and siderophile elements crystallize, but the lighter, more volatile minerals, which do not fit into the crystal lattice of iron and iron-like elements, will move upwards by the mere effect of buoyancy. The newly formed solid iron is now also a part of the inner core, and as a net result the ICB has been shifted upwards as well.

Buffett *et al.* (1996) do not provide a general expression for the function $f(\eta)$. However, as long as η remains small enough, we may approximate $f(\eta)$ by the following expression, valid to terms of $\mathcal{O}(\eta^3)$:

$$f(\eta) \approx (\,2\eta + 3\,A\,\eta^2\,)\mathcal{M}. \tag{17}$$

The constant A is the algebraic sum of three terms which are not known very accurately. For the present epoch [*i.e.* for $t = 0$, $\eta(0) = \frac{r_i(0)}{r_o} = \frac{1221}{3480} \approx 0.35$], the values of the parameters provided by Buffett *et al.* (1996) yield $A = 1.57$. The multiplication constant $\mathcal{M}$ represents the heat that must be extracted to cool the entire core to its solidification temperature. Buffett *et al.* (1996) furnish $\mathcal{M} = 1.88 \times 10^{30}$ J. It should be clear, however, that there exists an overall uncertainty in all parameter values which reaches at least several percent, but could be much larger. Owing to this uncertainty, neglecting terms of $\mathcal{O}(\eta^3)$ [$= \mathcal{O}(0.35^3) \approx 0.06$ for the present size of the inner core] in the function $f(\eta)$ seems to be justified.

We substitute formula (17) into Eqn.(16) and integrate with respect to time t from the instant $t = 0$ at which the temperature first fell below the liquidus at the centre of the Earth and the inner core started to grow ($\eta(0) = 0$), up to an arbitrary instant t :

$$\eta^2 + A\,\eta^3 = \mathcal{M}^{-1} \int_0^t Q(t)\,dt\,. \tag{18}$$

Thus, specific predictions for the growth of the inner core necessitate an estimate of the heat flux function $Q(t)$. According to Stacey (1992, p. 301), the current net cooling rate of the Earth is about 10^{13} W, *i.e.* approximately one fourth of the heat flux measured at the Earth's surface (Sclater *et al.*, 1980). Stacey attributes a large fraction of the 10^{13} W to the cooling of the mantle, the rest stemming from the cooling of the crust and core. His preferred estimate of the present-day heat loss Q from the core is 3.0×10^{12} W, in good agreement with an estimate derived by Sleep (1990) on the basis of the heat transported by mantle plumes. Stevenson *et al.* (1983) and Mollett (1984) find that the heat flux from the core may have slowly decreased with time, the amount depending on poorly known initial conditions. Accordingly, Buffett *et al.* (1996) consider $Q = 4.0 \times 10^{12}$ W as a typical time-averaged value, but they also consider $Q = 6.0 \times 10^{12}$ W and $Q = 2.5 \times 10^{12}$ W as high and low values to span a plausible range of solutions. It is obvious that a variation of Q with time could easily be included in the solution of Eqn.(18), although such a level of detail seems unwarranted given the present uncertainty in the time-averaged value.

$\langle Q \rangle$	2.5×10^{12} W	4.0×10^{12} W	6.0×10^{12} W
$\mathcal{M}$	1.88×10^{30} J	1.88×10^{30} J	1.88×10^{30} J
B	0.042 Ga^{-1}	0.067 Ga^{-1}	0.101 Ga^{-1}
A	1.57	1.57	1.57
$\eta(0.0)$	0.000	0.000	0.000
$\eta(0.5)$	0.132	0.163	0.196
$\eta(1.0)$	0.181	0.223	0.267
$\eta(1.5)$	0.217	0.266	0.318
$\eta(2.0)$	0.246	0.302	0.359
$\eta(2.5)$	0.271	0.332	0.395
$\eta(3.0)$	0.294	0.359	0.426
$\eta(3.5)$	0.314	0.383	0.454
$\eta(4.0)$	0.332	0.405	0.480
$\eta(4.5)$	0.349	0.425	0.504
$\eta(5.0)$	0.365	0.444	0.526

Table 2.– Some typical values for the growth of the inner core

In terms of such a time-averaged value of Q, which we write $\langle Q \rangle$, Eqn.(18) yields

$$\eta^2 + A\,\eta^3 = B\,\tau\,, \qquad (18')$$

where $B = \langle Q \rangle\,\mathcal{M}^{-1}$ and τ is the time, expressed in units of 10^9 years, since the temperature at the Earth's centre fell below the liquidus of iron.

The general solution of the cubic equation (18′) consists of two complex conjugate roots, and one real root. They can be obtained with no effort using the `Mathematica` software. However, their symbolic expressions, even that of the physically acceptable real root, are quite involved. We shall not reproduce them here. Table 2 provides some typical values of the different parameters used, as well as values of the solutions $\eta(\tau)$ corresponding to $\langle Q \rangle = 2.5 \times 10^{12}$ W, 4.0×10^{12} W, and 6.0×10^{12} W, respectively, for values of τ ranging from 0.0 to 5.0 Ga by steps of 0.5 Ga. Assuming that after the Earth's formation, at least 10^9 years were needed for the core to cool down sufficiently at the centre for inner core formation to begin, Table 2 obviously indicates that $\langle Q \rangle \approx 3.5 \times 10^{12}$ W seems to be a minimal average heat flux for the inner core to have reached its present radius of 1221 km. In fact, we can obtain a more precise estimate of this minimum heat flux by means of Eqn.(18′) and assuming $\tau = 3.5\,\mathrm{Ga} \approx 1.10 \times 10^{17}\,\mathrm{s}$, $\eta = 0.35$: $\langle Q \rangle = \mathcal{M}\,(\eta^2 + A\,\eta^3)\,\tau^{-1} \approx 3.19 \times 10^{12}$ W. Fig. 4 shows the minimum average heat flux across ICB, $\langle Q \rangle_{min}$, expressed in units of 10^{12} W, needed for the inner core to reach its present size, as a function of the age of the inner core, expressed in aeons.

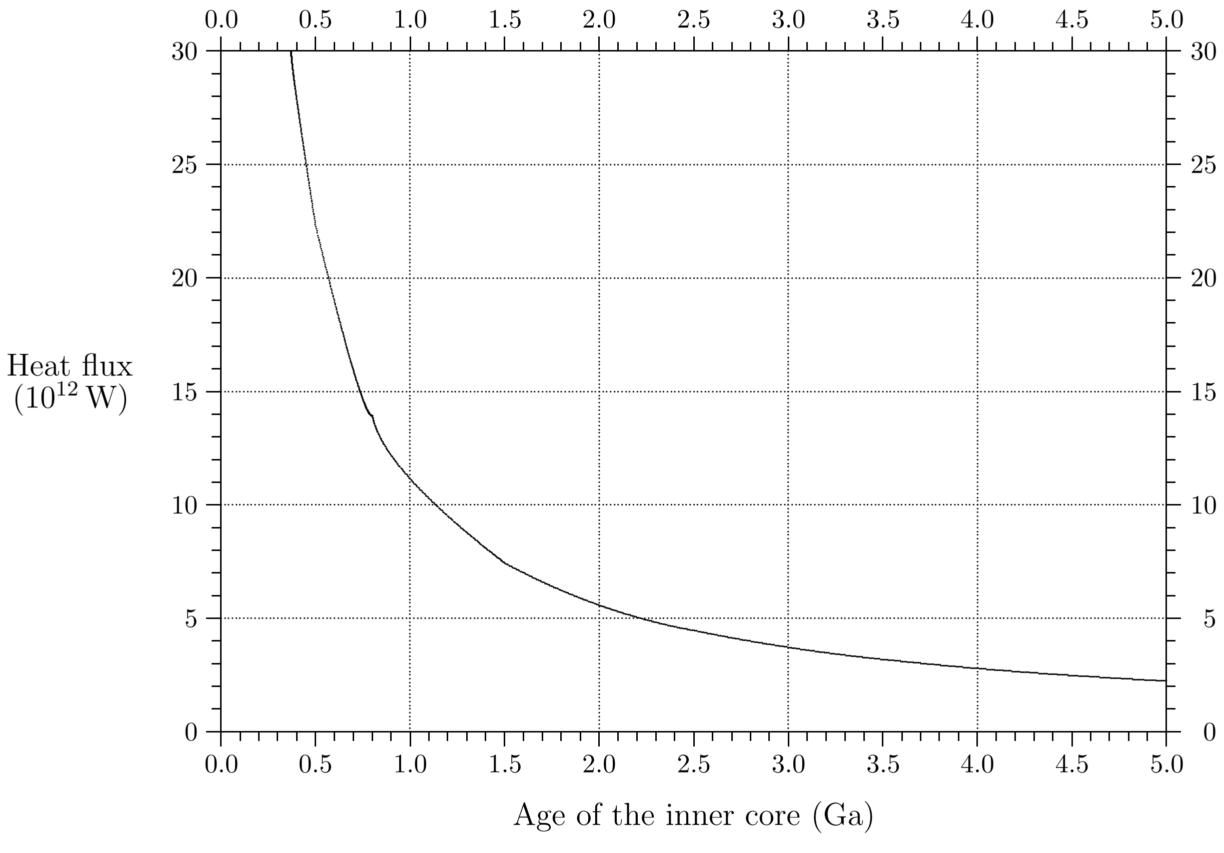

Figure 4.– Average heat flux across the ICB needed to yield the present size of the inner core (1221 km), as a function of a prescribed age for the inner core.

Let us assume that the inner core started to form 2.5 aeons ago, in the early Proterozoic, and adopt an average rate of heat loss from the inner core of $\langle Q\rangle \approx 4.46\times 10^{12}\,\mathrm{W}$, consistent with the present radius of the inner core corresponding to $\eta = 0.35$. With the latter value of $\langle Q\rangle$, the constant B in Eqn.(18′) becomes $B = (0.35^2 + 1.57\,0.35^3)/2.5 \approx 0.076\,\mathrm{Ga}^{-1}$ and Eqn.(18′) reads

$$\eta^2 + 1.57\,\eta^3 = 0.076\,\tau\,. \tag{18''}$$

Differentiating the latter relation, we obtain $(2\,\eta + 4.71\,\eta^2)\,d\eta = 0.076\,d\tau$, *i.e.*

$$\frac{d\eta}{d\tau} = \frac{0.076}{2\,\eta + 4.71\,\eta^2}\,. \tag{19}$$

Thus, for the present epoch, we have

$$\tau = 0.00, \quad \eta = 0.35, \quad \frac{d\eta}{d\tau} \approx 0.060\,\mathrm{Ga}^{-1}, \quad r_i = 1221\,\mathrm{km}, \quad r_o = 3480\,\mathrm{km}, \quad \frac{dr_i}{d\tau} \approx 207\,\mathrm{km\,Ga}^{-1}.$$

Characteristic parameter	PREMM value
Radius of inner core, r_i (10^6 m)	1.2215
Radius of outer core, r_o (10^6 m)	3.4800
Present value of $\eta = r_i/r_o$	0.3510
Mass of inner core, M_i (10^{24} kg)	0.0946
Mass of outer core, M_o (10^{24} kg)	1.8150
Total mass of core, M_c (10^{24} kg)	1.9096
Average density of inner core, ρ_i ($10^4\,\mathrm{kg\,m^{-3}}$)	1.2391
Average density of outer core, ρ_o ($10^4\,\mathrm{kg\,m^{-3}}$)	1.0746
Average density of core, $\bar{\rho}_c$ ($10^4\,\mathrm{kg\,m^{-3}}$)	1.0817
Inertia moment of inner core, I_i ($10^{37}\,\mathrm{kg\,m^2}$)	0.0056
Inertia moment of outer core, I_o ($10^{37}\,\mathrm{kg\,m^2}$)	0.8956
Total inertia moment of core, I_c ($10^{37}\,\mathrm{kg\,m^2}$)	0.9012
Inertia coefficient of the core, y_c	0.3897
Mean equivolumetric radius of Earth, R (10^6 m)	6.3710
Total mass of Earth, M (10^{24} kg)	5.9737
Mean density of Earth, $\bar{\rho}$ ($10^3\,\mathrm{kg\,m^{-3}}$)	5.5150
Mean inertia moment of Earth, I ($10^{37}\,\mathrm{kg\,m^2}$)	8.0200
Mean inertia coefficient of Earth, y	0.3322

Table 3.– Some characteristic parameters of Earth model PREMM (Denis *et al.*, 1997)

By means of formulae (12′) and (19), and using the values of the different parameters listed in Table 3 for Earth model PREMM (Denis *et al.*, 1997), we are now in a position to estimate the present rate of change in the Earth's inertia moment caused by the growth of the inner core.

With $\beta = 0.873$, $\frac{8\pi}{15}\,\rho_i\, r_o^5 = 1.06\times 10^{37}\,\mathrm{kg\,m^2}$, $\eta = 0.351$ and $\frac{d\eta}{dt} = 0.06\,\mathrm{Ga}^{-1}$, formula (12$'$ yields a rate of change of I of $-2.60\times 10^{34}\,\mathrm{kg\,m^2}$ per aeon. The spin angular momentum being conserved throughout the process of inner core formation, we have $I(\tau)\,\Omega(\tau) = I(0)\,\Omega(0) = \text{constant}$ or, equivalently, $\Omega\Delta I = -I\Delta\Omega$. Thus

$$\frac{(\Delta\texttt{LOD})_{ic}}{\texttt{LOD}} = -\frac{(\Delta\Omega)_{ic}}{\Omega} = \frac{(\Delta I)_{ic}}{I} \approx -\frac{2.60\times 10^{34}\,[\mathrm{kg\,m^2\,Ga^{-1}}]}{8.02\times 10^{37}\,[\mathrm{kg\,m^2}]} = -3.24\times 10^{-4}\,\mathrm{Ga}^{-1}\,. \qquad (20)$$

We conclude that, at the present epoch, the crystallization of the inner core gives rise to a relative secular *decrease* Δ`LOD` in the length of day by approximately 28 seconds per aeon, or 2.8 μs/cy.

6. Effect of macrodiffusion of iron compounds across the CMB

Majewski (1995) used non-equilibrium thermodynamic theory to investigate the interactions occurring at the core-mantle boundary (CMB) between the upper layers of the outer core and the deepest layers of the lower mantle, specifically the D$''$ region. The latter seems to contain a proportion of iron oxides and iron sulphides that are not in phase equilibrium with the rest of the bulk composition of region D$''$. Therefore, according to Majewski (1995), there exists a tendency for the heavier iron compounds to cross the CMB and integrate the core material, increasing slowly but steadily the radius of the outer core. The actual mass transfer process invoked by the author is a kind of macroscopic diffusion called *macrodiffusion.* In his interesting paper, Majewski unfortunately ends up with a diffusion coefficient too large by several orders of magnitude and, hence, with a much too large mass transfer rate and the completely unlikely growth rate for the outer core of 22 cm/year.

In her study on palaeorotation and evolution of the Earth's internal structure, Beghein (1997) considered Majewski's theory and was able to trace two kinds of errors, namely an algebraic mistake and some arithmetical errors. After making the necessary corrections, Beghein finds that the macrodiffusion of iron oxides across the CMB leads to an annual increase of the core radius by 13.2 μm, value to which the iron sulphides still add 0.25 μm. In the following, we assume that 'macrodiffusion à la Majewski' leads to a growth rate of the core of 13.5 μm/year, *i.e.* 13.5 km/Ga. This amount seems plausible enough. It corresponds to $d\eta/d\tau \approx 0.00212\,\mathrm{Ga}^{-1}$.

The present-day rate of change of the Earth's inertia moment caused by macrodiffusion at the CMB can also be estimated by means of Eqn.(12$''$), taking here $r_i = 3480\,\mathrm{km}$, $r_o = 6371\,\mathrm{km}$, $\eta = 0.546$, $M = 5.974\times 10^{24}\,\mathrm{kg}$, $\rho_i = 10817\,\mathrm{kg/m^3}$, $\beta = 0.510$. We calculate that $dI/d\tau = -1.35\times 10^35\,\mathrm{kg\,m^2}$, and

$$\frac{(\Delta\texttt{LOD})_{oc}}{\texttt{LOD}} = -\frac{(\Delta\Omega)_{oc}}{\Omega} = \frac{(\Delta I)_{oc}}{I} \approx -\frac{1.35\times 10^{35}\,[\mathrm{kg\,m^2\,Ga^{-1}}]}{8.02\times 10^{37}\,[\mathrm{kg\,m^2}]} = -1.68\times 10^{-3}\,\mathrm{Ga}^{-1}\,. \qquad (21)$$

Thus, the contribution of macrodiffusion of heavy iron compounds across the CMB to the rate of change of `LOD` at the present epoch can be estimated to be about 145 seconds per aeon, or 14.5 μs/cy.

7. What if the core had been growing continuously?

Birch (1965) has estimated that the average moment of inertia of a hot, undifferentiated Earth would be about $9.09\times 10^{37}\,\mathtt{kg\,m}^2$, that for a cold, undifferentiated Earth would be $8.79\times 10^{37}\,\mathtt{kg\,m}^2$. These values are 13% respectively 10% larger than the present value, $8.02\times 10^{37}\,\mathtt{kg\,m}^2$. The change Δ`LOD` caused by internal segregation processes ranges, therefore, from 2.4 to to 3.1 hours, roughly.

A rather more intricate problem is to infer from the value of $\Delta\texttt{LOD}$ the value of the *rate of change* of LOD, for the time and duration of the segregation processes, particularly that of core formation, are still not too well understood and remain to a large extent controversial (Dicke, 1966; Poirier, 1996; Stevenson, 2002). Thus, albeit our hypothesis seems unlikely because it is in conflict with the currently prevailing idea that core formation *must* have been a thermal run-away process which occurred very early in the Earth's history, we take account of the important fact that '... hypotheses, like cats, have nine lives' (Dietz and Holden, 1973) and assume with Urey (1952), Runcorn (1962a; 1962b; 1965) and others, that the core has been forming rather slowly and continuously over geological time. More specifically, we adopt Runcorn's idea that core growth is part of mantle-wide convection, iron and iron compounds being brought down to the core-mantle boundary together with the other mantle material by convection, and settling there whereas the rest of material, now partially depleted of iron, is brought up again within the same convective cell.

Let M denote here the total amount of iron in the Earth, ρ the density of iron, $r(t)$ the radius of the core at any instant of time t, and R the total mean radius of the Earth. Runcorn does not suggest a specific physical or physico-chemical process for the fixation of iron at the CMB, but he makes the plausible assumptions that (1) the mass of iron received by the core per unit time, at any instant t, namely $4\pi\, r^2 \rho\, dr/dt$, is proportional to the surface area of the core, $4\pi\, r^2$, and (2) to the mass of iron remaining in the mantle, $M - \frac{4\pi}{3}\, r^3\, \rho$. Of course, this is a simplified model because it neglects different disturbing effects, *e.g.*, compression and thermal effects. Runcorn's assumptions (1) and (2) can be expressed mathematically by the equation

$$4\pi\, r^2 \rho\, \frac{dr}{dt} = 4\pi C_1\, r^2 \left(M - \frac{4\pi}{3}\, r^3 \rho \right) , \tag{22}$$

where C_1 is a proportionality constant. Dividing both sides of Eqn.(22) by the quantity $4\pi\, r^2 \rho$, introducing the *final core radius* r_f such that $\frac{4\pi}{3} \rho\, r_f^3 = M$, *i.e.* $r_f = [3M/(4\pi\rho)]^{1/3}$, and putting $\xi = r/r_f$, we obtain

$$\frac{d\xi}{dt} = C_2\, (1 - \xi^3) \qquad \text{or} \qquad \frac{d\xi}{1 - \xi^3} = C_2\, dt\,, \tag{22$'$}$$

with $C_2 = \frac{4\pi}{3}\, C_1\, r_f^2$. Integrating Eqn.(22$'$) yields

$$\frac{1}{\sqrt{3}}\, \tan^{-1} \left(\frac{1 + 2\xi}{\sqrt{3}} \right) - \frac{1}{3}\, \log\,(1 - \xi) + \frac{1}{6}\, \log\,(1 + \xi + \xi^2) = C_2\, t + C_3\,. \tag{23}$$

Because ξ on the l.h.s. of Eqn.(23) comes in as the argument of transcendental functions, it is not possible to express the solution $\xi = \xi(t)$ in terms of elementary functions. It is possible, however, to obtain a numerical solution $\xi = \xi(t)$. Such a solution, plotted as a graph, can be found in a paper by Runcorn (1961a). In later papers, Runcorn (1962b; 1965) adapted this solution, which depends on two arbitrary parameters, to fit some *a priori* data which he considered critical but which might as well be considered speculative. On this evidence, he concluded that the final core radius is $r_f = 0.56\, R \approx 3568\,\text{km}$, implying that with the present core radius $r_c = 0.546\, R = 3480\,\text{km}$, there is still left in the mantle about 6% of iron which is slowly settling into the core.

These results seem plausible to us, and we adopt them here as a working hypothesis. From Runcorn's (1962b) solution, we infer that $C_2 \approx 0.93\,\text{Ga}^{-1}$ and $C_3 = 3^{-1/2}\, \tan^{-1}(3^{-1/2}) \approx 0.30$. Thus, at the present epoch for which $\xi = 0.975$, we obtain from Eqn.(22$'$) that $d\xi/d\tau \approx 0.068\,\text{Ga}^{-1}$, or $dr_c/d\tau \approx 243\,\text{km}\,\text{Ga}^{-1}$. Comparing the latter value for the core growth rate 'à la Runcorn' with the value computed earlier for the core growth rate 'à la Majewski', we find that for the present epoch Runcorn's theory would predict a core growth 18 times faster than Majewski's theory. Consequently, in the process imagined by Runcorn, the rate of change of the inertia moment as well as the rate of change of the length of day would also be 18 times larger than in the case of Majewski's macrodiffusion theory. [It is noteworthy that the two theories are not exclusive of one another. Therefore, the total effect could eventually be the sum of both contributions.] Assuming Runcorn's theory to be essentially correct, we find that the contribution of mantle-wide convection to the rate of change of `LOD` at the present epoch would be 43.5 minutes per aeon, or 0.26 ms/cy.

Moreover, Eqn.(22′) shows that during the larger part of the Proterozoic, when the values of ξ were significantly smaller than during the Phanerozoic, the growth rate of the core should have been much larger. Tomecka-Suchoń and Denis (1999) argued on this premise that the core formation curve published by Runcorn (1962b) would, together with a strong reduction of tidal friction caused by the Proterozoic supercontinent, explain the main features of Fig. 2, particularly the much lower despinning rate during the Ptz.

8. Discussion and preliminary conclusions

Denis (1985; 1986) discussed briefly the hypothesis that core growth would be going on at the present epoch at a rate sufficient to contribute significantly to time changes of the inertia moments and related geometric, kinetic and dynamic parameters of the Earth. Disregarding crystallization of the inner core and macrodiffusion across the CMB, effects which had not yet been thoroughly investigated in 1985, he concluded that the mechanisms of core formation and core growth were too speculative to make very meaningful statements about the rates of change of inertia moment and core radius.

In this paper, we have estimated the possible implications of core evolution on the changes of `LOD` for the present epoch. If we give some credit to Runcorn's (1962a,b) theory of a slow, continuous core formation process which would have been particularly active during the Proterozoic, we are able to explain the very small despinning rate which we know to have occurred all along the Ptz. Moreover, according to Runcorn's theory, core growth would still be going on nowadays at a rate which we can estimate at approximately 243 km per aeon, corresponding to a *relative decrease* of `LOD` of 0.26 ms per century, to be compared with the *increase* of `LOD` by approximately 2 ms per century caused by tidal friction involving principally the semi-diurnal lunar partial tide M_2.

Incidentally, it seems that Urey (1952) was the first to suggest that the Earth's iron core had separated gradually from the silicate mantle throughout the Earth's life, and that its radius was still increasing. His suggestion was based, at least partially, on the fact that there is still more iron present in the Earth's mantle than is compatible with mineral phase equilibria; moreover, it assumes implicitly that the Earth was born out of the solar nebula as a cold body and got heated up at a later stage, mainly from the heat released by the decay of long-lived radioactive elements and from the gravitational energy released by the formation of the core. A decade later, in order to explain the results of palaeomagnetic measurements, Runcorn (1962a; 1962b) pioneered the idea that the continents were drifting apart as a consequence of mantle-wide convection. As a by-product of his continental drift theory (Runcorn, 1962c), he lent support to Urey's hypothesis that the core had been growing slowly and, in fact, came up with a specific law of growth rate [see our Eqn.(23)]. Runcorn's theory tried to fit a number of ideas and observations concerning continental drift, large-scale mantle convection, distribution of tectonic features, topography and hypsometric curve, radioactive age peaks of ancient orogenies, *etc.* into a single framework. Because it touches on so many aspects of geodynamics, it obviously bears some flavour of speculation. Morgan's (1968) and Le Pichon's (1968) hypothesis of plate tectonics, as well as important geochemical discoveries, seem to have made Runcorn's theory appear obsolete before it became almost totally forgotten. Nevertheless, as far as we can see, its main features have never been formally disproved. Two other theories of core evolution, which are independent of but not incompatible with Runcorn's theory, are less controversial.

Thus, the theory of a growing solid inner core is nowadays almost universally accepted, and is supposed to account for the energy supply to the liquid outer core necessary to drive the geodynamo. In this paper, we estimated the contribution of the growing inner core on the secular change of `LOD`. We found that it compensates the despinning caused by tidal friction only by a rather tiny amount, namely 2.8 μs per century. On the other hand, Majewski (1995) and Beghein (1997) suggested that a net flux of iron oxides and sulphides through the core-mantle boundary augments the core radius each year by 13.5 μm, leading to a *relative secular decrease* of `LOD` by 14.5 μs per century. Thus, the combined effect of both inner and outer core evolution should amount to something like 15 μs per century.

There seems to be a large consensus that core formation took place very early in the Earth's history (Oversby and Ringwood, 1971), and that throughout geological times Earth rotation had been ruled essentially by tidal friction, *not* by segregation, sedimentation and fractionation phenomena. Thus, core formation seems to result from the migration of iron particles to regions of lesser gravity by means of a run-away process which was either contemporaneous with or finished soon (say, 200 million years) after the formation of the Earth itself. Elsasser (1963) suggested that iron falling through the mantle is slowed down when it reaches a region of high viscosity. It there forms a coherent layer which, however, is gravitationally unstable and results in the formation of quite large 'drops'. The latter fall rapidly to the centre, giving rise to a proto-core in a few hundred million years. The diameter of the core after this event is only slightly smaller than that of the present core although, according to Elsasser (1963), the fall of iron from the lower mantle might have been much slower. Tozer (1965) considered two distinct models of physical separation of the iron phase from the silicate phase, assuming a cold origin of the Earth. In his first model, the metallic phase is contained within a continuous silicate phase as discrete masses, whereas in his second model, silicate and metal are well mixed. The two models lead to completely different conclusions as far as the time of core formation is concerned. Tozer (1965) argues that his first model should be rejected and, on the basis of the results obtained for his second model, he favours the view that core formation by differentiation was virtually complete already in the remote past.

The empirical evidence for an early and fast formation of the core is provided by geochemists, not by geophysicists. The former argue that the observed partitioning of the siderophile elements and, moreover, different isotopic ratios for crustal and upper mantle material plead in favour of very early core formation and do not seem consistent with intense core formation occurring much later than the end of the accretion process which formed the Earth and the other planets of the solar system (*e.g.* Harper and Jacobsen, 1996). Nevertheless, the potential importance of segregation and fractionation processes for changes of `LOD` should be borne in mind when discussing long-term changes of the Earth's inertia moment (Varga and Denis, 1990). The suggested scenario varies to some extent from author to author, but the essential features remain similar (Poirier, 1996). It assumes implicitly that during these early epochs of the Earth's existence, the terrestrial material was relatively hot and ductile, either originally or by very strong early heating. It is usually supposed that a large part of the heat necessary to heat up the whole Earth to a point where iron could "drop out" of the mantle stemmed from the decay of the short-lived radioisotope ^{26}Al injected into the well mixed proto-solar nebula by the explosion of a supernova. Incidentally, the latter is supposed to have triggered the formation of the solar system. Another part of the required heat came from the bombarding of the early Earth by large meteorites and asteroids which marked the end of the accretion phase. The process of core formation is, therefore, thought to be a run-away process: The settling of some iron releases heat which increases the temperature and makes the mantle material become more ductile. A greater ductility enhances the process of iron settling, which again releases more heat, which still raises the temperature and ductility. Again, this enhances the fall of iron, and so on. Early advocates of this theory of fast core formation are Elsasser (1963), Tozer (1965), Dicke (1966), Ringwood (1970), Oversby and Ringwood (1971), followed by many others. Shannon and Agee (1996) consider the possibility that the core has been formed by percolation.

Core formation and evolution have obviously played a very important role in the evolution of the Earth's internal structure. The release of gravitational energy involved in core formation can be estimated to be 1.72×10^{31} J (Beghein, 1997). According to the virial theorem, half of this amount (namely 8.6×10^{30} J) has been converted into internal heat, and half of it has been radiated away. Assuming an average heat capacity $C_p \approx 10^3$ J kg^{-1} K^{-1}, core formation must have raised the average temperature inside the Earth by 1100 K. If we adopt the rather typical average thermal expansion coefficient $\alpha_p \approx 10^{-5}$ K^{-1}, the Earth's volume was expanded by 1.1%, resulting in an increase of the Earth's radius of 23 km. These order of magnitude estimates are enough to establish the fact that core formation had a dramatic effect on the Earth's internal structure and the subsequent evolution and dynamics but, owing to the small value of the thermal expansion coefficient, changed the Earth's dimensions only insignificantly. This conclusion is important if we consider the decrease in `LOD` brought about by core formation which, in agreement with investigations performed by Birch (1965), we estimate between 2.4 and 3.1 hours. Thus, if the core formed, indeed, as fast as the modern scenarios suggest, say within 50 to 200 Ma, the early Earth could have spun up quite appreciably in the Hadaean or early Pre-Archaean, instead of spinning down as in the later epochs. Typically, `LOD` could have been decreasing at an average rate comprised between 0 and 20 ms per century.

Considering presently available data and hypotheses with a critical eye, we conclude tentatively that core formation seems to have had a major effect on the changes of `LOD` only at the very beginning of Earth's history. Throughout the rest of geological times, changes of `LOD` are likely to have been ruled solely by tidal dissipation effects. If we rule out the possibility that the Earth's core formed in the slow way considered by Urey (1952) and Runcorn (1962a,b), we are led to the conclusion the rate of change of `LOD` contributed by core evolution is of the order of a few microseconds per century, whereas tidal friction yields a rate of the order of several milliseconds per century. In fact, a recent study on geomagnetic palaeointensities performed by Denis *et al.* (2002) does not give much support to Runcorn's theory, but does not rule it out.

Tidal energy dissipation is associated with an increasing Earth–Moon distance. In section 2, we have given some thought to this problem. We conclude that the proto-Moon probably stayed only a very short time (a few hours to a few days) in a close vicinity of the Earth, right after it had been formed as the result of a collision of the young Earth with a large asteroid which may have had the size of Mars. We tentatively suggest that our curve #6 of Fig. 3 depicts most plausibly the history of the changes of the average Earth-Moon distance.

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